EconS 503 - Microeconomic Theory II
Midterm Exam #2, April 3rd 2020, via email.

Instructions:

- Please read all exercises carefully.
- Answer each exercise in a formal and concise manner, but include all your steps. This will allow you to obtain partial credit.
- If possible, please write your answer to each question in a different page to facilitate grading.
- The exam has 4 required exercises, for a total of 100 points, and a bonus exercise (15 points) based on a published paper. I encourage all students to answer all exercises including the bonus exercise.
- This is a take-home exam. It was posted on the course website, and announced via email to all students, on April 3rd, at 11:15am. It is due on April 4th, at 11:00am. No late submission will be allowed. You cannot contact the TA of the class for questions. You can contact me, but only for clarifications if you find that some question is unclear.

Good luck!!

1. **Temporary punishments in Bertrand competition**. Consider an industry with two firms competing in prices a la Bertrand, facing a linear inverse demand function $p(Q) = 100 - Q$, where Q denotes aggregate output. Firms face a common marginal cost $c = 10$. For simplicity, assume that both firms have the same discount factor $\delta \in (0, 1)$.

   (a) *Bertrand equilibrium.* Find equilibrium prices in Nash equilibrium of the Bertrand game when firms interact only once.

   (b) *Infinitely repeated game - Permanent reversion.* Consider now a grim-trigger strategy (GTS) where firms start setting a collusive price that maximizes their joint profits and continue to do so if both firms chose collusive prices in all previous periods. Otherwise, every firm permanently reverts to the Bertrand equilibrium you found in part (a). Under which conditions on δ this GTS can be sustained as a SPNE of the infinitely repeated game?

   (c) *Infinitely repeated game - Temporary reversion.* Consider again the GTS of part (b), but assume that, upon a deviation, firms *temporary* revert to the Bertrand equilibrium of part (a) during T periods. Under which conditions on δ can this GTS be sustained as a SPNE of the infinitely repeated game? [*Hint*: Rather than solving for the minimal δ sustaining cooperation, solve for the length of the temporary punishment T.]

2. **Cournot competition when all firms are uninformed - Allowing for cost correlation.** Consider an industry with two firms competing a la Cournot and inverse demand function $p(Q) = 1 - Q$ where $Q = q_1 + q_2$ denotes aggregate output. Every firm i privately observes its marginal cost of production, $MC_i = 1/4$ or $MC_i = 0$, both equally likely, but it does not observes its rival's marginal costs, MC_j. The probability distribution is, however, common knowledge among firms. Assume that firms' costs are *positive correlated*, that is, when firm i observes that its costs are high ($MC_i = 1/4$), it assigns a probability $p^H > 1/2$ to its rival's costs being high and $1 - p^H < 1/2$ to its rival's costs being low. Similarly, after observing that its own costs are low, $MC_i = 0$, firm i assigns a probability $p^L < 1/2$ to its rival's costs being high and $1 - p^L > 1/2$ to its rival's costs being low. Intuitively, observing that its own costs are high (low) increases the probability that its rival's costs are high (low) as well.

   (a) Find the best response function for every firm i, $q_i^k(q_j^H, q_j^L)$, where $k = \{H, L\}$ denotes firm i's marginal cost (high or low).

   (b) Use your results from part (a) to find the Bayesian Nash Equilibrium (BNE) of the game.

   (c) Evaluate your results in the special cases of perfect positive cost correlation, where $p^H = 1$ and $p^L = 0$. Interpret.

3. **First-price auction with entry fees.** Consider a first-price auction with N bidders. Every bidder i's valuation, v_i, is distributed according to a uniform distribution function, that is, $F(v_i) = v_i$ for all $v_i \sim U[0, \overline{v}]$. Consider the following two-stage game: in the first stage, the seller sets an entry fee $E \geq 0$ that every participating bidder must pay, otherwise his bid is ignored; in the second stage, every bidder i independently and simultaneously submit his bid for the object.

   (a) *Second stage.* In this part of the exercise, let us focus on the second stage of the game. For a given entry fee E, find the optimal bidding function that bidder i chooses in the second stage, $b_i(v_i, E)$. [*Hint*: Assume that there exists a critical bidder whose valuation v_e makes him indifferent between participation or not, given a positive entry fee E].

   (b) How are equilibrium bids affected by an increase in the entry fee E? Does a higher E limit participation in the auction?

   (c) *First stage.* Anticipating the optimal bidding function $b_i(v_i)$ you found in part (a), what is the optimal entry fee E^* that the seller sets in the first stage to maximize his expected revenue from the auction? For simplicity, assume that the critical bidder, who is indifferent between participation or not, submits a bid, $b_e(v_e) = 0$.

4. **Selten's horse.** Consider the "Selten's Horse" game depicted in Figure 1. Player 1 is the first mover in the game, choosing between C and D. If he chooses C, player 2 is called on to move between C' and D'. If player 2 selects C' the game is over. If player 1 chooses D or player 2 chooses D', then player 3 is called on to move without being informed whether player 1 chose D before him or whether it was player 2 who chose D'. Player 3 can choose between L and R, and then the game ends.

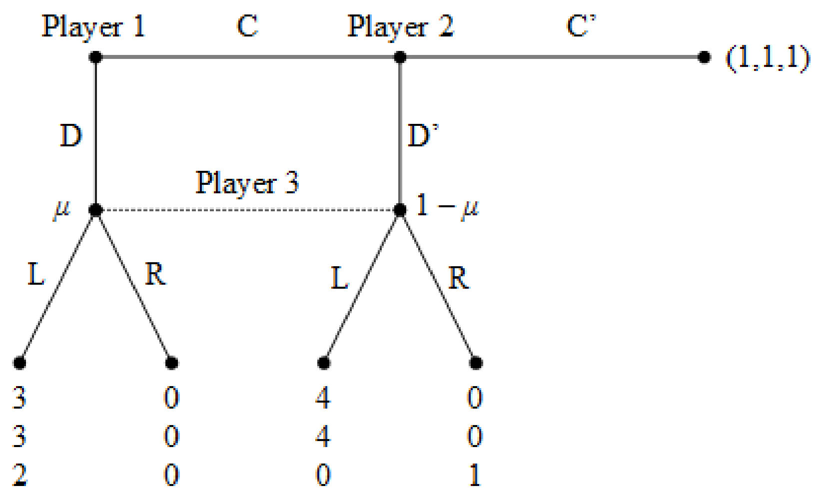

Figure 1. Selten's horse.

(a) Define the strategy spaces for each player. Then find all pure strategy Nash equilibria (psNE) of the game.

(b) Argue that one of the three psNEs you found in part (a) is not sequentially rational. A short verbal explanation suffices.

(c) Show that strategy profile $\{C, C', R\}$ can be sustained as a PBE of the game. (You don't need to show that this is actually the unique PBE we can sustain in this game.)

5. **BONUS EXERCISE - A model of sales, based on Varian (1980).**[1] Firms offer sales at different times. In this exercise, we show that offering sales (or, more generally, randomizing over prices) is a strategy that helps firms maximize their expected profits. This exercise belongs to the literature on "price dispersion" where firms face a share of consumers who are uninformed about prices, and offer different prices, either at different locations (spatial price dispersion) or at different points in time (temporal price dispersion, as we analyze in this exercise). Price discrimination models, in contrast, assume that consumers can perfectly observe prices.

   Consider an industry with N firms and free entry, so firms enter until the profits from doing so are zero. Consumers have a reservation price r for an homogeneous good and purchase at most one unit. A share α^I of consumers is informed about prices, buying from the cheapest firm, and a share $1-\alpha^I$ are uninformed, who purchase from any firm. Therefore, there are $\alpha^U = \frac{1-\alpha^I}{N}$ uninformed consumers per firm. Firms face a symmetric cost function $C(q) = F + cq$, where $F > 0$ denotes fixed costs and c represents its marginal cost. Every firm can only charge one price for its product.

   As a reference, note that $C(\alpha^I+\alpha^U) = F+c(\alpha^I+\alpha^U)$ denotes the cost from serving the maximum amount of customers (both informed and uninformed consumers). Therefore, the ratio

   $$p_L \equiv \frac{F + c(\alpha^I + \alpha^U)}{\alpha^I + \alpha^U}$$

   represents the average cost in this setting.

   We next show that, in the above context, every firm has incentives to randomize its pricing over a certain interval. The following questions should help you find the specific cumulative distribution function $F(p)$ that every firm uses in the mixed-strategy Nash equilibrium of the game.

   (a) Show that $F(p) = 0$ for all $p < p_L$, and that $F(p) = 1$ for all $p > r$.

   (b) Show that the cumulative distribution function $F(p)$ is non-degenerated, that is, there is no pure strategy Nash equilibrium.

   (c) For simplicity, assume that $F(p)$ is continuous.[2] Find expected profits from the pricing strategy $F(p)$.

   (d) Using the no entry condition, find the cumulative distribution function $F(p)$ with which every firm randomizes.

   (e) Show that the cumulative distribution function $F(p)$ has full support in $p \in [p_L, r]$. That is, $F(p_L + \varepsilon) > 0$ and $F(r - \varepsilon) < 1$ for any $\varepsilon > 0$.

   (f) Taking into account that $\pi_f(r) = 0$, find the equilibrium number of firms in the industry, n^*.

   (g) Taking into account that $\pi_s(p_L) = 0$, and the equilibrium number of firms N^*, find the lower bound of firms' randomization strategy, p_L.

[1] Varian, H. (1980) "A model of sales," American Economic Review, 70, pp. 651–59.

[2] That is, there is no "mass point" in the pricing strategy $F(p)$ that every firm uses. Intuitively, the firm chooses all prices in the $[p_L, r]$ interval with positive probability. More compactly, this means that the density function $f(p) > 0$ for all $p \in [p_L, r]$.

(h) Evaluate your above results in the special case in which all consumers are uninformed.

(i) *Numerical example.* Evaluate your results in parts (d), (f), and (g) at parameter values $r = 1$, $F = \frac{2}{9}$, $c = 0$, and $\alpha^I = \frac{1}{3}$.

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