EconS 503 - Final exam
Due date: May 2nd 2020, at 9:00am, via email.

Instructions:

- Please read all exercises carefully.
- Answer each exercise in a formal and concise manner, but include all your steps. This will allow you to obtain partial credit.
- If possible, please write your answer to each question in a different page to facilitate grading.
- The exam has 6 required exercises, for a total of 100 points.
- This is a take-home exam. It was posted on the course website, and announced via email to all students, on April 30th, at 9:00am. It is due on May 2nd, at 9:00am. No late submission will be allowed. You cannot contact the TA of the class for questions. You can contact me, but only for clarifications if you find that some question is unclear.

Good luck!!

1. **Cheap talk vs. Delegation, based on Dessein (2002).**[1] Consider the Crawford and Sobel (1982) cheap talk game, where an expert privately observes the realization of parameter $\theta \sim U[0,1]$, sends a message $m \in [0,1]$ to the uninformed politician who updates his beliefs about θ, and responds with a policy $p \in [0,1]$. The politician's utility function is given by $u_P(p,\theta) = -(p-\theta)^2$, while the expert's utility function is $u_E(p,\theta) = -(p-(\theta+\delta))^2$, where parameter $\delta > 0$ represents the divergence between the preferences of the politician and the expert (also known as the expert's bias parameter).

   Consider now an alternative communication setting, where the politician delegates the decision of policy p to the expert. Intuitively, the politician seeks to minimize the information loss from the cheap talk setting, at the cost of having the expert implementing a policy that considers his own bias. We next examine which policy the expert implements in under delegation, and in which cases the politician is better off delegating the decision to the expert rather than operating in the standard cheap talk context.

   (a) *Delegation.* Which policy the expert chooses under delegation? Which are the expected utilities for expert and politician?

   (b) *Do-it-yourself.* Consider now that the receiver (politician) ignores the sender's messages, which is often known as if the politician took a "do-it-yourself" approach. Find the policy he responds with, and his expected utility in this setting. Under which value of the preference divergence parameter δ the politician prefers the "do-it-yourself" approach than the delegation approach studied in part (a) of the exercise?

   (c) From previous exercises, we found that expected utilities in the Crawford and Sobel (1982) cheap talk game are

$$u_P^{CT} = -\frac{1}{12N^2} - \frac{\delta^2(N^2-1)}{3}$$

   for the politician, where the superscript CT indicates "cheap talk"; and $u_E^{CT} = u_P^{CT} - \delta^2$ for the expert. In addition, recall that N denotes the number of partitions in the $[0,1]$ interval, or different messages that the expert sends; where this N-partition equilibrium can be sustained if the preference divergence parameter is sufficiently small, that is, $\delta \leq \frac{1}{2N(N-1)}$. Show that the expert prefers delegation rather than sending messages to the politician in the cheap talk game, and that this result holds under all parameter conditions. Then show that the politician prefers delegation only if the expert's bias, as captured by δ, is sufficiently small.

[1] Dessein, W. (2002) "Authority and communication in organizations." *Review of Economic Studies*, 69, pp. 811–38.

2. **Reputation on the job, based on Bénabou and Tirole (2006).**[2] Consider a continuum of risk-neutral agents, each of whom can make a decision, $a = \{0, 1\}$, where $a = 1$ corresponding to taking a prosocial action (such as donation to charities, community services, or working assiduously in a group project, etc.), and $a = 0$ corresponding to not taking such an action. Those agents who participate in prosocial activities incur a cost of c but receive a financial reward of w (such as free meals, tax credits, gift cards for volunteering work) and an intrinsic satisfaction of v that is privately observable by every agent. However, the distribution of $v \in [0, 1]$, $G(v)$, with density $g(v)$, is common knowledge among players. We consider participation decisions to be monotonic in v, such that exists a critical agent with intrinsic satisfaction v^* who is indifferent between participation or not, that is

$$a = \begin{cases} 1 & v \geq v^* \\ 0 & v < v^*. \end{cases}$$

In this context, each participant enjoys a positive utility from good reputation, defined as the difference between the expected value of intrinsic satisfaction among those who participate and the intrinsic satisfaction of the critical agent v^*. On the contrary, each non-participant receives a disutility from bad reputation, defined as the difference between the expected value of intrinsic satisfaction among those who do not participate and the intrinsic satisfaction of the critical agent v^*. In this exercise, we analyze whether giving out more or less financial reward w would impact agents' willingness to participate in socially desirable activities.

(a) Assume that each agent places a common weight of μ on reputation (that is, $\mu = \mu_i = \mu_j$), what is the typical agent i's utility from taking or not taking the prosocial action?.

(b) Solve for the critical agent v^*.

(c) *Comparative Statics.* Does giving out a higher financial reward w induce more agents to participate? For simplicity, evaluate your answer in the case of a uniformly distributed v, where $G(v) = v$. [*Hint*: Consider the labor supply function, $L(w) = 1 - G[v^*(w)]$, which is the mass of agents who participate in the prosocial activities as induced by wage w.]

(d) Evaluate the intrinsic satisfaction of the critical agent v^*. How does a cost increase affect him? What if the agent places a higher emphasis on reputation (that is, μ increases)?

[2] Bénabou R. and Tirole J. (2006). Incentives and Prosocial Behavior. *American Economic Review*, 96(5), 1652-78.

3. **Emission fees and mechanisms.** Consider an industry with N polluting firms producing a homogenous good. Let the profit function of firm i be $\pi_i(q_i) = \ln q_i$, which is increasing and concave in its pollutants q_i. The social cost from pollution is

$$C(q_1, \cdots, q_n) = \sum_{i=1}^{n} \frac{\gamma_i}{2} q_i^2,$$

which is also increasing but convex in the pollutants q_i emitted by firm i. Finally, a regulator (e.g., government agency) considers the following welfare function

$$W(q_1, \cdots, q_n) = \sum_{i=1}^{n} \pi_i(q_i) - C(q_1, \cdots, q_n)$$

(a) *Complete information.* Assume that the regulator can observe pollution levels and sets an emission fee t_i per unit of emissions. Find the following: (i) firm i's profit-maximizing pollution level as a function of fee t_i, $q_i(t_i)$; (ii) the socially optimal pollution from firm i, q_i^{SO}; and (iii) the emission fee t_i that induces firm i to produce q_i^{SO}, i.e., the fee t_i that solves $q_i(t_i) = q_i^{SO}$.

(b) *Incomplete information.* Assume that the level of pollution is unobservable to the regulator but observable among all firms. Then, the regulator can devise a circular monitoring mechanism, in which firm i reports the observed pollution level of firm $i-1$, $\overline{q}_{i-1}$, firm $i-1$ reports the observed pollution of firm $i-2$, $\overline{q}_{i-2}$, and firm 1 reports that of firm n, $\overline{q}_n$. This allows the regulator to set an emission fee per unit of pollution

$$t_i = \frac{\partial C(\overline{q}_i, q_{-i})}{\partial q_i},$$

where $\overline{q}_i$ denotes firm i's pollution (reported by firm $i+1$), and q_{-i} represents the true pollution level of all other firms. In addition, firm i faces a penalty of $(\overline{q}_{i-1} - q_{i-1})^2$ for misporting his neighbor's pollution level not at q_{i-1}.

1. Will firm i misreport the output of firm $i-1$? Why or why not?
2. Write down firm i's profit-maximization problem and solve for its optimal output.
3. Find the tax revenue generated by the mechanism, and the social cost of pollution.

4. **Designing Optimal Taxation using Mechanisms.** Consider a government needing to raise a fixed sum \$$S$ through income tax. There are two types of workers, high productivity (H) and low productivity (L), and the output (gross income) produced by each is given by

$$q^k = \theta^k e^k, \text{ where } k = H, L$$

where e^k is the amount of effort exerted by a worker of type k and the productivity parameter satisfies $\theta^H > \theta^L$. Hence, for a given effort level, the high-productivity worker generates a larger amount of output than the low-productivity worker. The utility function of a worker with type k is

$$v^k = q^k - t^k - g(e^k)$$

where t^k is the tax on a worker of type k, and $g(\cdot)$ is a strictly increasing and convex function in effort, i.e., $g' > 0$ and $g'' > 0$. The government has no interest in the inequality of utility outcomes and so just seeks to maximize the expected social welfare

$$W = pv^H + [1 - p]v^L$$

where p is the proportion of H-type workers.

(a) What is the government's budget constraint?

(b) *Complete information.* If the government was perfectly informed about the worker's type, find the socially optimal taxes, and the associated output levels.

(c) *Parametric example (Complete information).* Assume that the cost of effort function is $g(e^k) = (e^k)^2$, so its derivatives are $g' = 2e^k \geq 0$ and $g'' = 2 > 0$; as required. Evaluate the FOCs found in part (b) for the complete information context assuming that productivity parameters are $\theta^H = 1$ and $\theta^L = \frac{1}{2}$. Find the optimal values of q^H and q^L.

(d) *Incomplete information.* Assuming that the government cannot observe the worker's type, write the government's objective function in terms of q^H, q^L, p, and S.

(e) Using the government's objective function you identified in part (d), write down the government's optimization problem. [*Hint*: You can use $y^k \equiv q^k - t^k$ to simplify your calculations. In that case, recall that the set of choice variables for the social planner changes from $\left(q^H, q^L, t^H, t^L\right)$ to $\left(q^H, q^L, y^H, y^L\right)$.]

(f) Find the solution to the government's problem in part (d). Compare your answer to the complete information solution found in part (b).

(g) *Parametric example (Incomplete information).* Assume the same cost of effort function as in the parametric example developed in part (c), $g(e^k) = (e^k)^2$, and the same set of productivity parameters $\theta^H = 1$ and $\theta^L = \frac{1}{2}$. In addition, consider that both types of workers are equally likely, i.e., $p = \frac{1}{2}$. Find the optimal values of q^H and q^L in the incomplete information setting. Then, find the optimal y^H and y^L, where $y^k \equiv q^k - t^k$.

5. **Gibbard-Satterthwaite theorem.** In this chapter, we analyzed the aggregation of individual preferences into a social preference relation satisfying a set of desirable properties. However, we assumed individual preferences were truthfully reported by each individual. In this exercise, we examine a setting in which individuals do not necessarily truthfully reveal their preferences. In particular, we are interested in social choice functions that are "strategy proof."

   First, note that a *social choice function* $c\left(\succsim^1, \succsim^2, ..., \succsim^N\right) \in X$ maps the profile of individual preferences $\left(\succsim^1, \succsim^2, ..., \succsim^N\right)$ into an alternative $x \in X$. That is, society uses the social choice function (scf) to "select" an alternative $x \in X$, using the information in the profile of individual preferences $\left(\succsim^1, \succsim^2, ..., \succsim^N\right)$. Hence, we say that a scf $c(\cdot)$ is *strategy-proof* if every individual i prefers the alternative that the scf selects when he reports his true preferences, $c(\succsim^i, \succsim^{-i}) = x$, than that arising when he misreports his preferences, $c(\succsim'^i, \succsim^{-i}) = y$, i.e., $x \succsim^i y$, where $\succsim^{-i}$ denotes the profile of individual preferences by all other agents $\left(\succsim^1, ..., \succsim^{i-1}, \succsim^{i+1}, ..., \succsim^N\right)$. In words, if a scf is strategy proof, individuals have no strict incentives to misreport their preferences, regardless of the preferences other individuals report, $\succsim^{-i}$; which holds true even if the other individuals misreport their preferences. We seek to show, in several steps, Gibbard-Satterthwaite's theorem, which says that: If there are three or more alternatives in X, then every strategy-proof scf is dictatorial.[3] In the next questions of this exercise, we will start showing that (1) a strategy-proof scf must exhibit two properties: Pareto efficiency and monotonicity; and (2) every Pareto efficient and monotonic scf must be dictatorial.

   We need to define what we mean by Pareto efficient scf: A scf is *Pareto efficient* when every individual i's strict preference for x over y, $x \succ^i y$, where $x, y \in X$, yields the scf to select x, i.e., $c\left(\succsim^1, \succsim^2, ..., \succsim^N\right) = x$. We also define what we mean by monotonic scfs: Consider a initial profile of individual preferences, $\left(\succsim^1, \succsim^2, ..., \succsim^N\right)$, yielding that alternative x is chosen by the scf, i.e., $c\left(\succsim^1, \succsim^2, ..., \succsim^N\right) = x$. Assume that the preferences of at least individual i change from $x \succsim^i y$ to $x \succ'^i y$, for every $y \in X$, i.e., alternative x rises to the only spot at the top of his ranking of alternatives, and the preference for x is not lowered for any individual, i.e., $x \not\prec y$. We then say that a scf is *monotonic* if the scf still selects x under the new profile of individual preferences, $c\left(\succ'^1, \succ'^2, ..., \succ'^N\right) = x$. Hence, loosely speaking, a scf is monotonic if it keeps selecting x as socially preferred when x becomes the top alternative for at least one individual.

   (a) Show that strategy-proofness implies monotonicity on the scf.

   (b) Use monotonicity to show that the scf must be Pareto efficient.

   After demonstrating that strategy-proofness implies monotonicity and Pareto efficiency, we are ready to show the main result of Gibbard-Satterthwaite's theorem (namely, that in a context where the set of alternatives has more than three elements,

[3] The definition of a dictatorial scf is similar to , in the definition in swf. In particular, we say that a scf $c(\cdot)$ is *dictatorial* if there is an individual d (the dictator) such that, if $x \succsim^d y$ for every two alternatives $x, y \in X$, then the scf selects x, i.e., $c\left(\succsim^1, \succsim^2, ..., \succsim^N\right) = x$. That is, a scf is dictatorial if there is an individual d such that $c(\cdot)$ chooses d's top choices, regardless of the preferences of all other individuals.

and where the scf satisfies monotonicity and Pareto efficiency, then such scf must be dictatorial). We will demonstrate that using five steps.

c. *Step 1.* Consider a profile of strict rankings in which alternative x is ranked highest and y lowest for every individual i; as illustrated in the next table. In this setting, Pareto efficiency implies that the scf must select x.

$\succsim^1$	...	$\succsim^{n-1}$	$\succsim^n$	$\succsim^{n+1}$	...	$\succsim^N$	Social choice
x	...	x	x	x	...	x	x
.		.	.	.		.	
.		.	.	.		.	
.		.	.	.		.	
y	...	y	y	y	...	y	

Consider now that we change individual 1's ranking by raising y in it one position at a time. Show that there must exist an individual n for which the social ranking changes when y is raised above x in individual n's ranking.

d. *Step 2.* Consider now a different profile of individual preferences in which: x is moved to the bottom of individual i's ranking, for all $i < n$, and x is moved to the second last position in individual i's ranking, for all $i > n$. Show that this change in individual preferences does not change the selection of the scf.

e. *Step 3.* In this step, we use the assumption that the number of elements in the set of alternatives X is equal or larger than 3. For that, we only need to consider an alternative $z \neq x, y$ in our above steps.

f. *Step 4.* Consider a profile of individual preferences compatible with those in Step 3. Switch the ranking of alternatives x and y for all individuals $i > n$; as depicted in the next table.

$\succsim^1$	...	$\succsim^{n-1}$	$\succsim^n$	$\succsim^{n+1}$	...	$\succsim^N$	Social choice
.		.	x	.	...	.	x
.		.	z	.		.	
.		.	y	.		.	
z	...	z		z	...	z	
y	...	y	.	y	...	y	
x	...	x	.	x	...	x	

Show that alternative x must be socially selected.

g. *Step 5.* Argue that the scf must be dictatorial.

6. **Checking properties on a social welfare function, based on Kaneko and Nakamura (1979).**[4] Consider the following social welfare function

$$SW(x) = x_1^{\alpha_1} \cdot x_2^{\alpha_2} \cdot \ldots \cdot x_N^{\alpha_N} = \prod_{i=1}^{N} x_i^{\alpha_i}$$

where $x = (x_1, x_2, ...x_N)$ denotes an alternative, where $x \in \mathbb{R}^N$, and $\alpha_i > 0$ represents the weight that the social planner assigns to agent i. Alternatively, this social welfare function can be represented in its linear form by applying logs, as follows,

$$\alpha_1 \ln x_1 + \alpha_2 \ln x_2 \cdot \ldots \cdot \alpha_N \ln x_N = \sum_{i=1}^{N} \alpha_i \ln x_i$$

In this exercise, we show that ordinal preferences among the alternatives can be represented by the above (cardinal) social welfare function that satisfies:

- Paretian (P),
- Anonymity (A),
- Neutrality (N), and
- Independence of Irrelevant Alternatives (IIA).

Show that the above social welfare function $SW(x)$ satisfies these five properties.

[4] Kaneko M. and Nakamura K. (1979). The Nash Social Welfare Function. *Econometrica,* 47(2), pp. 423-35.