EconS 503 - Advanced Microeconomics II
Homework #9 - Due date: April 22nd, via email.

1. **Moral hazard with multiple tasks, based on Holmström-Milgrom (1991).**[1] Let us consider a moral hazard problem between a principal and an agent. However, let us now allow the agent to take two effort levels e_1 and e_2. This represents, for instance, a salesman choosing how much effort to exert visiting potential customers, how much time to spend creating a more attractive website for online sales, investigating new sales strategies, etc. In this exercise we seek to understand how the multidimensionality in the agent's effort affects our results in the standard moral hazard problem analyzed in this chapter.

   Assume that the cost of exerting effort levels e_1 and e_2 is

   $$c(e_1, e_2) = \frac{1}{2}e_1^2 + \frac{1}{2}e_2^2$$

   These effort levels produce output y with output function

   $$y = f_1 e_1 + f_2 e_2 + \varepsilon$$

   with performance $p = g_1 e_1 + g_2 e_2 + \phi$. Random shocks in output, ε, and performance, ϕ, follow distributions of $G(\phi)$ and $H(\varepsilon)$, respectively, with zero expectations, that is, $E(\varepsilon) = E(\phi) = 0$.

   For simplicity, assume that both principal and agent are risk neutral with payoff functions of $\pi = y - w$ for the principal (e.g., firm), where w denotes the salary she pays to the agent; and $U = w - c(e_1, e_2)$ for the agent (e.g., worker). Consider that the principal offers a salary $w = F + bp$ where F is fixed component of the contract and b is the bonus which provides a higher salary to the agent as his performance p increases. In particular, the timing of the game is as follows:

   - The principal and agent sign a contract $w = F + bp$.
   - The agent takes effort levels e_1 and e_2 which are unobservable to the principal.
   - Random shocks ε and ϕ, are realized, affecting the agent's output and performance, respectively.
   - Output y and performance p are observed by the principal and agent.
   - The agent receives wage $w = F + bp$.

   Answer the following questions.

   (a) Find the agent's optimal efforts and indirect utility as a function of the bonus parameter b.

   (b) Find the principal's optimal contract w^* and his equilibrium profits.

[1] Holmström-Milgrom (1991) "Multitask Principal-Agent Analyses: Incentive Contracts, Asset Ownership, and Job Design," *Journal of Law, Economics, & Organization*, vol. 7, pp. 24-52. For a more readable presentation, see Bolton and Dewatripont (2005), pp. 216-28.

(c) *Comparative Statics.* How is the optimal contract you found in part (b) affected by the output rates f_1 and f_2? How is it affected by the performance rates g_1 and g_2? Explain.

(d) Given the optimal contract found above, what are the principal's expected payoff, the agent's expected utility, and the expected social welfare in equilibrium?

(e) What is the socially optimal contract? Compare it against the contract that emerges in the subgame perfect equilibrium of the game you found in part (b).

(f) What is the deadweight loss in this contractual setting?

(g) *Numerical example.* Consider output rates $f_1 = \frac{1}{2}$ and $f_2 = \frac{1}{3}$, and performance rates $g_1 = \frac{2}{3}$ and $g_2 = \frac{1}{4}$. In this context, evaluate the equilibrium bonus b^*, efforts e_1^* and e_2^*, wage w^*, the agent's expected utility $U(w^*)$, the principal's expected profit $\pi(w^*)$, and social welfare SW^*. Then, evaluate the socially optimal bonus b^{**}, social welfare SW^{**} at b^{**}, and deadweight loss due to unobservability of effort.

2. **Principal agent model with two principals (Common agency problem), based on Stole (1990).**[2] Consider two principals, 1 and 2, who assign different tasks to a common agent (e.g., worker). There are 2 output levels in principal j's task, where $j = \{1, 2\}$, denoted as q_j^H and q_j^L, where $q_j^H > q_j^L > 0$. The probability of output q_j^H coincides with the agent's effort on task j, e_j , where $e_j \in [0, 1]$, such that the probability of the opposite output level, q_j^L, becomes $(1 - e_j)$. Intuitively, the more effort the agent puts in task j, the more likely it is to realize a high output level on that task.

   The agent incurs a cost of effort, $c(e_1, e_2)$, which is increasing and convex in both arguments. Additionally, the agent enjoys a utility $v(l)$ from leisure l, which is increasing and concave in leisure. Assume that the amount of leisure he enjoys and the amount of labor he supplies to both principals add up to 1, that is, $e_1 + e_2 + l = 1$. Every principal j observe the output but not the effort the agent exerts, and pays w_j^H and w_j^L to the agent when high and low levels of output are realized, respectively. For simplicity, assume that the principals are risk-neutral but the agent is risk averse, with utility function $u(\cdot)$ where $u(0) = 0$, $u' > 0$, and $u'' < 0$.

   (a) *Independent task assignment.* Write down principal 1's objective function and the constraints that he faces, assuming that the two principals act independently in assigning different tasks to the agent.

   (b) Setup the Lagrangian function and solve for the optimal effort that the agent exerts on task 1 (you can keep the Lagrangian multipliers).

   (c) *Joint task assignment.* Write down the principal's Lagrangian function if they jointly assign tasks to the agent, and solve for the optimal effort that the agent exerts on the two tasks.

   (d) Compare the level of effort that the agent exerts in parts (b) and (c). Interpret.

[2] Stole, L. (1990) Mechanism design under common agency, Mimeo.