

# ***Strategy and Game Theory: Practice Exercises with Answers,***

**by Felix Munoz-Garcia and Daniel Toro-Gonzalez, Springer-Verlag, 2019**

**Errata in Second Edition, Updated on March 24<sup>th</sup>, 2020**

## **Chapter 1 – Dominance Solvable Games**

- Page 14, The last paragraph of Exercise #7 should be deleted, from “Therefore, our most...” to “...equilibrium outcome.)”
- Page 30, the displayed equation at the top should read

$$S(h_i, h_j) = \alpha \sum_{i=1}^I h_i + \beta \prod_{i=1}^I h_i.$$

This expression should also be the displayed equation at the beginning of the answer key, inside the large bracket.

## **Chapter 2 – Pure strategy Nash equilibrium and simultaneous-move games with complete information**

- Page 51, last displayed equation at the bottom of the page should end with  $-4r_i$ , rather than with  $+4r_i$ .
- Page 62, last displayed equation at the bottom of the page should read  $(X, X)$ ,  $(X, Y)$ , and  $(Y, X)$ .
- Page 81.
  - Seventh displayed equation (which describes equilibrium prices) should read  $\frac{1+3c}{4}$  at the end.
  - Last displayed equation of the page (describing equilibrium profits) should read

$$\pi_c = \left( \frac{1+3c}{4} - c \right) \frac{1-c}{4} = \frac{(1-c)^2}{16}.$$

- Page 81.

## **Chapter 3 – Mixed strategies, strictly competitive games, and correlated equilibria**

- Page 100, last displayed equation (immediately before Exercise #5) should read
$$msNE = \{pB, (1-p)NB; qS, (1-q)S\}$$
$$= \left\{ \left( \frac{1}{2} \right) B, \left( \frac{1}{2} \right) NB; \left( \frac{1}{5} \right) S, \left( \frac{4}{5} \right) NS \right\}.$$
- Page 104, last displayed equation (immediately before Exercise #6) should read
$$msNE = \{pU, (1-p)D; qL, (1-q)R\}$$
$$= \left\{ \frac{3}{5} U, \frac{2}{5} D; \frac{2}{5} L, \frac{3}{5} R \right\}.$$
- Page 113, the section describing “*Mixing between T and C*” at the top of the page should finish reporting the msNE of the game, as follows:

$$msNE = \left\{ \left( \frac{1}{2}T, \frac{1}{2}C, 0B \right), \left( \frac{1}{3}L, \frac{2}{3}R \right) \right\}.$$

- Page 114. The paragraph after the first displayed equations should read: “Therefore, player 2 plays  $R$  in pure strategies. Player 1 is then indifferent between  $C$  and  $B$  since both yield a payoff of \$2. As a consequence, we found a third msNE which we report as follows:

$$msNE = \left\{ \left( 0T, \frac{1}{2}C, \frac{1}{2}B \right), R \right\}$$

Note that player 1 is not choosing  $T$  with probability  $p_1 = \frac{1}{2}$  since this probability only holds when player 2 is indifferent between  $L$  and  $R$ . In this context, player 2 is choosing a pure strategy ( $R$ ) while player 1 randomizes.”

- Page 129. Figure 3.45 should have a label  $u_2$  in the vertical axis rather than  $u_1$ .
- Page 134. Figure 3.52 should read “(1,5), psNE ( $U,R$ )” at the top left-hand label, and “(5,1), psNE ( $D,L$ )” at the bottom right-hand label.
- Page 136. Figure 3.55 at the top of the page. This matrix should have  $\frac{1}{2}$  in the cell corresponding to ( $U,R$ ) rather than in ( $U,L$ ). In other words, the content of the cells at the top of the matrix should switched.
- Page 139. Last paragraph should delete the sentence in parenthesis starting at “(See the payoff matrix in...” until “...which only entails a payoff of zero.)”
- Page 143.

- Figure 3.62 should have the payoffs at the bottom left cell and at the top right-hand cell switched, so it reads “-10, 0” at the bottom left cell and “0, -10” at the top right-hand cell.
- The second paragraph of part (b) should read “When comparing ( $C, NC$ ) and ( $NC, C$ ), we find that player 1 prefers the former since

$$u_1(C, NC) = 0 > -10 = u_1(NC, C).$$

while player 2 prefers the latter because

$$u_2(C, NC) = -10 < 0 = u_2(NC, C).$$

Similarly, when comparing ( $C, C$ ) and ( $C, NC$ ), in the left-hand column of the matrix, we find that player 1 prefers the latter given that

$$u_1(C, C) = -5 < 0 = u_1(C, NC).$$

- Page 148. The second displayed equation should read

$$6 - 6q = 2q \Leftrightarrow q = \frac{3}{4}.$$

#### Chapter 4 – Sequential-move games with complete information

- Page 163, Exercise 4.
  - Part (a) should read “Find the husband’s best responses, for each of his wife’s actions in the first stage.”
  - Part (b) should read “Find the wife’s equilibrium action and...”
- Page 165. Part c, line 4, should read “we found that only the former can be sustained in equilibrium.”
- Page 174. Exercise 8, line 2, should read “The game is similar to that in Exercise 7, but with...”
- Page 184. Fourth displayed equation should read

$$P(Q^{Cournot}) = 1 - \frac{2(1-c)}{3} = \frac{1+2c}{3}.$$

- Page 189. Exercise 13, part c, should read “Repeat part (b), but assuming that...”
- Page 197. Last displayed equation at the bottom of the page should read

$$(q_1^S, q_2(q_1), q_3(q_1, q_2)) = \left(8, 8 - \frac{1}{2}q_1, \frac{16 - q_1 - q_2}{2}\right).$$

## Chapter 5 – Applications to industrial organization

- Page 250, figure 5.14 should read “Quantities” rather than “Quatities” in both the bottom row and right-hand column.

## Chapter 9 – Perfect Bayesian Equilibrium and signaling games

- Page 435, in the second displayed equation, and the rest of the exercise, should read as follows:

$$1 - 2p_1^1 + p_2^1 + c_1 + 2\left(1 - \frac{1}{3}c_1\right)\frac{1}{3}\frac{1}{f'(f^{-1}(p_1^1))} = 0.$$

Simplifying, and solving for  $p_1^1$ , we find firm 1’s best response function in the first-period game

$$p_1^1(p_2^1) = \frac{6 + c_1(9[f'(f^{-1}(p_1^1))] - 2) + 9[f'(f^{-1}(p_1^1))](1 + p_2^1)}{18[f'(f^{-1}(p_1^1))]}$$

Inserting this expression of  $p_1^1$  into  $p_2^1 = \frac{1+p_1^1}{2}$ , we obtain the optimal second-period price for firm 1

$$p_2^1 = \frac{3 + c_1}{3} \mp \frac{2(3 - c_1)}{27[f'(f^{-1}(p_1^1))]}$$

Plugging this result into the best response function  $p_1^1(p_2^1)$ , yields the optimal first-period price for firm 1

$$p_1^1 = 1 + \frac{2c_1}{3} \mp \frac{4(3 - c_1)}{27f'(f^{-1}(p_1^1))}$$

As suggested in the exercise, let us now assume that there exists a linear function  $p_1^1 = f(c_1)$ , that generates the previous strategy profile. That is:

$$p_1^1 = A_0 + A_1c_1 = f(c_1)$$

where  $A_0$  and  $A_1$  are positive constants. Intuitively, a firm with zero-unit costs,  $c_1 = 0$ , would charge a first-period price of  $A_0$ , while a marginal increase in its unit costs,  $c_1$ , would entail a corresponding increase in prices of  $A_1$ . Figure 9.52 illustrates this pricing function for firm 1. (Note that this is a separating strategy profile, as firm 1 charges a different first-period price depending on its unit cost as long as  $A_1 > 0$ . A pooling strategy profile would exist if  $A_1 = 0$ .)

Setting it equal to the expression for  $p_1^1$  we found above and letting  $A_1 = \frac{1}{f'(f^{-1}(p_1^1))}$ , we obtain:

$$A_0 + A_1 \cdot c_1 = 1 + \frac{2c_1}{3} + A_1 \frac{4(3 - c_1)}{27}$$

since  $A_1$  measures the slope of the pricing function (see Fig. 9.52), thus implying  $A_1 = \frac{1}{f'(f^{-1}(p_1^1))}$ .

Rearranging the above expression, we find

$$27A_0 + 31A_1c_1 = 3(9 + 4A_1 + 6c_1)$$

or, after solving for  $A_1$ , we obtain

$$A_1 = \frac{9(3 - 3A_0 + 2c_1)}{31c_1 - 12}$$

In addition, when firm 1’s costs are nil,  $c_1 = 0$ , the above expression becomes,

$$A_0 = 1 + \frac{4}{9}A_1.$$

Inserting this equation into the expression for  $A_1$  found above,  $A_1 = \frac{9(3-3A_0+2c_1)}{31c_1-12}$ , yields

$$A_1 = \frac{9\left(3 - 3\left(1 + \frac{4}{9}A_1\right) + 2c_1\right)}{31c_1 - 12}$$

Solving for  $A_1$ , we obtain  $A_1 = \frac{18}{31} \approx 0.58$ . Therefore, the intercept of the pricing function,  $A_0$ , becomes:

$$A_0 = \frac{39}{31} \approx 1.26.$$

Hence, the pricing function  $p_1^1$  of firm 1,  $p_1^1 = A_0 + A_1c_1$ , becomes:

$$p_1^1 = \frac{18}{31} + \frac{39}{31}c_1.$$