

Errata file for

“Practice Exercises for Advanced Microeconomic Theory” MIT

Press

March 4, 2020

1. Chapter 1 - Preferences and utility

2. Chapter 2 - Demand Theory

- Page 49. Exercise #15. Part (c). Add the following explanation at the beginning of the answer for part (c), as a new bullet point: "In this proof, we seek to use the "Squeeze Theorem", which entails finding two terms, one above and one below $[\alpha x_1^\rho + \alpha x_2^\rho]^{\frac{1}{\rho}}$, both of them converging to the same number. By doing that, we will be able to claim that $[\alpha x_1^\rho + \alpha x_2^\rho]^{\frac{1}{\rho}}$ must also converge to that number."

3. Chapter 3 - Demand Theory-Applications

4. Chapter 4 - Production Theory

5. Chapter 5 - Choice under Uncertainty

- Exercise #9.

- Page 116. The line immediately below the displayed equation at the bottom should read "where $\alpha \neq 0$ and $\beta \neq 1$. Find the..."
- Page 118, The vertical axis of Figure 5.7 (top of the page) should be $r_A(x, u)$ instead of $r_A(w, u)$.

- Exercise #24. Page 128.

- Part (b). The last displayed equation of this part has an r missing, so it should read $r_R(x) = \dots$
- Part (c). Second displayed equation in this section should read

$$-u'(w_0 - s^{**}) + \delta \rho E[u'(w_1 + \tilde{x} + \rho s^{**})] > -u'(w_0 - s^{**}) + \delta \rho u'(w_1 + E(\tilde{x}) + \rho s^{**})$$

In addition, the third displayed equation in this section has an unnecessary bracket at the end, so it should read

$$-u'(w_0 - s^{**}) + \delta \rho u'(w_1 + \rho s^{**}) = 0$$

6. Chapter 6 - Partial and General Equilibrium

- Page 133.

- Second paragraph, line 11 should read "Exercise 13 examines a context with positive externalities in consumption, which..."

- Second paragraph, line 14 should read "Exercises 16-20 focus on excess demand functions..."
- Exercise #1:
 - Page 134. Part (b) of the exercise. The last sentence of the first bullet point should have a comma between $F = \frac{1}{16}$ and N^* .
 - Page 134. Part (b) of the exercise. The displayed equation at the bottom of the page should have $(N + 1)$, rather than $(N - 1)$, in the denominator.
- Exercise #3:
 - Page 136. Paragraph immediately below first displayed equation should read "...he receives $(1 - \tau)p$ for every unit..."
 - Page 136. Third displayed equation has an extra " \geq " sign at the end of the second line. Please delete.
 - Page 137. Label in Figure 6.1 should read $(1 - \tau)p^P$ on the vertical axis, and $x((1 - \tau)p^P)$ on the right-hand side of the figure.
 - Page 137. Paragraph immediately below Figure 6.1 should have $x((1 - \tau)p)$ rather than $x((1 + \tau)p)$ in the second line, and $(1 - \tau)p^P$ rather than $(1 + \tau)p^P$ in the fourth line.
- Exercise #5:
 - Page 137. The third line of the question should read "an inverse demand function" rather than "a inverse demand function".
- Exercise #7:

- Page 138. Second line of the question should have an apostrophe so it reads "solve the individual's UMP".
- Exercise #9:
 - Page 140. First bullet point should read "consumer B is made better off while consumer A reaches..." rather than "consumer 2 is made better off while consumer 1 reaches..."
 - Page 141. First bullet point should read "Using good y as the numeraire, i.e., $p_y = \$1$, the price ratio becomes $\frac{p_x}{p_y} = p_x$. The budget line..." The following sentence should read "has a slope $-p_x$ and crosses..."
 - Page 141. End of the first bullet point (before part b of the exercise) should read "Therefore, the WEA is given by the vector $\{(5, 5), (5, 5)\}$, where every consumer enjoys 5 units of every good."
 - Page 142. Figure 6.4 (top of the page) should have $-p_x$ at the lower right-hand corner rather than $-p_1$, to be consistent with the above edits.
- Exercise #11:
 - Page 144. Last sentence should read "increases by GSP, which induces..."
- Exercise #15:
 - Page 147. Second line after first displayed equation of the question should read "for each consumer, whereas the second commodity is..."
 - Page 148. The seventh line of the first bullet point should read "Finally, plugging this into our result..."
 - Page 149. The fifth line should read "Finally, plugging this into our result..."

- Page 150. The third line of part (d) should read as "prefer not to have good 2..."
 - Page 150. Figure 6.7 should have the axes relabelled so the superscripts are either A or B for each consumer. Specifically, x_1^1 should be x_1^A and x_2^1 should be x_2^A , both for consumer A ; and similarly, x_1^2 should be x_1^B and x_2^2 should be x_2^B , both for consumer B .
 - Page 151. Figure 6.8 should incorporate the same changes as Figure 6.7 described above.
- Exercise # 17:
 - Page 153. Part (b), last paragraph. Fourth line should read "is $p_1 = \$0.6$. However, when γ increases..."
- Exercise # 21:
 - Page 156. The first line should read "and two goods $l = \{1, 2\}$."
 - Page 159. Second bullet point. The second line should have x_1^B instead of x_2^B in the middle of the line since we are talking about good 1 both each consumer.
 - Page 160. There should be a full-stop at the end of the exercise, that is, after $x_2^B = \frac{3}{5}$.
- Exercise # 25:
 - Page 166. In figure 6.12, the superscript A in the equality $x_1^A = x_2^A$ in the left-hand corner of the figure is misplaced.
 - Page 169. Second bullet point. Sixth line should read "...which is lower than

the utility that individual A could receive by consuming his original bundle..."

7. Chapter 7 - Monopoly

- Exercise #3. Page 173, Parametric example. Second line. The cost function $c(p)$ should read $c(q)$.
- Exercise #5. Page 176, part (c). The line after the displayed equation comparing profits should read "... which can be further simplified to $0 < \delta(3 + \delta)$, a condition that holds for all discount factors $\delta \in [0, 1]$. Hence, the monopolist prefers to sell rather than leasing the durable good."
- Exercise #9.
 - Page 182. The displayed equation before the second bullet (center of the page) should read as follows

$$\hat{q} = \frac{4(a-c)}{8b-3\beta^2} \quad \text{and} \quad \hat{A} = \left[\frac{3(a-c)\beta}{8b-3\beta^2} \right]^2.$$

- Page 182. The second bullet point of the page should read "*Comparing first- and second-best policies.* Comparing \hat{A} and A^{sp} , we see that $\hat{A} < A^{sp}$ since

$$\left[\frac{3(a-c)\beta}{8b-3\beta^2} \right]^2 < \left[\frac{(a-c)\beta}{2b-\beta^2} \right]^2$$

simplifies to $6b < 8b$, which holds given that $b > 0$. In the second-best policy, the social planner selects a smaller cost-reducing investment..."

- Page 183. Figure 7.2 has labels switched, that is, " A^{SP} , first best" should be next to the upper curve, and " A , second best" should go next to the lower

curve.

8. Chapter 8 - Game Theory and Imperfect Competition

- Page 189.
 - Second paragraph. Third line. Delete the sentence "In particular, exercise 10 analyzes necessary and sufficient conditions in a Cournot model with N firms."
 - Second paragraph. Fifth line should read "Afterwards, we allow the Cournot model for product differentiation (exercise 13) and introduce fixed costs (which can significantly change some results, as described in exercise 14). A recurrent topic..."
 - Second paragraph. Eighth line should read "We analyze those issues in exercise 15 and explore..."

- Exercise #5.

- Page 192. First displayed equation. Strategy s_2 should be replaced by s_1 , so the displayed equation reads

$$u_2(s_1, L) \geq u_2(s_1, R) \quad \text{for all } s_1 \in \{U, C\}.$$

- Page 192. Third paragraph, fifth line. The inequality should read as follows
$$u_2(s_1, L) \geq u_2(s_1, M).$$
- Page 192. Fourth paragraph. Add a comma in the last sentence so it reads:
"strategies (IDWDS), we obtain..."

- Exercise #7.

- Page 194. Last line on the first paragraph should read "Prisoner's Dilemma game."

- Page 195. The second displayed equation should read as

$$qu_1(A, A) + (1 - q)u_1(A, B) = qu_1(B, A) + (1 - q)u_1(B, B)$$

and the third displayed equation should read as follows

$$q = \frac{u_1(B, B) - u_1(A, B)}{[u_1(B, B) - u_1(A, B)] + [u_1(A, A) - u_1(B, A)]}$$

The subsequent paragraph should read as follows "The numerator is positive since $u_1(B, B) > u_1(A, B)$ by definition. Furthermore, the denominator is larger than the numerator given that the second term satisfies $u_1(A, A) > u_1(B, A)$ by assumption. Hence, probability..."

- Page 195. Part (c). Last sentence should read "...resembles a Prisoner's Dilemma game."

- Page 196. First line should read "player 2 indifferent between A and B is".

- Page 196. The fifth displayed equation should read as follows

$$qu_1(A, A) + (1 - q)u_1(A, B) = qu_1(B, A) + (1 - q)u_1(B, B)$$

and the sixth displayed equation should read as follows

$$q = \frac{u_1(B, B) - u_1(A, B)}{[u_1(B, B) - u_1(A, B)] + [u_1(A, A) - u_1(B, A)]}$$

The subsequent paragraph should read as follows "The numerator is negative since $u_1(B, B) < u_1(A, B)$ by definition. The denominator is also negative given that both of its terms are negative, i.e., $u_1(B, B) < u_1(A, B)$ and $u_1(A, A) < u_1(B, A)$ by assumption. In addition, the absolute value of the denominator is greater than..."

- Exercise #9.
 - Page 198. First bullet point. Third sub-bullet should read as "best response functions entail a..."
- Exercise #11.
 - Page 198. The title of the exercise should read "Cournot with equity swaps (Reynolds and Snapp, 1986)" in bold font. In addition, the reference should be added to the list of references at the back of the book, as follows: Reynolds, Robert J., and Bruce R. Snapp (1986) "The competitive effects of partial equity interests and joint ventures." *International Journal of Industrial Organization* 4, no. 2: 141-153.
 - Page 199. Part (c), last sentence should read "...yields an output $q^C = \frac{1-\gamma}{3-2\gamma}$."
 - Page 200. First bullet point. Last sentence should read "...which are larger than those under a standard Cournot model."
- Exercise #13.

- Page 201. The second paragraph of the question (below the first displayed equation) should read "where $j \neq i$ and parameter θ satisfies $\theta \in [0, 1]$, that is, if..."

- Exercise #15.

- Page 204. Part (a). First displayed equation should read

$$V^C = \underbrace{p \left(\pi_m - F + \frac{\delta}{1 - \delta} \pi_n \right)}_{\text{Collusion is detected today}} + \underbrace{(1 - p) (\pi_m + \delta V^C)}_{\text{Collusion is not detected today}}$$

- Page 205. Part (a). First displayed equation (top of the page) should read as follows

$$\delta > \frac{\pi_d - \pi_m + Fp}{(1 - p) (\pi_d - \pi_n)} \equiv \bar{\delta}$$

Please add the following sentence immediately after the above displayed equation: "Cutoff discount factor $\bar{\delta}$ is positive since deviating profit π_d satisfies $\pi_d > \pi_m$ and $\pi_d > \pi_n$, parameter F is positive, and probability $p \in (0, 1)$. In addition, cutoff $\bar{\delta} < 1$ if $\pi_d - \pi_m + Fp < (1 - p) (\pi_d - \pi_n)$, which simplifies to $p (\pi_d - \pi_n + F) < \pi_m - \pi_n$."

- Page 205. Part (b). First displayed equation of this part should read as follows

$$\frac{\partial \bar{\delta}}{\partial F} = \frac{p}{(1 - p) \underbrace{(\pi_d - \pi_n)}_+} > 0$$

Second displayed equation of part (b) should read as follows

$$\begin{aligned} \frac{\partial \bar{\delta}}{\partial p} &= \frac{F(1-p)(\pi_d - \pi_n) + (\pi_d - \pi_n)(\pi_d - \pi_m + Fp)}{(1-p)^2(\pi_d - \pi_n)^2} \\ &= \frac{(\pi_d - \pi_n)(\pi_d - \pi_n + F)}{(1-p)^2(\pi_d - \pi_n)^2} \\ &= \frac{\overbrace{(\pi_d - \pi_n)}^+ + F}{(1-p)^2 \underbrace{(\pi_d - \pi_n)}^+} > 0. \end{aligned}$$

- Exercise #17.

- Page 206, part (b). The first bullet of answer key, after the displayed equation, should read “Solving for n , we obtain that $n < m + \sqrt{m} - 1$, as depicted in the (n, m) -pairs below the line $m + \sqrt{m} - 1$ in figure 8.3.” The legend of Figure 8.3 should also be edited so it reads “Figure 8.3. Profitable mergers satisfy $n < m + \sqrt{m} - 1$.”
- Page 207, part (c). Equation $m - \sqrt{m} - 1$ should read $m + \sqrt{m} - 1$.
- Page 207. Part (b). Last sentence in the paragraph below Figure 8.3 should read "... as depicted in the points above the 45-degree line. For instance, in an industry with $n = 10$ firms, the condition we found determines that this merger is only profitable if and only if $10 < m + \sqrt{m} - 1$, which solving for m yields $m > 8.2$. That is, the merger is profitable if at least 9 firms (rounding to the next integer) merge."

- Exercise #19.

- Page 208. First bullet point. The first displayed equation should read as

$$(1 - q_i - q_j) q_i - F = \left(1 - \left(\frac{1 - q_j}{2}\right) - q_j\right) \left(\frac{1 - q_j}{2}\right) - F > 0,$$

- Page 208. First bullet point. Fourth line. Add a space between "off" and "staying" so it reads "...better off staying inactive..."
- Page 208. First bullet point. Last paragraph should read "We find Cournot equilibria wherever best response functions cross. Then we distinguish..."
- Page 210. The caption of figure 8.7 should read "Figure 8.7 Equilibrium when fixed costs are moderately high, $\frac{1}{9} < F \leq \frac{1}{4}$."
- Page 210. Last bullet point. Last sentence should read "both firms producing zero output at the origin..."
- Exercise #21.
 - Page 211. The fourth paragraph should read as "Intuitively, note that, relative to a standard Cournot..."
 - Page 212. Footnote #3 should read "In particular, figure 8.10 depicts $\max\{0, \frac{6c-1}{5c}\}$ since profit share λ^* must..."
- Exercise #23.
 - Page 213. The fourth line of the question should read "...both firms set the monopoly price and share demand equally. Assume that..."
 - Page 214. The last line of the exercise should read "...as a function of F when $c = \frac{1}{3}$, i.e., $\bar{\delta} = \frac{4}{8-72F}$." Figure 8.11 should be redrawn to depict this new cutoff.

- Exercise #25.
 - Page 215. Last sentence of the page should read "Figures 8.12a and 8.12b show two separate cases of..."
 - Page 216. Last displayed equation (immediately before Exercise #27) should read as follows

$$q^* = \frac{a - 4c + \sqrt{a^2 - 8ac}}{8bc}$$

- Exercise #27.
 - Page 217. Second bullet point. Last sentence of the exercise should read "... and thus, competition a la Cournot is optimal during the punishment phase for all values of δ ."

9. Chapter 9 - Externalities and Public Goods

- Exercise #1, Page 220. First displayed equation should not list q^i as its second argument. This equation should read $u_i(s^i) = v^i(s^i) + \alpha w^i$.
- Exercise #3, Page 226. Second bullet should read "... is significantly reduced, from 6.82 to 5.7 units. Aggregate supply is..."
- Exercise #5.
 - Page 227. In part (a), the third displayed equation should not have a second argument, a , so it should read

$$\frac{\partial \pi(q^E)}{\partial q} = 10 - 2q^E \leq 0, \text{ with equality if } q^E > 0.$$

- Page 228. Part (c), last displayed equation of the page. The expectation operator should have a subscript a so it reads E_a rather than E .
- Page 229. Part (c), first and second displayed equation of the page. The expectation operator should have a subscript a so it reads E_a rather than E . Part (d), first sentence should read "Figure 9.2 illustrates the welfare loss associated with tax t^* , which induces..."
- Exercise #7. Page 234. The last displayed equation of this exercise should read

$$q_i^E(x_i^E) = q_j^E(x_i^E) = \frac{\theta - (1 - \beta) \left(\frac{\theta[9 - (1 - \beta)\delta]}{18} \right)}{3} = \frac{\theta [9 + 9\beta + \delta(1 - \beta)^2]}{54}.$$

- Exercise #9.
 - Page 234. The second line of the question should read "Assume that each individual i has wealth, $\omega_i \geq 0$, and a Cobb-Douglas utility function..."
 - Page 234. The last part of question (a) should read "Find the demand functions denoted $(x_i(\cdot), g_i(\cdot))$, for the private and public good."
 - Page 235. The last sentence of the question (before part a) should read "For simplicity, normalize the price of the public good to one, and denote the price of the private good as $p \geq 0$."
 - Page 237. Part (c2). The second displayed equation should finish with "for the public good, where $i, j \in \{1, 2, \dots, k\}$ and $i \neq j$."
- Exercise #11. Page 238. Third line should read "...the same wealth, $M \geq 1$, and that the price for both goods is 1."
- Exercise #13. Page 241. In the first displayed equation, there should be a space

between the max operator and w .

- Exercise #15.
 - Page 243. Third displayed equation should have $(N - 1)$ rather than $(n - 1)$ in the second term.
 - Page 244. Fifth displayed equation should include $= 0$ at the end of the expression, so it reads $a - c - (b + 2d)Q = 0$. In addition, the paragraph immediately below this equation should read "an equal share" rather than "a equal share".

10. Chapter 10 - Contract Theory

- Exercise #17.
 - Page 273. Paragraph immediately below the third displayed equation (Exercise 10.17) should read "...denotes his preference for quality (where $\alpha_H > \alpha_L > 1$), x represents the quality..."
 - Page 274. The second displayed equation should read

$$\max_{p \geq 0} \frac{(p - 1)}{2} \left(\frac{\alpha_H^2}{4p^2} + \frac{\alpha_L^2}{4p^2} \right)$$

The fourth displayed equation should read

$$\pi^U = \frac{(p^U - 1)}{2} \left(\frac{\alpha_H^2}{4p^2} + \frac{\alpha_L^2}{4p^2} \right) = \frac{\alpha_H^2 + \alpha_L^2}{32} > 0.$$

– Page 275. The second displayed equation should read

$$\frac{\alpha_H^2}{8p^2} - \frac{\alpha_H^2}{4p^2} - \frac{\alpha_L^2}{4p^2} + \frac{\alpha_H^2}{4p^2} + \frac{\alpha_L^2}{4p^2} = 0.$$

The third displayed equation should read

$$p^{ST} = \frac{2(\alpha_H^2 + \alpha_L^2)}{2\alpha_L^2 + \alpha_H^2},$$

The fourth displayed equation should read

$$T^{ST} = \frac{\alpha_L^2}{4p^{ST}} = \frac{\alpha_L^2 (2\alpha_L^2 + \alpha_H^2)}{8(\alpha_H^2 + \alpha_L^2)}$$

The fifth displayed equation should read

$$\begin{aligned} \pi^{ST} &= \frac{\alpha_L^2 (2\alpha_L^2 + \alpha_H^2)}{8(\alpha_H^2 + \alpha_L^2)} + \left(\frac{2(\alpha_H^2 + \alpha_L^2)}{2\alpha_L^2 + \alpha_H^2} - 1 \right) \left(\frac{\alpha_H^2 + \alpha_L^2}{8} \right) \left(\frac{2\alpha_L^2 + \alpha_H^2}{2(\alpha_H^2 + \alpha_L^2)} \right)^2 \\ &= \frac{(4\alpha_L^2 + \alpha_H^2)(2\alpha_L^2 + \alpha_H^2)}{32(\alpha_H^2 + \alpha_L^2)}. \end{aligned}$$

The sixth displayed equation should read

$$\max_{p \geq 0} \frac{1}{2} [T + (p-1)x_H(p)]$$

The last displayed equation should read

$$\max_{p \geq 0} \frac{1}{2} \left[\frac{\alpha_H^2}{4p} + (p-1) \frac{\alpha_H^2}{4p^2} \right]$$

– Page 276. The first displayed equation should read

$$-\frac{\alpha_H^2}{2p^2} - \frac{\alpha_H^2(p-2)}{4p^3} = 0$$

solving for p yields $p^H = \frac{2}{3}$. Then, the fee in this contract is $T^H = \frac{\alpha_H^2}{4p^2} = \frac{3\alpha_H^2}{8}$, entailing profits of

$$\pi^H = \frac{3\alpha_H^2}{32}.$$

The third displayed equation should read

$$\frac{(4\alpha_L^2 + \alpha_H^2)(2\alpha_L^2 + \alpha_H^2)}{32(\alpha_L^2 + \alpha_H^2)} < \frac{3\alpha_H^2}{32}$$

The fourth displayed equation should read

$$2x^2 - 3x - 8 > 0$$

The paragraph after the fourth displayed equation should read “where, for compactness, we denote $x \equiv \left(\frac{\alpha_H}{\alpha_L}\right)^2$. Solving for x , we obtain that $2x^2 - 3x - 8 > 0$ holds for all $x > \frac{\sqrt{3+\sqrt{73}}}{2} \simeq 1.7$. Intuitively, when the high-value customer assigns a sufficiently higher valuation than the low-value customer, that is, the differences in valuation satisfy $\frac{\alpha_H}{\alpha_L} > 1.7$, the monopolist earns a higher profit selling to the high-value customer than to all types (i.e., $\pi^H > \pi^{ST}$). In order to illustrate the above result, we next provide a numerical example.”

The fifth displayed equation should read

$$p^{ST} = \frac{26}{17} \simeq 1.529, \quad T^{ST} = \frac{1}{p^{ST}} = \frac{17}{26} \simeq 0.654, \text{ and}$$

$$\pi^{ST} = \frac{425}{416} \simeq 1.022.$$

The sixth displayed equation should read

$$p^H = \frac{2}{3}, \quad T^H = 3.375, \quad \text{and} \quad \pi^H = 0.84375.$$

The last displayed equation should read

$$\max_{x_H, T_H, x_L, T_L} \frac{1}{2} (T_H - x_H + T_L - x_L)$$

– Page 277. Ninth displayed equation should read

$$\max_{x_H, x_L} \frac{1}{2} \left(\underbrace{\alpha_L \sqrt{x_L} + \alpha_H (\sqrt{x_H} - \sqrt{x_L})}_{T_H} - x_H + \underbrace{\alpha_L \sqrt{x_L} - x_L}_{T_L} \right)$$

– Page 278. First sentence should end with "...tariffs are" rather than "...tariffs is".

The second displayed equation should read

$$\begin{aligned} \pi^{MT} &= \frac{1}{2} [(T_H - x_H) + (T_L - x_L)] \\ &= \frac{1}{2} \left[\left(\frac{2\alpha_H^2 - 3\alpha_L\alpha_H + 2\alpha_L^2}{2} - \frac{\alpha_H^2}{4} \right) + \left(\frac{\alpha_L(2\alpha_L - \alpha_H)}{2} - \frac{(2\alpha_L - \alpha_H)^2}{4} \right) \right] \\ &= \frac{2\alpha_L^2 + \alpha_H^2 - 2\alpha_L\alpha_H}{4}. \end{aligned}$$

The last sentence of part (c) should read "And profits are $\pi^{MT} = 1.25$."

In part (d), the first paragraph should end with "more profitable when $\frac{\alpha_H}{\alpha_L} > 1.7$ ".

The first displayed equation should read

$$\frac{\alpha_L^2 + \alpha_H^2}{32} < \frac{3\alpha_H^2}{32} < \frac{2\alpha_L^2 + \alpha_H^2 - 2\alpha_L\alpha_H}{4}$$

The paragraph immediately below the first displayed equation of part (d) should read "For the first inequality to hold, we need $2\alpha_H^2 > \alpha_L^2$, or $\frac{\alpha_H}{\alpha_L} > \sqrt{2}$, which is satisfied since $\frac{\alpha_H}{\alpha_L} > 1.7 > \sqrt{2}$. For the second inequality to hold, we need

$$13\alpha_H^2 - 16\alpha_L\alpha_H + 16\alpha_L^2 > 0,$$

which is always positive. Therefore, the profit ranking $\pi^U < \pi^H < \pi^{MT}$ holds when $\frac{\alpha_H}{\alpha_L} > 1.7$."

Add two bullet points immediately below that read

* However, the profit ranking becomes $\pi^U < \pi^{ST} < \pi^{MT}$ when $\frac{\alpha_H}{\alpha_L} < 1.7$.

Specifically,

$$\frac{\alpha_L^2 + \alpha_H^2}{32} < \frac{(4\alpha_L^2 + \alpha_H^2)(2\alpha_L^2 + \alpha_H^2)}{32(\alpha_L^2 + \alpha_H^2)} < \frac{2\alpha_L^2 + \alpha_H^2 - 2\alpha_L\alpha_H}{4}$$

For the first inequality to hold, we need

$$\alpha_H^4 + 2\alpha_L^2\alpha_H^2 + \alpha_L^4 < 8\alpha_L^4 + 6\alpha_L^2\alpha_H^2 + \alpha_H^4,$$

which simplifies to $4\alpha_L^2\alpha_H^2 + 7\alpha_L^4 > 0$ that is always positive since $\alpha_H > \alpha_L > 1$.

For the second inequality to hold, we need

$$\alpha_H^4 + 6\alpha_L^2\alpha_H^2 + 8\alpha_L^4 < 8(\alpha_L^2 + \alpha_H^2)(2\alpha_L^2 + \alpha_H^2 - 2\alpha_L\alpha_H),$$

which simplifies to

$$7x^4 - 16x^3 + 18x^2 - 16x + 8 > 0.$$

where, for compactness, we denote $x \equiv \frac{\alpha_H}{\alpha_L}$. This inequality is positive for all $x > 1$, which holds since $\alpha_H > \alpha_L$ by definition.

* Therefore, the monopolist derives the highest profit by practicing menu pricing for all values of α_H and α_L .

The last displayed equation at the bottom of the page should read

$$\pi^U = 0.406 < \pi^{ST} = 1.022 < \pi^{MT} = 1.25.$$