EconS 503 - Microeconomic Theory II
Homework #7 - Due date: April 1st, scanned via email.

1. **Exercise from Bolton and Dewatripont:**

   (a) Exercise 6 (see page 650 since all exercises are at the end of the book).

2. **PBEs in bargaining.** A buyer and a seller are bargaining. The seller owns an object for which the buyer has a value v; the seller's value is zero. The buyer knows v but the seller does not. The seller's beliefs about v, which are common knowledge, are that $v = 30$ with probability $\frac{1}{2}$ and $v = 10$ with probability $\frac{1}{2}$. There are two periods of bargaining; there is no discounting (i.e., $\delta = 1$).

   - In the first period, the seller makes an offer p_1 that represents a price that the buyer will need to pay to buy the object. The buyer can accept or reject the offer. If the buyer accepts, the offer is implemented and the game ends. If the buyer rejects, the game continues to the second period.
   - In the second period, the seller (again) makes an offer p_2, which is the price the buyer will need to pay to buy the object. The buyer can accept or reject the offer. If the buyer accepts, the offer is implemented and the game ends. If the buyer rejects, then the seller keeps the object and the game ends.

   If the buyer buys the object in the first period, then the payoffs are p_1 for the seller and $v - p_1$ for the buyer. Similarly, if the buyer buys the object in the second period, then the payoffs are p_2 for the seller and $v - p_2$ for the buyer. If the buyer does not buy the object, then the payoffs are zero for each player.

   (a) Provide an extensive-form representation of this game.

   (b) Find a Perfect Bayesian equilibrium in which the seller believes that any buyer that rejects a first-period offer is the type with valuation $v = 10$ with probability 1. (Justify your answer, and remember to fully specify the Perfect Bayesian equilibrium.)

3. **Cheap talk with three types.** Consider the cheap talk model with three types discussed in class (Investing recommendations game). Let us focus on the partially separating strategy profile where the Analyst (sender) recommends Buy both when the stock outperforms the market and when its neutral, but recommends Hold when the stock underperforms the market. In class, we made a simplifying assumption on off-the-equilibrium beliefs (after the Investor receives a Sell recommendation), denoted by γ_1, γ_2, and $1 - \gamma_1 - \gamma_2$.

   (a) Without restricting off-the-equilibrium beliefs, find under which conditions the above partially separating strategy profile can be sustained as a PBE of this game.

(b) Consider now the pooling strategy profile where the Analyst recommends Buy regardless of the stock's type. Under which conditions can this strategy profile be supported as a PBE?

4. **Signaling when the expert receives imprecise signals.** Consider the following signaling model between an expert (E), such a special interest group, and a decision maker (DM), such as a politician. For simplicity, assume that the state of the world is discrete, either $\theta = 1$ or $\theta = 0$ with prior probability $p \in (0, 1)$ and $1 - p$, respectively. The expert privately observes an informative but noisy signal s, which also takes two discrete values $s \in \{0, 1\}$. The precision of the signal is given by the conditional probability

$$\text{prob}(s = k|\theta = k) = q,$$

where $k = \{0, 1\}$, and $q > \frac{1}{2}$. In words, the probability that the signal s coincides with the true state of the world θ is q (precise signal), while the probability of an imprecise signal where $s \neq \theta$ is $1 - q$. The time structure of the game is as follows:

1) Nature chooses θ according to the prior p.
2) Expert observes signal s and reports a message $m \in \{0, 1\}$
3) Decision maker observes m and responds with $x \in \{0, 1\}$
4) θ is observed and payoffs are realized

The payoff function for the decision maker is

$$u(x, \theta) = \left(\theta - \frac{1}{2}\right) x$$

while that of the expert is

$$v(m, \theta) = \begin{cases} 1, & \theta = m \\ 0, & \theta \neq m \end{cases}$$

which, in words, indicates that the expert's payoff is 1 when the message she sends coincides with the true realization of the state of the world, but becomes zero otherwise. Importantly, her payoff is unaffected by the signal, which she only uses to infer the actual realization of parameter θ. Intuitively, $v(m, \theta)$ is often understood as a "reputation function" since it provides the expert with a payoff of 1 only when his message was an accurate representation of the true state of the world (which in this model he does not precisely observe).

(a) Is there a Perfect Bayesian equilibrium (PBE) in which the expert reports his signal truthfully?