EconS 503 - Microeconomic Theory II
Homework #6 - Due date: Monday, March, 23rd 2020

1. **Cournot competition when all firms are uninformed - General setting**. Consider again the setting in Example 8.10 about two firms competing a la Cournot under incomplete information about their production costs. Let us now provide a more general analysis by considering marginal costs c_H and c_L for firm 2, where $c_H > c_L \geq 0$, occurring with probability p and $1-p$ respectively.

   (a) Find the best response function for every firm i, $q_i^k(q_j^L, q_j^H)$, where $k = \{L, H\}$ denotes firm i's marginal cost (high or low).

   (b) Use your results from part (a) to find the Bayesian Nash Equilibrium (BNE) of the game.

   (c) How do the equilibrium output levels you found in part (a) are affected by changes in c_H, c_L, and p? Interpret.

2. **Sequential version of a first-price auction.** Consider an auction with two bidders. Every bidder $i = \{1, 2\}$ privately observes his valuation v_i for the object, drawn from a uniform distribution $U[0, 1]$, which is common knowledge among players. Assume that all bidders are risk neutral. Unlike in the standard first-price auction, where players simultaneously and independently submit their bids, let us consider its sequential-move version with the following time structure: Bidder 1 submits his bid, b_1; bidder 2 observes b_1 and responds either buying the good at this price or steps out (which implies that bidder 1 purchases the object for the price he originally submitted b_1).

   (a) Find equilibrium bidding strategies for bidder 2 (follower) and bidder 1 (leader).

   (b) Is this auction efficient?

3. **Bidders receiving independent signals**. Consider the following *second-price* sealed-bid auction with two bidders competing for the object. Players receive private and independent signals, t_1 and t_2, drawn from uniform distribution $U[0, 1]$. Player i's valuation for the object is defined as

$$v_i = \alpha_i t_i + \alpha_j t_j,$$

   where $i \neq j$ and parameters α_i and β satisfy $\alpha_i > \beta \geq 0$. Intuitively, every player's valuation is a function of the signal he receives, t_i, and (to a lesser extent) the signal that his rival receives, t_j.

   (a) Show that bidding function $\beta(t_i) = (\alpha_i + \alpha_j)t_i$, for every bidder $i = \{1, 2\}$ is a Bayesian Nash equilibrium of this game.

   (b) Characterize all symmetric equilibria in which player i bids $\beta(t_i)$, where β is strictly increasing and differentiable.

4. **Third-price auction (Bonus question).** Consider an auction with $N \geq 3$ bidders, each of them privately observing his valuation of the good, $v_i \in [0, 1]$. Each bidder independently and simultaneously submit his own bid, b_i, and the bidder submitting the highest bid is selected as the winner of the auction and receives the good. In this case, however, let us assume that the winning bidder pays the *third* highest bid; thus justifying why this auction format is referred to as "third-price auction."

   (a) Assume a cumulative distribution function $F(v_i)$, with positive density for all valuations $f(v_i) > 0$ for all $v_i \in [0, 1]$. Find bidder i's equilibrium bidding function $b_i(v_i)$ in this auction.

   (b) For the remainder of the exercise, consider that valuations are distributed according to a uniform distribution, so $F(v_i) = v_i$. Evaluate the equilibrium bidding function you found in part (a) using this distribution function. How does $b_i(v_i)$ change in the number of bidders N?

   (c) Show that the equilibrium bidding function satisfies $b_i(v_i) > v_i$. Justify.

   (d) Compare the equilibrium bidding functions in the first-, second-, and third-price auction. Interpret.