EconS 503 - Microeconomic Theory II
Homework #5 - Due date: March 6th, 2020

1. **Exercises from Tadelis:**

   - Exercise 9.3 from Chapter 9.
   - Exercises 10.3, 10.7, 10.9, 10.12 from Chapter 10.

2. **Collusion with probability of being caught - Harrington (2014).**[1] Consider an industry with N firms. For generality, we do not assume whether they compete in quantities or prices yet, nor the inverse demand function or costs they face. Consider that firms are symmetric and in the Nash equilibrium of the unrepeated game, every firm earns profits π^N, so we label the present value of the noncollusive stream as

$$V^N \equiv \pi^N + \delta\pi^N + ... = \frac{1}{1-\delta}\pi^N.$$

   When firms collude, each of them earns profit π^C, where $\pi^C > \pi^N$. When a firm unilaterally deviates from the collusive outcome, it earns a deviating profit of π^D, where $\pi^D > \pi^C$ in that period. Consider a standard Grim-Trigger strategy (GTS) where every firm chooses to collude in period $t = 1$, and continues to do so in subsequent periods $t > 1$ if all firms colluded in previous periods. If one firm did not cooperate in previous periods, however, all firms revert to the Nash equilibrium of the unrepeated game, earning π^N thereafter (permanent punishment scheme). For simplicity, assume that all firms exhibit the same discount factor $\delta \in (0, 1)$.

   (a) Find the minimal discount factor δ that sustains this GTS as a subgame perfect equilibrium of the game.

   (b) For the rest of the exercise, let us assume that the cartel faces a exogeneous probability p of being discovered, prosecuted, and convicted, by a regulatory agency such as the Federal Trade Commission. If caught and convicted in period t, a firm must pay a fine F^t, where $F^t = \beta F^{t-1} + f$. Parameter $1 - \beta$ can be understood as the depreciation rate, which we assume to satisfy $\beta \in (0, 1)$ to guarantee that the penalty is bounded. In addition, assume that $F^0 = 0$, so that $F^1 = f$, $F^2 = \beta F^1 + f$, and similarly for subsequent periods. Find the collusive value $V^C(F)$ given an accumulated penalty F. [*Hint*: Solve for $V^C(F)$ recursively.]

   (c) Write down the condition (inequality) expressing that every firm has incentives to collude, obtaining $V^C(F)$ rather than deviating. For simplicity, you can assume that if the cartel is convicted during the deviation period, it has no chances of being caught during the permanent punishment phase.

[1] Harrington, Joseph E. Jr. (2014) "Penalties and the Deterrence of Unlawful Collusion," *Economic Letters*, 124, pp. 33-36.

(d) The steady-state penalty is $F = \frac{f}{1-\beta}$, which is found by solving $F = \beta F + f$. Evaluate the collusive value $V^C(F)$ at this penalty, and insert your result in the condition you found in part (c) of the exercise. Rearrange and interpret.

(e) *Bertrand competition.* Assume that firms compete a la Bertrand, selling homogeneous products with inverse demand function $p(Q) = 1 - Q$ where Q denotes aggregate output. All firms face a symmetric marginal cost $c > 0$. In this setting, every firm obtains zero profits in the Nash equilibrium of the unrepeated game, entailing $\pi^N = 0$. If a firm unilaterally deviates from the collusive price (charging a price infinitely close, but below, the collusive price), it captures all industry sales, earning a profit $\pi^D = N\pi^C$ during the deviating period. Evaluate your results from part (d) of the exercise in this context. Then discuss whether collusion becomes easier to sustain when the penalty f increases; and when the number of firms N increases.