

EconS 503 - Microeconomic Theory II
Homework #4 - Due date: Friday, February 21st, in class

1. **Exercises from Tadelis:** Exercises 7 and 9 from Chapter 8.
2. **Strategic pre-commitment.** Consider the following sequential-move game. In the first stage, every firm $i = \{1, 2\}$ chooses its investment in cost-reducing technologies, k_i . This investment decreases its marginal production cost from c_0 to $c_0 - \alpha k_i$, where $\alpha \in [0, 1]$ denotes the effectiveness of the investment. Intuitively, when $\alpha = 0$, investment is futile but when $\alpha = 1$ every dollar invested reduces the initial marginal cost of the firm by one dollar. In the second stage, both firms observe the investment profile (k_i, k_j) selected in the previous stage and then compete a la Cournot. Firms face inverse demand function $p(Q) = 1 - Q$, where $Q = q_1 + q_2$ represents aggregate output.
 - (a) *Second stage.* Find the best response function in the second period, $q_i(q_j)$. [Hint: It should be a function of firm i 's investment in cost-reducing technologies, k_i]
 - (b) Find the second-period output function, $q_i(k_i, k_j)$.
 - (c) *First stage.* To simplify our analysis, assume that $c_0 = \frac{1}{2}$ and $\alpha = \frac{1}{4}$ for the remainder of the exercise. Find firm i 's best response function in the first stage, $k_i(k_j)$. Is it positively or negatively sloped? Interpret.
 - (d) Find the equilibrium investment in cost-reducing technologies, k_i^* .
 - (e) Summarize the subgame perfect equilibrium of the game.
3. **Entry deterrence before Bertrand competition.** Consider a market with inverse demand $p = a - Q$, where Q denotes aggregate output. The incumbent monopolist is present in the market and a potential entrant considers entering the industry at a fixed setup cost $K > 0$. For simplicity, assume that both firms face no production costs. If entry does not occur, the incumbent (firm 1) keeps its monopoly position. If entry ensues, firms compete a la Bertrand, with sales going to the firm setting the lowest price (and in the case that both firms set the same price, assume that consumers buy from firm 1).
 - (a) Find the SPNE of the game. Does firm 2 chooses to enter in equilibrium?
 - (b) Let us now allow firm 2 to choose, before the beginning of the game, a pair (c_2, p_2) of a capacity c_2 and price p_2 , where capacity c_2 sets an upper bound on firm 2's production. For simplicity, assume that each unit of capacity costs one dollar. If entry occurs, now firm 1 observes the pair (c_2, p_2) , responds to the pair setting a price p_1 before consumers choose which firm to buy from. Find firm 1's profits if it chooses to deter entry. Find firm 1's profits from accommodating entry. Compare its profits. Under which parameter values does firm 1 choose to deter entry?
4. **Stackelberg with m leaders and n followers, Huck et al. (2001).** Consider a market with two firms with $m+n$ firms, inverse demand function given by $p(Q) = a - Q$, and all firms facing a (constant) unit cost c .

- (a) Find the Subgame Perfect Nash Equilibrium of the following two-stage game. In the first stage, m firms (leaders) decide their output. In the second stage, the remaining n firms (followers) decide their output.
- (b) Study the profitability of the merger of two leaders and the profitability of the merger of two followers.