EconS 594 - Industrial Organization
Midterm Exam - Due date: Tuesday, October 22nd, in class.

Instructions: This exam has 5 exercises. Write your answers to each exercise in a different page. Show all your work and be as clear as possible in your answer. You can work in groups, but each student must submit his/her exam. The due date of this take-home exam is Tuesday, October 22nd, in class. I strongly recommend you work a few exercises every day, rather than trying to solve all exercises in one day. Since this is a take-home exam, late submission will be subject to significant grade reduction.

1. **Switching costs, based on Klemperer (1995).**[1] Consider an industry with N consumers distributed along the $[0, 1]$ line, which measures consumers' linear cost of learning how to use a product. Firms A and B sells the same homogeneous product but are located at 0 and 1, respectively, and have the same unit cost $c = 0$ in each period. Therefore, a consumer located at point x has a learning cost tx of using firm A's product and $t(1 - x)$ of using B's product, where $t > 0$. Consumers do not have any physical transportation cost.

   The time structure of the game is the following:

   i) At period 1, consumer utility is given by

   $$u = r - p_i - t\,|l_i - x|$$

   for every firm $i = \{A, B\}$, where l_i denotes the location of firm i.

   ii) At period 2, each consumer has a reservation price r to buy the good.

   Goods cannot be stored and consumers do not discount future payoffs. Goods are perceived as perfectly homogeneous, but there are switching costs: if a consumer changes provider, he has to pay a switching cost $s > 0$, which for simplicity is assumed to be independent of the distance that separates the consumer from the firm. Firms set simultaneously prices in each period. Assume that $r > c$, that $s \geq r - c$, and that $r - 2t > c$.

   (a) Find the equilibrium prices of each period of the game, p^1_{iS} and p^2_{iS} for every firm i where subscript S denotes that our setting considers switching costs.

   (b) Find the equilibrium price for the game where the first period is like the above, but in the second period there are no switching costs at all. Label prices p^1_{iNS} and p^2_{iNS} for every firm i where subscript NS denotes that our setting considers no switching costs.

   (c) Show that second-period prices are higher with switching costs, $p^2_{iS} > p^2_{iNS}$; but first-period prices are lower, $p^1_{iS} < p^1_{iNS}$ for every firm i.

[1] Klemperer, P. (1995) "Competition when consumers have switching costs: An overview" Review of Economic Studies, 62, pp. 515-39.

2. **Over-investment in R&D.** Consider an industry with an incumbent firm, I, facing a potential entrant, E. If it enters, the entrant produces the same homogeneous good as the incumbent, and market demand is $p(q) = 1 - q$, where q denotes aggregate output.

   (a) Consider the following game where firms choose their actions simultaneously. In the first stage, every firm $i = \{I, E\}$ simultaneously decide their investment in R&D, x_i, which yields total production cost $C(q_i, x_i) = (c - x_i)q_i$. The total cost of investing x_i dollars is x_i^2. At this stage, firm E also decides whether to enter, and pays a fixed entry cost $F > 0$. In the second stage, active firms observe each other's investment decisions and respond with their output. Find the subgame perfect equilibrium of the game.

   (b) Consider a variation of the game in part (a), where now firm I is the only firm investing in R&D in the first stage. In the second stage, observing firm I's investment decision, firm E decides whether to enter and its investment in R&D, x_E. In the last stage, active firms observe each other's investment decisions and respond with their output. Find the subgame perfect equilibrium of the game. [Hint: You will need to find for which values of entry cost F, the incumbent accommodates entry, deters entry, and for which values of F entry is blockaded.]

3. **Cartels and renegotiation.** Consider an industry with two firms selling a homogeneous product and competing in prices.

   (a) Find the minimal discount factor sustaining a collusive monopoly price.

   (b) Upon a defection, assume now that firms could meet, at no cost, and renegotiate their collusive agreement. Can collusion be sustained under a symmetric GTS?

   (c) Consider now an asymmetric GTS with temporary punishment lasting T periods. In particular, every firm starts setting monopoly price p^m. If a deviation is detected, the deviator sets a price p and the non-deviator sets a price p^P during T periods, where $p > p^P$ and the non-deviator profits become $\pi(p^P, p) = \frac{\pi(p^m, p^m)}{2}$. That is, prices during the punishment phase are chosen so that the non-deviator makes the same profits as before the deviation occurred. After T periods, firms revert to the collusive price p^m. Write the incentive compatibility condition for every firm to collude after a history of collusion, and for the defecting firm to go along with the punishment rather than deviating from it. Show that collusion cannot be sustained with this asymmetric GTS either.

   (d) In the asymmetric GTS considered in part (c), assume that collusive firms face a probability θ of being found colluding and, if so, a fine $F > 0$. The first meeting of the cartel will then take place if its expected penalty, θF, satisfies $\theta F < \frac{\pi(p^m)}{2}\frac{1}{1-\delta}$. For which value of θF collusion arises in equilibrium? Interpret.

4. **Pay to switch or pay to stay?, based on Schaffer and Zhang (2000).**[2] Consider an industry with two firms, A and B, selling a homogeneous good, and both facing the

[2] Shaffer, G. and Zhang, Z. (2000), "Pay to Switch or Pay to Stay: Preference-Based Price Discrimination in Markets with Switching Costs," Journal of Economics & Management Strategy 9, pp. 397-424.

same marginal cost of production, $c > 0$. Consumers buy at most one unit of either firm. Consumers are partitioned in the following way: Group a of consumers represents a fraction $\theta \in \left[\frac{1}{2}, 1\right]$ of all consumers, while group b represents a fraction $1 - \theta$ of all consumers. If both firms charge the same price, all consumers in group a (group b) purchase from firm A (firm B, respectively). This entails that, if both firms charge the same price to both groups, firm A captures a larger market share.

In addition, let $F_k(x)$ denote the fraction of consumers in group $k = \{a, b\}$, with loyalty l_k less or equal to x, and

$$F_k(x) = \begin{cases} 0 & \text{if } x < 0 \\ \frac{x}{l_k} & \text{if } 0 \leq x \leq l_k \\ 1 & \text{if } x > l_k. \end{cases}$$

To understand the intuition behind this probability, consider, as an illustration, consumers in group a. If firm A charges a price premium $x = p_A - p_B$, then consumers in group a purchase from firm A if and only if $x < l_a$ (that is, if $p_A - p_B < l_a$ or $p_A < l_a + p_B$), which occurs with probability $1 - F_a(x)$. In other words, the fraction of consumers in group a who purchase from firm A is $1 - F_a(x)$ while the remaining fraction of consumers purchase from firm B, that is, $F_a(x)$. In contrast, the fraction of consumers in group b who purchase from firm A is $F_b(x)$ while those buying from firm B is $1 - F_b(x)$.

Let p_i denote the price that firm $i = \{A, B\}$ sets on group a, and let $\widetilde{p}_i$ represent the price that firm i sets on group b. Firm A's demand is then

$$D_A = \theta \left[1 - F_a(p_A - p_B)\right] + (1 - \theta) F_b(\widetilde{p}_B - \widetilde{p}_A)$$

where the first (second) term indicates the sales from group a (group b). Similarly, firm B's demand is

$$D_B = \theta F_a(p_A - p_B) + (1 - \theta) \left[1 - F_b(\widetilde{p}_B - \widetilde{p}_A)\right]$$

where, similarly, the first (second) term indicates the sales from group a (group b).

(a) *No price discrimination.* If firms cannot price discriminate, $p_i = \widetilde{p}_i$ for every firm i, show that firm A sets a strictly higher price than firm B if and only if $\theta > 1/2$.

(b) Still in the setting of part (a), write each firm's profit maximization problem and find equilibrium prices and profits.

(c) Evaluate equilibrium prices you found in part (b) at $\theta = 1/2$. Interpret your results.

(d) Evaluate equilibrium prices you found in part (b) at $\theta = 1$. Interpret your results.

(e) *Price discrimination.* When firms can price discriminate, $p_i \neq \widetilde{p}_i$ for every firm i, show that $p_A > \widetilde{p}_A$ for firm A but $p_B < \widetilde{p}_B$ for firm B.

(f) Still in the setting of part (e), write each firm's profit maximization problem and find equilibrium prices and profits.

(g) *Comparison.* Compare equilibrium prices from parts (b) and (f), to compute the price discount that firm A offers, $d_A = p_A - \widetilde{p}_A$. When positive, firm A applies a discount to consumers in group b, which Schaffer and Zhang (2000) refer to as firm A "pays to switch". In contrast, when $d_A < 0$, firm A applies a discount to consumers in its own group a, informally known as that the firm "pays to stay." Show that firm A practices "pay to stay" only when the loyalty ratio $\frac{l_b}{l_a}$ satisfies $\frac{l_b}{l_a} \geq 2$.

(h) Compute the price discount that firm B offers, $d_B = \widetilde{p}_B - p_B$, showing that it "pays to stay" only when the loyalty ratio $\frac{l_b}{l_a}$ satisfies $\frac{l_b}{l_a} < \frac{1}{2}$. Combine your results with those in part (e) to argue that it is never optimal for both firms to practice "pay to stay." Interpret.

5. **Buyer power coordination, Fumagalli and Motta (2008).**[3] Consider an industry with an incumbent firm, I, which already invested sunk cost, $F > 0$, and a potential entrant E which has not incurred this sunk cost yet. If it enters, the entrant produces the same homogeneous good as the incumbent. The potential entrant is more efficient than the incumbent, with marginal cost c_E satisfying $c_E < c_I$. Buyers have a unit demand for the good and their maximum willingness to pay is v.

   The time structure of the game is the following:

   i) At $t = 0$, N buyers call a procurement auction for the good.

   ii) At $t = 1$, the incumbent and the potential entrant simultaneously make their (public) bids to all the buyers.

   iii) At $t = 2$, each buyer observes the bids and, independently of the other buyers, decides whether to accept the incumbent's or the entrant's offer.

   iv) At $t = 3$, the incumbent fulfils all the orders it has received. The entrant observes the number of buyers who addressed it, and decides whether to actually enter the industry or not. If it does not enter, the entrant's payoff is zero.

   v) At $t = 4$, the buyers whose orders have not been fulfilled by the entrant can turn to the incumbent.

   We assume that $F > v - c_E$, i.e., a single buyer is not enough to trigger entry, and $F < N(c_I - c_E)$, i.e., entry is viable if the entrant charges a price $p \geq c_I$ and is addressed by all buyers. For which parameter conditions you can support a subgame perfect equilibrium where the potential entrant does not enter the industry? Interpret.

[3] Fumagalli, C. and M. Motta (2008) "Buyers' Miscoordination, Entry and Downstream Competition," Economic Journal, 118, pp. 1196-1222.