EconS 594 - Industrial Organization
Price Competition with Uncertain Costs[1]

1. **Price competition with uncertain costs.** Consider an industry with $n \geq 2$ firms selling a homogeneous good. Firms face linear demand $Q(p) = 1 - p$. Every firm i privately observes its marginal cost c_i, independently drawn from a uniform distribution $U \sim [0, 1]$. The lowest price charged by the competitors of firm i is $\widehat{p_{-i}} = \min\{p_1, .., p_{i-1}, p_{i+1}, .., p_n\}$.

   (a) *Demand.* Describe firm i's demand function.

   - We need to separately consider three cases:
     - When firm i sets a price p_i strictly below the lowest competing price, $\widehat{p}_{-i} = \min\{p_1, .., p_{i-1}, p_{i+1}, .., p_n\}$, firm i captures all the market, which means that its demand is given by $q_i = 1 - p_i$.
     - When firm i sets a price p_i strictly above the lowest competing price, $\widehat{p}_{-i}$, firm i makes no sales, entailing a demand $q_i = 0$.
     - Finally, when firm i's price coincides with the lowest competing price, $\widehat{p}_{-i}$, market demand is evenly shared among the m firms charging the lowest price, that is, $q_i = \frac{1-p_i}{m}$.
   - In summary, firm i's demand is

$$q_i(p_i, \widehat{p}_{-i}) = \begin{cases} 1 - p_i & \text{if } p_i < \widehat{p}_{-i}, \\ \frac{1-p_i}{m} & \text{if } p_i = \widehat{p}_{-i}, \text{ and} \\ 0 & \text{otherwise.} \end{cases}$$

   (b) *Profit Maximization.* We seek to find Bayesian Nash Equilibria (BNE) where every firm i uses a price function $p^*(c_i)$ mapping its marginal cost c_i into a price p_i. Write firm i's expected profit-maximization problem.

   - Every firm i solves

$$\max_{p_i \geq 0} \quad \underbrace{(p_i - c_i)}_{\text{Margin per unit}} \underbrace{(1 - p_i)}_{\text{Demand}} \Pr(p_i < \widehat{p}_{-i}) \tag{1}$$

     where the last term represents the probability that firm i charges a price strictly below the lowest competing price. This occurs, in particular, when firm i's price lies below each of its competitor's prices, which we write as follows

$$\begin{aligned} \Pr(p_i &< \widehat{p}_{-i}) = \Pr\left(p_i < p^*(c_1)\right) \times ... \times \Pr(p_i < p^*(c_{i-1})) &(1) \\ \times \Pr(p_i &< p^*(c_{i+1})) \times ... \times \Pr(p_i < p^*(c_n)). &(2) \end{aligned}$$

[1] Felix Munoz-Garcia, Associate Professor in Economics, Address: 103H Hulbert Hall, Washington State University, Pullman, WA 99164, USA. E-mail: fmunoz@wsu.edu.

To express each of these $n-1$ probabilities, we now claim that the price function is strictly increasing (we confirm this point below). Therefore, the probability that firm i's price lies strictly below that of firm j is

$$\begin{aligned} \Pr\left(p_i < p^*(c_j)\right) &= \Pr\left(p^{*-1}(p_i) < c_j\right) \\ &= 1 - p^{*-1}(p_i) \end{aligned}$$

where, in the first equality, we applied the inverse on both sides of the inequality; and, in the second equality, we use the property that costs are uniformly distributed, so $p^{*-1}(p_i) < c_j$ is represented by $1 - p^{*-1}(p_i)$. Intuitively, the inverse $p^{*-1}(p_j)$ denotes the marginal cost of firm j when setting price p_j and following the equilibrium price function $p^*(c_j) = p_j$, so that $c_j = p^{*-1}(p_j)$.

- Therefore, probability in (2) can be rewritten as

$$\begin{aligned} \Pr\left(p_i < \widehat{p}_{-i}\right) &= \left[1 - p^{*-1}(p_i)\right] \times ... \times \left[1 - p^{*-1}(p_i)\right] \\ &\quad \times \left[1 - p^{*-1}(p_i)\right] \times ... \times \left[1 - p^{*-1}(p_i)\right] \end{aligned}$$

  We are then ready to rewrite the expected-profit maximization problem in (1) as follows

$$\max_{p_i \geq 0} (p_i - c_i)(1 - p_i)[1 - p^{*-1}(p_i)]^{n-1}.$$

(c) Differentiate the expected-profit maximization problem with respect to price p_i.

- Differentiating expression (1) with respect to price p_i, we obtain

$$(1+c_i-2p_i)[1-p^{*-1}(p_i)]^{n-1}-(p_i-c_i)(1-p_i)(n-1)[1-p^{*-1}(p_i)]^{n-2}\frac{\partial p^{*-1}(p_i)}{\partial p_i} = 0 \tag{3}$$

  To simplify this first-order condition, recall that, in a symmetric equilibrium, $p^*(c_i) = p_i$, and that the derivative of the inverse price-setting function is the inverse of the derivative. Therefore,

$$p^{'*}(c_i)(1+c_i-2p^*(c_i))[1-c_i]^{n-1} - (p^*(c_i)-c_i)(1-p^*(c_i))(n-1)[1-c_i]^{n-2} = 0 \tag{4}$$

  Dividing equation (4) by $[1-c_i]^{n-2}$, yields

$$p^{'*}(c_i) = \frac{(n-1)(p^*(c_i)-c_i)(1-p^*(c_i))}{(1-c_i)(1+c_i-2p^*(c_i))} \tag{5}$$

(d) *Linear price function.* Suppose that this price function is linear, that is, $p(c_i) = a + bc_i$, where a and b are parameters we need to find. In this context, $p^{'*}(c_i) = b$. Assume also that $p^*(1) = 1$, which you should confirm at the end of the exercise.

- Assumption $p^*(1) = 1$ is relatively reasonable, as it means that the firm with the highest marginal cost $c_i = 1$ charges a price equal to this marginal cost. A lower price would entail losses per unit and a higher price will clearly yield no sales since the probability of another firm having the highest marginal cost too is negligible. Using assumption $p^*(1) = 1$ in the linear price function $p(c_i) = a + bc_i$, we obtain

$$a + b1 = 1,$$

or $b = 1 - a$. Therefore, the price function $p(c_i) = a + bc_i$ can be rewritten as $p(c_i) = a + (1-a)c_i$, entailing that

$$p - c_i = a(1 - c_i).$$

Our above results also imply that

$$1 - p = (1-a)(1-c_i)$$

and that

$$1 + c_i - 2p = (1-2a)(1-c_i).$$

Each of the above results is one of the terms in expression (5), so we can now insert them, where appropriate, to obtain

$$\underbrace{1-a}_{b} = \frac{(n-1)\overbrace{a(1-c_i)}^{p^*(c_i)-c_i}\overbrace{(1-a)(1-c_i)}^{1-p^*(c_i)}}{(1-c_i)\underbrace{(1-2a)(1-c_i)}_{1+c_i-2p^*(c_i)}}.$$

This expression is only a function of one unknown, a. Solving for a, yields $a = \frac{1}{n+1}$. Therefore, parameter b is

$$b = 1 - a = 1 - \frac{1}{n+1} = \frac{n}{n+1}$$

ultimately yielding a price function

$$p(c_i) = \frac{1}{n+1} + \frac{n}{n+1}c_i.$$

When $c_i = 1$, the price becomes \$1, as predicted, since

$$p(1) = \frac{1}{n+1} + \frac{n}{n+1}1 = \frac{1+n}{n+1} = \$1.$$

(e) Does firm i charge a price above its marginal cost c_i in the price function found in part (d)? Interpret.

- Graphically, price function $p(c_i)$ originates at $a = \frac{1}{n+1}$ when firm i's marginal cost is the lowest, $c_i = 0$, and increases as the firm's cost increases (as it becomes less efficient), at a rate of $\frac{n}{n+1}$.
- In other words, the price function $p(c_i)$ lies above the 45-degree line, especially when the firm is relatively efficient (low c_i). In particular, every firm i sets a price strictly above its marginal cost c_i for all $0 \leq c_i < 1$, and a price that coincides with its marginal cost when $c_i = 1$.