Vertical differentiation and natural monopoly

Consider the following model of vertically differentiated products with two firms. Every consumer when buying from firm \( i \) enjoys utility

\[ r - p_i + \theta s_i \]

where parameter \( \theta \sim U[\theta, \bar{\theta}] \) denotes how much this consumer cares about quality. Intuitively, a consumer with \( \theta = \bar{\theta} \) does not assign any concern to quality, while \( \theta = \theta \) assigns the maximal importance to quality. Assume that, for simplicity, both firms’ marginal production cost is \( c = 0 \). Consider that qualities are given, where \( s_2 > s_1 \), but firms compete in prices.

**Third stage - Finding demand.** If a consumer purchases from firm 1, his utility is \( r - p_1 + \theta s_1 \), while purchasing from firm 2 yields \( r - p_2 + \theta s_2 \). Therefore, the indifferent consumer \( \theta \) solves

\[ r - p_1 + \theta s_1 = r - p_2 + \theta s_2 \]

which yields

\[ \hat{\theta} = \frac{p_2 - p_1}{s_2 - s_1} \]

Therefore, firm 1’s demand is \( \hat{\theta} - \theta \), while firm 2’s demand is \( \bar{\theta} - \hat{\theta} \). Graphically, cutoff \( \hat{\theta} \) shifts rightward along the interval \( [\theta, \bar{\theta}] \) when the price differential \( p_2 - p_1 \) increases (meaning that firm 2 sets higher prices than firm 1), expanding the demand for firm 1 while shrinking that of firm 2. In contrast, cutoff \( \hat{\theta} \) shifts leftward when the quality differential \( s_2 - s_1 \) increases (that is, firm 2 offers a higher quality than firm 1), shrinking the demand of firm 1 and expanding that of firm 2.

**Second stage - Prices.** Firm 1 chooses the price \( p_1 \) that solves

\[
\max_{p_1} (p_1 - 0) \left( \frac{p_2 - p_1}{s_2 - s_1} - \frac{\theta}{\theta - \theta} \right) \\
\text{(Demand, } \hat{\theta} - \theta) 
\]

Differentiating with respect to \( p_1 \), we obtain

\[
\frac{\partial 
\pi_1}{\partial p_1} = \frac{p_2 - p_1}{S_2 - S_1} - \theta - \frac{p_1}{s_2 - s_1} = 0 
\]

Solving for \( p_1 \), we find firm 1’s best response function

\[
p_1(p_2) = \frac{p_2 - \theta(s_2 - s_1)}{2} 
\]

with vertical intercept at \( \frac{-\theta(s_2 - s_1)}{2} \) and slope \( \frac{p_2}{2} \). Intuitively, when firm 2 increases its price by $1, firm 1 responds increasing its own by $0.5.

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Operating similarly for firm 2, we have that this firm chooses price $p_2$ to solve

$$\max_{p_2} (p_2 - 0) \left( \frac{\bar{\theta} - p_2 - p_1}{s_2 - s_1} \right)$$

Demand, $\bar{\theta}$ - $\bar{\theta}$

Differentiating with respect to $p_2$, we obtain

$$\frac{\partial \pi_2}{\partial p_2} = \frac{p_1 - 2p_2}{S_2 - S_1} + \bar{\theta} = 0$$

Solving for $p_2$, we find firm 2’s best response function

$$p_2(p_1) = \frac{p_1 + \bar{\theta}(s_2 - s_1)}{2}$$

with vertical intercept at $\frac{\bar{\theta}(S_2 - S_1)}{2}$ and slope $\frac{\bar{\theta}}{2}$. Intuitively, when firm 1 increases its price by $1$, firm 2 responds increasing its own by $0.5$.

Simultaneously solving for $p_1$ and $p_2$ in the above best response functions, we find equilibrium prices

$$p_1^*(s_1, s_2) = \frac{(\bar{\theta} - 2\bar{\theta})(s_2 - s_1)}{3}, \text{ and}$$

$$p_2^*(s_1, s_2) = \frac{(2\bar{\theta} - \bar{\theta})(s_2 - s_1)}{3}.$$ 

Since quality is given and satisfies $s_2 > s_1$ by definition, we can claim that $p_1^*(s_1, s_2) > 0$ if and only if $\bar{\theta} > 2\bar{\theta}$. Similarly, $p_2^*(s_1, s_2) > 0$ if and only if $2\bar{\theta} > \bar{\theta}$, which can be rewritten as $\bar{\theta} > \frac{\bar{\theta}}{2}$. These two conditions entail that we can identify three regions of parameter $\bar{\theta}$:

1. When $\bar{\theta} > 2\bar{\theta}$, condition $\bar{\theta} > \frac{\bar{\theta}}{2}$ also holds, implying that both firms set positive prices. Intuitively, individuals with the highest concern for quality (those with $\theta = \bar{\theta}$) have such a high concern, relative to the individuals with the lowest concern (those with $\theta = \bar{\theta}$), that both firms can sell positive units and make a profit.

2. When $\frac{\bar{\theta}}{2} < \bar{\theta} \leq 2\bar{\theta}$, the price of firm 1 is zero while that of firm 2 is positive (i.e., only firm 2 is active). In this case, the “quality concern differential” between the individuals with the highest and lowest quality concern, $\bar{\theta} - \theta$, is lower, implying that only firm 2 (the firm with the highest quality) can make a profit.

3. When $\bar{\theta} \leq \frac{\bar{\theta}}{2}$, both firms charge a zero price (both firms are inactive). In this setting, the quality “quality concern differential” is even smaller, entailing that no firm can make a positive profit.

When parameter $\bar{\theta}$ takes intermediate values, $\frac{\bar{\theta}}{2} < \bar{\theta} \leq 2\bar{\theta}$, we can then claim that firm 2 is a “natural monopoly” since firm 1 voluntarily exits the market.