Vertical differentiation and natural monopoly[1]

Consider the following model of vertically differentiated products with two firms. Every consumer when buying from firm i enjoys utility

$$r - p_i + \theta s_i$$

where parameter $\theta \sim U[\underline{\theta}, \overline{\theta}]$ denotes how much this consumer cares about quality. Intuitively, a consumer with $\theta = \underline{\theta}$ does not assign any concern to quality, while $\theta = \overline{\theta}$ assigns the maximal importance to quality. Assume that, for simplicity, both firms' marginal production cost is $c = 0$. Consider that qualities are given, where $s_2 > s_1$, but firms compete in prices.
Third stage - Finding demand. If a consumer purchases from firm 1, his utility is $r - p_1 + \theta s_1$, while purchasing from firm 2 yields $r - p_2 + \theta s_2$. Therefore, the indifferent consumer $\widehat{\theta}$ solves

$$r - p_1 + \widehat{\theta} s_1 = r - p_2 + \widehat{\theta} s_2$$

which yields

$$\widehat{\theta} = \frac{p_2 - p_1}{s_2 - s_1}.$$

Therefore, firm 1's demand is $\widehat{\theta} - \underline{\theta}$, while firm 2's demand is $\overline{\theta} - \widehat{\theta}$. Graphically, cutoff $\widehat{\theta}$ shifts rightward along the interval $[\underline{\theta}, \overline{\theta}]$ when the price differential $p_2 - p_1$ increases (meaning that firm 2 sets higher prices than firm 1), expanding the demand for firm 1 while shrinking that of firm 2. In contrast, cutoff $\widehat{\theta}$ shifts leftward when the quality differential $s_2 - s_1$ increases (that is, firm 2 offers a higher quality than firm 1), shrinking the demand of firm 1 and expanding that of firm 2.
Second stage - Prices. Firm 1 chooses the price p_1 that solves

$$\max_{p_1} \; (p_1 - 0) \underbrace{\left(\frac{p_2 - p_1}{s_2 - s_1} - \underline{\theta} \right)}_{\text{Demand, } \widehat{\theta} - \underline{\theta}}$$

Differentiating with respect to p_1, we obtain

$$\frac{\partial \pi_1}{\partial p_1} = \frac{p_2 - p_1}{S_2 - S_1} - \underline{\theta} - \frac{p_1}{s_2 - s_1} = 0$$

Solving for p_1, we find firm 1's best response function

$$p_1(p_2) = \frac{p_2 - \underline{\theta}(s_2 - s_1)}{2}$$

with vertical intercept at $\frac{-\underline{\theta}(S_2 - S_1)}{2}$ and slope $\frac{p_2}{2}$. Intuitively, when firm 2 increases its price by \$1, firm 1 responds increasing its own by \$0.5.

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Operating similarly for firm 2, we have that this firm chooses price p_2 to solve

$$\max_{p_2} (p_2 - 0) \underbrace{\left(\overline{\theta} - \frac{p_2 - p_1}{s_2 - s_1}\right)}_{\text{Demand, } \overline{\theta} - \widehat{\theta}}$$

Differentiating with respect to p_2, we obtain

$$\frac{\partial \pi_2}{\partial p_2} = \frac{p_1 - 2p_2}{S_2 - S_1} + \overline{\theta} = 0$$

Solving for p_2, we find firm 2's best response function

$$p_2(p_1) = \frac{p_1 + \overline{\theta}(s_2 - s_1)}{2}$$

with vertical intercept at $\frac{\overline{\theta}(S_2 - S_1)}{2}$ and slope $\frac{p_1}{2}$. Intuitively, when firm 1 increases its price by \$1, firm 2 responds increasing its own by \$0.5.
Simultaneously solving for p_1 and p_2 in the above best response functions, we find equilibrium prices

$$\begin{aligned} p_1^*(s_1, s_2) &= \frac{\left(\overline{\theta} - 2\underline{\theta}\right)(s_2 - s_1)}{3}, \text{ and} \\ p_2^*(s_1, s_2) &= \frac{\left(2\overline{\theta} - \underline{\theta}\right)(s_2 - s_1)}{3}. \end{aligned}$$

Since quality is given and satisfies $s_2 > s_1$ by definition, we can claim that $p_1^*(s_1, s_2) > 0$ if and only if $\overline{\theta} > 2\underline{\theta}$. Similarly, $p_2^*(s_1, s_2) > 0$ if and only if $2\overline{\theta} > \underline{\theta}$, which can be rewritten as $\overline{\theta} > \frac{\underline{\theta}}{2}$. These two conditions entail that we can identify three regions of parameter $\overline{\theta}$:

1. When $\overline{\theta} > 2\underline{\theta}$, condition $\overline{\theta} > \frac{\underline{\theta}}{2}$ also holds, implying that both firms set positive prices. Intuitively, individuals with the highest concern for quality (those with $\theta = \overline{\theta}$) have such a high concern, relative to the individuals with the lowest concern (those with $\theta = \underline{\theta}$), that both firms can sell positive units and make a profit.

2. When $\frac{\underline{\theta}}{2} < \overline{\theta} \leq 2\underline{\theta}$, the price of firm 1 is zero while that of firm 2 is positive (i.e., only firm 2 is active). In this case, the "quality concern differential" between the individuals with the highest and lowest quality concern, $\overline{\theta} - \underline{\theta}$, is lower, implying that only firm 2 (the firm with the highest quality) can make a profit.

3. When $\overline{\theta} \leq \frac{\underline{\theta}}{2}$, both firms charge a zero price (both firms are inactive). In this setting, the quality "quality concern differential" is even smaller, entailing that no firm can make a positive profit.

When parameter $\overline{\theta}$ takes intermediate values, $\frac{\underline{\theta}}{2} < \overline{\theta} \leq 2\underline{\theta}$, we can then claim that firm 2 is a "natural monopoly" since firm 1 voluntarily exits the market.