Mergers with synergies[1]

1. **Mergers with synergies.** Consider an industry of $n \geq 2$ firms competing a la Cournot. Let us analyze the following sequential-move game where, in the first stage, every firm chooses whether or not to merge with other $k-1$ firms (so k firms merge and the remaining $n-k$ firms do not); and, in the second stage, firms compete a la Cournot. The k firms that merge coordinate their production decision to maximize their joint profits while the remaining $n-k$ firms do not coordinate. Firms face an inverse demand curve $p(Q) = 1 - Q$, where $Q \geq 0$ denotes aggregate output. Firms are symmetric in their marginal cost of production c, where $1 > c \geq 0$.

   In this setting, we allow for the k firms that merge to benefit from a cost-reduction effect, so their marginal cost of production decreases to $c - x$. In contrast, the firms that did not merge still face a marginal cost c.

   (a) As a benchmark, find the profits of every firm i before the merger.

   - If no merger occurs, every firm solves

   $$\max_{q_i \geq 0} (1 - q_i - Q_{-i})q_i - cq_i$$

   where Q_{-i} denotes the aggregate output of firm i's rivals. Differentiating with respect to q_i, and solving for q_i, we find firm i's best response function

   $$q_i(Q_{-i}) = \frac{1-c}{2} - \frac{1}{2}Q_{-i}$$

   which is negatively sloped in its rivals' aggregate output, Q_{-i}. In a symmetric equilibrium, $q_i = q_j = q$ for every two firms $i \neq j$, which entails $Q_{-i} = (N-1)q$. Therefore, the above best response function simplifies to

   $$q = \frac{1-c}{2} - \frac{1}{2}(N-1)q$$

   and, solving for q, we obtain the equilibrium output of every firm i when no merger occurs:

   $$q = \frac{1-c}{n+1}.$$

   Then, equilibrium profits become

   $$\pi^{NM} = \left(\frac{1-c}{n+1}\right)^2$$

   where the superscript NM indicates "no merger."

   (b) Find the output that the merged and unmerged firms produce.

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- After the merger, we need to solve a Cournot problem where firms face asymmetric costs. The merged entity solves

$$\max_{q_M \geq 0} (1 - q_M - Q_{-i})q_M - (c - x)q_M$$

where q_M denotes the production level of the merged entity and Q_{-i} represents the aggregate output of all outside firms that did not merge. Differentiating with respect q_M, yields

$$1 - 2q_M - Q_{-i} - c + x = 0$$

and solving for q_M, we obtain the best response function

$$q_M(Q_{-i}) = \frac{1 - c + x}{2} - \frac{1}{2}Q_{-i}.$$

- For the unmerged firms, each chooses its individual production q_i to solve

$$\max_{q_i \geq 0} (1 - q_i - q_M - Q^U_{-i})q_i - cq_i$$

where Q^U_{-i} denotes the aggregate production level of all other $(n-k) - 1$ firms that did not merge. Differentiating with respect to q_i, we find

$$1 - 2q_i - q_M - Q^U_{-i} - c = 0$$

and solving for q_M, we obtain the best response function

$$q_i(q_M, Q^U_{-i}) = \frac{1 - c}{2} - \frac{1}{2}\left(q_M + Q^U_{-i}\right)$$

At this point, we can invoke symmetry: in a symmetric equilibrium all unmerged firms produce the same output, $q_i = q_j = q_{UM}$ for all $n - k$ firms, where subscript UM denotes unmerged firms. Therefore, Q^U_{-i} can be expressed as $Q^U_{-i} = [(n-k) - 1]\, q_{UM}$. Inserting this result in the above best response function, we find that

$$q_{UM} = \frac{1 - c}{n - k + 1} - \frac{1}{n - k + 1}q_M$$

In addition, $Q_{-i} = (n - k)q_{UM}$, which inserted in the merged entity's best response function, yields

$$q_M = \frac{1 - c + x}{2} - \frac{n - k}{2}q_{UM}.$$

- Combining the above two best response functions, we can solve for the equilibrium values of q_M and q,

$$q_M = \frac{1 - c + (n - k + 1)x}{n - k + 2} \quad \text{and} \quad q_{UM} = \frac{1 - c - x}{n - k + 2}$$

(c) For which value of the cost-reduction effect, x, the unmerged firms produce a positive output?

- Setting $q_{UM} \geq 0$, we obtain

$$\frac{1-c-x}{n-k+2} \geq 0$$

rearranging, yields

$$\frac{x}{1-c} < 1.$$

Intuitively, the cost-reduction effect, x, as a percentage of per-unit margins, $1-c$, must be relatively small. Otherwise, the cost-reduction effect that the merged entity experiences forces the exit of those firms that did not merge. For compactness, let $\theta \equiv \frac{x}{1-c}$, so we can write the above inequality as $\theta < 1$.

(d) Find the profits for the merged and unmerged firms.

- Inserting the equilibrium values of q_M and q_{UM} into their corresponding profits, we obtain

$$\pi_M = \left(\frac{1-c+(n-k+1)x}{n-k+2}\right)^2, \text{ and}$$
$$\pi_{UM} = \left(\frac{1-c-x}{n-k+2}\right)^2$$

(e) For which values of n and k can a merger be sustained in equilibrium? Interpret.

- A merger between k out of n firms is profitable if the post-merger profits exceed the pre-merger profits (for all firms that merged, as a whole), that is,

$$\pi_M \geq k\pi^{NM}$$

or

$$\left(\frac{1-c+(n-k+1)x}{n-k+2}\right)^2 \geq k\left(\frac{1-c}{n+1}\right)^2$$

which we can start simplifying as

$$\frac{1-c+(n-k+1)x}{n-k+2} \geq \sqrt{k}\frac{1-c}{n+1}$$

or further rearrange as

$$\theta \equiv \frac{x}{1-c} \geq \frac{(n-k+2)\sqrt{k}-(n+1)}{(n-k+1)(n+1)} \equiv \theta_M.$$

Intuitively, for the merger to be profitable, the cost-reduction effect, as captured by θ in the left-hand side of the inequality, must be sufficiently strong (higher than θ_M, where subscript M denotes that firms merge).

- For instance, when the merger does not produce cost reduction effects, $x = 0$, the left-hand side collapses to zero, and then the merger is only profitable if $0 \geq \theta_M$, which solving for the number of firms that merge, k, yield

$$k \geq \frac{2n + 3 - \sqrt{4n + 5}}{2} \equiv \widehat{k}$$

as in our analysis of mergers between several firms (where the merger does not produce synergies). If, instead, $x > 0$, and only two firms merge, $k = 2$, condition $\theta \geq \theta_M$ becomes

$$\theta \geq \frac{\left[\left(\sqrt{2} - 1\right) n - 1\right]}{n^2 - 1} \equiv \theta_M(2, n).$$

Cutoff $\theta_M(2, n)$ lies below 0.05 for most values of n. For instance, if $n = 3$, cutoff $\theta_M(2, 3) = 0.03$, indicating that, for the merger to be profitable, the cost-reduction effect (relative to the market size) must be larger than 3%.