Mergers between several firms[1]

1. **Mergers between several firms.** Consider an industry of $n \geq 2$ firms competing a la Cournot. Let us analyze the following sequential-move game where, in the first stage, every firm chooses whether or not to merge with other $k-1$ firms (so k firms merge and the remaining $n-k$ firms do not); and, in the second stage, firms compete a la Cournot. The k firms that merge coordinate their production decision to maximize their joint profits while the remaining $n-k$ firms do not coordinate. Firms face an inverse demand curve $p(Q) = a - Q$, where $a > 0$ and $Q \geq 0$ denotes aggregate output. Firms are symmetric in their marginal cost of production c, where $a > c \geq 0$.

   (a) As a benchmark, find the profits of every firm i before the merger.

   - If no merger occurs, every firm solves

   $$\max_{q_i} \ (a - q_i - Q_{-i})q_i - cq_i$$

   where Q_{-i} denotes the aggregate output of firm i's rivals. Differentiating with respect to q_i, and solving for q_i, we find firm i's best response function

   $$q_i(Q_{-i}) = \frac{a-c}{2} - \frac{1}{2}Q_{-i}$$

   which is negatively sloped in its rivals' aggregate output, Q_{-i}. In a symmetric equilibrium, $q_i = q_j = q$ for every two firms $i \neq j$, which entails $Q_{-i} = (N-1)q$. Therefore, the above best response function simplifies to

   $$q = \frac{a-c}{2} - \frac{1}{2}(N-1)q$$

   and, solving for q, we obtain the equilibrium output of every firm i when no merger occurs:

   $$q = \frac{a-c}{n+1}.$$

   Then, equilibrium profits become

   $$\pi^{NM} = \left(\frac{a-c}{n+1}\right)^2$$

   where the superscript NM indicates "no merger."

   (b) Find the profits that every merging firm earns.

[1] Felix Munoz-Garcia, Associate Professor in Economics, Address: 103H Hulbert Hall, Washington State University, Pullman, WA 99164, USA. E-mail: fmunoz@wsu.edu.

- When k firms merge, leaving $n-k$ unmerged firms, the total number of firms becomes $(n-k)+1$. In this setting, equilibrium profits are

$$\pi^M = \left(\frac{a-c}{\underbrace{(n-k)+1}_{\text{Number of firms}} + 1} \right)^2 = \left(\frac{a-c}{n-k+2} \right)^2$$

where the superscript M indicates "merger."

(c) For which values of n and k can a merger be sustained in equilibrium? Interpret.

- A merger between k out of n firms is profitable if the post-merger profits exceed the pre-merger profits (for all firms that merged, as a whole), that is,

$$\pi^M \geq k\pi^{NM}$$

or

$$\left(\frac{a-c}{n-k+2} \right)^2 \geq k \left(\frac{a-c}{n+1} \right)^2$$

which we can start simplifying as

$$(n+1)^2 \geq k(n-k+2)^2$$

or, further rearrange as

$$(k-1)\left[-k^2 + (2n+3)k - (n+1)^2\right] \geq 0.$$

Since $k \geq 2$ by definition (a merger must include at least two firms), we can solve for k in the second term to obtain

$$k \geq \frac{2n+3-\sqrt{4n+5}}{2} \equiv \widehat{k}.$$

Intuitively, for a merger to be profitable, it must be large enough, $k \geq \widehat{k}$. Otherwise, the merger is not profitable.

- **Interpretation.** Recall the positive and negative effect of a merger on profits when firms compete a la Cournot:
  - *Positive effect.* On one hand, a merger generates a positive effect on profits since firms reduce their output when coordinating their actions, which raises prices, and thus margins.
  - *Negative effect.* On the other hand, however, this output reduction leads the unmerged $n-k$ firms to respond by increasing their output levels (since best response functions are negatively sloped in this setting), which decreases the profits of the k firms that merged.

Our above result says that the positive effect of the merger dominates the negative effect when the number of firms merging is sufficiently high, $k \geq \widehat{k}$, since in that case the negative effect from the merger is relatively low.

- An alternative presentation of the above result divides both sides of the inequality by n, to obtain

$$\frac{k}{n} \geq \frac{\widehat{k}}{n}.$$

This rearranged inequality says that, for the merger to be profitable, the market share that the merged firms represent $\frac{k}{n}$ (left-hand side) must be larger than $\frac{\widehat{k}}{n}$ (right-hand side). In particular,

$$\frac{\widehat{k}}{n} = \frac{2n + 3 - \sqrt{4n + 5}}{2n}$$

which is a ratio extremely close to 0.8. Indeed, when $n = 2$, cutoff $\frac{\widehat{k}}{n}$ becomes 0.848, when $n = 3$ this cutoff is 0.812, when $n = 4$ the cutoff is 0.802, and when $n = 5$ the cutoff becomes 0.8. Intuitively, this means that, for the merger to be profitable, at least 80% of the firms must merge; which explains why this result is informally known as the "80% rule" after Salant et al. (1983).

References

[1] Salant, S. W., Switzer, S., and Reynolds, R. J. (1983) "Losses from horizontal merger: The effects of an exogenous change in industry structure on Cournot-Nash equilibrium." The Quarterly Journal of Economics, 98(2), 185–199