1. **Mergers between several firms.** Consider an industry of \( n \geq 2 \) firms competing \( a la \) Cournot. Let us analyze the following sequential-move game where, in the first stage, every firm chooses whether or not to merge with other \( k - 1 \) firms (so \( k \) firms merge and the remaining \( n - k \) firms do not); and, in the second stage, firms compete \( a la \) Cournot. The \( k \) firms that merge coordinate their production decision to maximize their joint profits while the remaining \( n - k \) firms do not coordinate. Firms face an inverse demand curve \( p(Q) = a - Q \), where \( a > 0 \) and \( Q \geq 0 \) denotes aggregate output. Firms are symmetric in their marginal cost of production \( c \), where \( a > c \geq 0 \).

(a) As a benchmark, find the profits of every firm \( i \) before the merger.

- If no merger occurs, every firm solves

\[
\max_{q_i} (a - q_i - Q_{-i})q_i - cq_i
\]

where \( Q_{-i} \) denotes the aggregate output of firm \( i \)'s rivals. Differentiating with respect to \( q_i \), and solving for \( q_i \), we find firm \( i \)'s best response function

\[
q_i(Q_{-i}) = \frac{a - c}{2} - \frac{1}{2}Q_{-i}
\]

which is negatively sloped in its rivals' aggregate output, \( Q_{-i} \). In a symmetric equilibrium, \( q_i = q_j = q \) for every two firms \( i \neq j \), which entails \( Q_{-i} = (N - 1)q \). Therefore, the above best response function simplifies to

\[
q = \frac{a - c}{2} - \frac{1}{2}(N - 1)q
\]

and, solving for \( q \), we obtain the equilibrium output of every firm \( i \) when no merger occurs:

\[
q = \frac{a - c}{n + 1}.
\]

Then, equilibrium profits become

\[
\pi^{NM} = \left(\frac{a - c}{n + 1}\right)^2
\]

where the superscript \( NM \) indicates “no merger.”

(b) Find the profits that every merging firm earns.
• When \( k \) firms merge, leaving \( n - k \) unmerged firms, the total number of firms becomes \((n - k) + 1\). In this setting, equilibrium profits are

\[
\pi^M = \left( \frac{a - c}{(n - k) + 1 + 1} \right)^2 = \left( \frac{a - c}{n - k + 2} \right)^2
\]

where the superscript \( M \) indicates “merger.”

(c) For which values of \( n \) and \( k \) can a merger be sustained in equilibrium? Interpret.

• A merger between \( k \) out of \( n \) firms is profitable if the post-merger profits exceed the pre-merger profits (for all firms that merged, as a whole), that is,

\[
\pi^M \geq k\pi^{NM}
\]

or

\[
\left( \frac{a - c}{n - k + 2} \right)^2 \geq k \left( \frac{a - c}{n + 1} \right)^2
\]

which we can start simplifying as

\[(n + 1)^2 \geq k(n - k + 2)^2\]

or, further rearrange as

\[(k - 1) [-(k^2 + 2n + 3)k - (n + 1)^2] \geq 0.\]

Since \( k \geq 2 \) by definition (a merger must include at least two firms), we can solve for \( k \) in the second term to obtain

\[k \geq \frac{2n + 3 - \sqrt{4n + 5}}{2} = \hat{k}.\]

Intuitively, for a merger to be profitable, it must be large enough, \( k \geq \hat{k} \). Otherwise, the merger is not profitable.

• **Interpretation.** Recall the positive and negative effect of a merger on profits when firms compete a la Cournot:

  - **Positive effect.** On one hand, a merger generates a positive effect on profits since firms reduce their output when coordinating their actions, which raises prices, and thus margins.
  
  - **Negative effect.** On the other hand, however, this output reduction leads the unmerged \( n - k \) firms to respond by increasing their output levels (since best response functions are negatively sloped in this setting), which decreases the profits of the \( k \) firms that merged.

Our above result says that the positive effect of the merger dominates the negative effect when the number of firms merging is sufficiently high, \( k \geq \hat{k} \), since in that case the negative effect from the merger is relatively low.
An alternative presentation of the above result divides both sides of the inequality by $n$, to obtain
\[ \frac{k}{n} \geq \frac{\hat{k}}{n}. \]
This rearranged inequality says that, for the merger to be profitable, the market share that the merged firms represent $\frac{k}{n}$ (left-hand side) must be larger than $\frac{\hat{k}}{n}$ (right-hand side). In particular,
\[ \frac{\hat{k}}{n} = \frac{2n + 3 - \sqrt{4n + 5}}{2n} \]
which is a ratio extremely close to 0.8. Indeed, when $n = 2$, cutoff $\frac{\hat{k}}{n}$ becomes 0.848, when $n = 3$ this cutoff is 0.812, when $n = 4$ the cutoff is 0.802, and when $n = 5$ the cutoff becomes 0.8. Intuitively, this means that, for the merger to be profitable, at least 80% of the firms must merge; which explains why this result is informally known as the “80% rule” after Salant et al. (1983).

References