HOMEWORK #2, ECON 594, ANSWER KEY

4. Suppose that $t_A > t_B$. Suppose now that only firm 1 observes consumer type t and that it can condition its price on this type – i.e., firm 1 set $p_1(t)$ – whereas firm 2 has to charge the same price to all consumers. (Consumers are assumed not to be able to trade among each other). Determine the demand function faced by the two firms. Determine the equilibrium in the simultaneous-move price game.

5. Suppose that $t_A = 2 > t_B = 1$. Consider the two-stage game in which, in the first stage, firms acquire the ability to identify a consumer's type t at cost C and, in the second stage, they compete in prices. Using your insights from parts 2 to 4, characterize the subgame-perfect equilibria as a function of C.

6. Discuss your findings.

Solutions to Exercise #1

1. Standard linear Hotelling model. Set $t \equiv t_A = t_B$. Demand for firm i:

$$\frac{1}{2} - \frac{p_i - p_j}{2t}, j \neq i$$

Firm i's maximization problem is

$$\max_{p_i} p_i \left(\frac{1}{2} - \frac{p_i - p_j}{2t} \right).$$

The first-order condition is

$$\frac{1}{2} - \frac{2p_i - p_j}{2t} = 0,$$

which gives rise to the best response

$$p_i = \frac{t}{2} + \frac{p_j}{2}.$$

In symmetric equilibrium, $p_1^* = p_2^* = t$. Equilibrium demand is 1/2, and equilibrium profits are $\pi_1^* = \pi_2^* = t/2$.

2. The maximization problem now becomes

$$\max_{p_i} \frac{1}{2} p_i \left(\frac{1}{2} - \frac{p_i - p_j}{2t_A} \right) + \frac{1}{2} p_i \left(\frac{1}{2} - \frac{p_i - p_j}{2t_B} \right).$$

The first-order condition is

$$\frac{1}{2} - \frac{2p_i - p_j}{4t_A} - \frac{2p_i - p_j}{4t_B} = 0,$$

which can be rewritten as

$$2 = \frac{2p_i - p_j}{t_A} + \frac{2p_i - p_j}{t_B}.$$

In symmetric equilibrium, $p_i^* = p_j^* = p^*$. Thus, we have

$$\begin{aligned} 2 &= p^* \left(\frac{1}{t_A} + \frac{1}{t_B} \right) \\ 2 &= p^* \frac{t_A + t_B}{t_A t_B} \\ p^* &= 2 \frac{t_A t_B}{t_A + t_B}. \end{aligned}$$

Equilibrium demand is $1/2$, and equilibrium profits are $\pi_1^* = \pi_2^* = t_A t_B/(t_A + t_B)$.

3. Using the results in 1, we obtain $p_1^{A*} = p_2^{A*} = t_A$ and $p_1^{B*} = p_2^{B*} = t_B$. Equilibrium demand is $1/4$ for each consumer type t, and equilibrium profits are $\pi_1^* = \pi_2^* = (t_A + t_B)/4$.

   Profits under uniform pricing are smaller than under discriminatory pricing if

$$\begin{aligned} \frac{t_A t_B}{(t_A + t_B)} &< \frac{t_A + t_B}{4} \\ 4 t_A t_B &< t_A^2 + 2 t_A t_B + t_B^2 \\ 0 &< t_A^2 - 2 t_A t_B + t_B^2 \\ 0 &< (t_A - t_B)^2 \end{aligned}$$

   which is always satisfied. Hence firms suffer from the regulatory intervention.

4. For firm 1, the maximization problem is

$$\max_{p_1^A, p_1^B} \frac{1}{2} p_1^A \left(\frac{1}{2} - \frac{p_1^A - p_2}{2t_A} \right) + \frac{1}{2} p_1^B \left(\frac{1}{2} - \frac{p_1^B - p_2}{2t_B} \right).$$

   For firm 2, the problem is

$$\max_{p_2} \frac{1}{2} p_2 \left(\frac{1}{2} - \frac{p_2 - p_1^A}{2t_A} \right) + \frac{1}{2} p_2 \left(\frac{1}{2} - \frac{p_2 - p_1^B}{2t_B} \right).$$

   The system of first-order conditions is then

$$\begin{aligned} \frac{1}{2} - \frac{2p_1^A - p_2}{2t_A} &= 0 \\ \frac{1}{2} - \frac{2p_1^B - p_2}{2t_B} &= 0 \\ \frac{1}{2} - \frac{2p_2 - p_1^A}{4t_A} - \frac{2p_2 - p_1^B}{4t_B} &= 0 \end{aligned}$$

   The first two equations can be written as $p_1^A = \frac{t_A}{2} + \frac{p_2}{2}$ and $p_1^B = \frac{t_B}{2} + \frac{p_2}{2}$. Substituting these expressions in the first-order condition of firm 2, we obtain that, in equilibrium,

$$
\begin{aligned}
\frac{1}{2}-\frac{2p_2^*-\left(\frac{t_A}{2}+\frac{p_2^*}{2}\right)}{4t_A}-\frac{2p_2^*-\left(\frac{t_B}{2}+\frac{p_2^*}{2}\right)}{4t_B} &= 0\\
\frac{1}{2}-\frac{3p_2^*-t_A}{8t_A}-\frac{3p_2^*-t_B}{8t_B} &= 0\\
\frac{3}{4}-\frac{3p_2^*}{8t_A}-\frac{3p_2^*}{8t_B} &= 0\\
1-\frac{p_2^*}{2t_A}-\frac{p_2^*}{2t_B} &= 0\\
\frac{1}{2}p_2^*\frac{t_A+t_B}{t_At_B} &= 1\\
p_2^* &= \frac{2t_At_B}{t_A+t_B}
\end{aligned}
$$

Substituting this equilibrium price in the best-response function of firm 1, we obtain

$$
\begin{aligned}
p_1^{A*} &= \frac{t_A}{2}+\frac{t_At_B}{t_A+t_B}\\
&= \frac{t_A^2+3t_At_B}{2(t_A+t_B)}
\end{aligned}
$$

and

$$
p_1^{B*}=\frac{t_B^2+3t_At_B}{2(t_A+t_B)}.
$$

Equilibrium demand for firm 1 among consumers of type t_A is

$$
\begin{aligned}
&\frac{1}{2}-\frac{p_1^{A*}-p_2^*}{2t_A}\\
=\ &\frac{1}{2}-\frac{1}{2t_A}\left(\frac{t_A^2+3t_At_B}{2(t_A+t_B)}-\frac{2t_At_B}{t_A+t_B}\right)\\
=\ &\frac{1}{2}-\frac{1}{2t_A}\frac{t_A^2-t_At_B}{2(t_A+t_B)}\\
=\ &\frac{1}{2}-\frac{1}{4}\frac{t_A-t_B}{t_A+t_B}
\end{aligned}
$$

Similarly, its demand among consumers of type t_B is

$$
\frac{1}{2}-\frac{1}{4}\frac{t_B-t_A}{t_A+t_B}.
$$

Hence, firm 1's equilibrium profit is

$$\begin{aligned}
& \frac{1}{2}\frac{t_A^2+3t_At_B}{2(t_A+t_B)}\left(\frac{1}{2}-\frac{1}{4}\frac{t_A-t_B}{t_A+t_B}\right)+\frac{1}{2}\frac{t_B^2+3t_At_B}{2(t_A+t_B)}\left(\frac{1}{2}-\frac{1}{4}\frac{t_B-t_A}{t_A+t_B}\right) \\
= & \frac{1}{16}\frac{t_A^2+3t_At_B}{t_A+t_B}\left(2-\frac{t_A-t_B}{t_A+t_B}\right)+\frac{1}{16}\frac{t_B^2+3t_At_B}{t_A+t_B}\left(2-\frac{t_B-t_A}{t_A+t_B}\right) \\
= & \frac{1}{16}\frac{t_A^2+3t_At_B}{(t_A+t_B)^2}(2(t_A+t_B)-(t_A-t_B))+\frac{1}{16}\frac{t_B^2+3t_At_B}{(t_A+t_B)^2}(2(t_A+t_B)-(t_B-t_A)) \\
= & \frac{1}{16}t_A\frac{(t_A+3t_B)^2}{(t_A+t_B)^2}+\frac{1}{16}t_B\frac{(t_B+3t_A)^2}{(t_A+t_B)^2} \\
= & \frac{1}{16(t_A+t_B)^2}(t_A(t_A+3t_B)^2+t_B(t_B+3t_A)^2) \\
= & \frac{1}{16(t_A+t_B)^2}(t_A[(t_A+t_B)^2+4t_B^2+4t_B(t_A+t_B)]+t_B(t_B+3t_A)^2)
\end{aligned}$$

Firm 2's equilibrium profit is

$$\begin{aligned}
& \frac{t_At_B}{t_A+t_B}\left[\left(\frac{1}{2}+\frac{1}{4}\frac{t_A-t_B}{t_A+t_B}\right)+\left(\frac{1}{2}+\frac{1}{4}\frac{t_B-t_A}{t_A+t_B}\right)\right] \\
= & \frac{t_At_B}{t_A+t_B}\left[1+\frac{1}{4}\frac{t_A-t_B}{t_A+t_B}+\frac{1}{4}\frac{t_B-t_A}{t_A+t_B}\right] \\
= & \frac{t_At_B}{t_A+t_B}
\end{aligned}$$

5. Denote equilibrium profits as a function of the regimes U or D for each of the two firms. Using parameter values $t_A = 2$ and $t_B = 1$, we have

$$\begin{aligned}
\pi_1^*(U,U) &= \pi_2^*(U,U) = t_At_B/(t_A+t_B) = 2/3 \\
\pi_1^*(D,D) &= \pi_2^*(D,D) = (t_A+t_B)/4 - C = 3/4 - C \\
\pi_1^*(D,U) &= \pi_2^*(U,D) = \frac{1}{16(t_A+t_B)^2}(t_A(t_A+3t_B)^2+t_B(t_B+3t_A)^2) - C \\
&= \frac{1}{16\times 9}(50+49) - C = \frac{11}{16} - C \\
\pi_1^*(U,D) &= \pi_2^*(D,U) = t_At_B/(t_A+t_B) = 2/3
\end{aligned}$$

We note that (U,U) is an equilibrium if $\pi_1^*(U,U) \geq \pi_1^*(D,U)$ which is equivalent to $C \geq 1/48$. Furthermore, (D,D) is an equilibrium if $\pi_1^*(D,D) \geq \pi_1^*(U,D)$ which is equivalent to $C \leq 1/12$. Hence, there is a unique equilibrium (D,D) for $C < 1/48$, (U,U) and (D,D) are equilibria for $C \in [1/48, 1/12]$, and (U,U) is the unique equilibrium if $C > 1/12$.

6. If it is not too costly, firms will opt to acquire information on consumer types t. With discrimination their profits will be larger than using a uniform price that applies to both consumer groups.

Exercise 7 *Price discrimination in duopoly with product returns*

Consider a duopoly market with two firms and a continuum of consumers. Each firm $i \in \{1,2\}$ sells its product at price p_i and incurs marginal costs of production equal to zero. Consumers are of measure 1 and have unit demand. With the purchase of one unit of product i a consumer of type x obtains utility $r - t|x - l_i| - p_i$ where l_i is the location of firm i and p_i is its price; if she does not buy her utility is set equal to $-\infty$. Half of consumers belong to the group that never returns a product—we call them "easy" consumers—and the other half ask for the replacement of the product with some probability, which firms have to provide—we call those consumers the "difficult" ones. The expected cost of selling to a consumer in this second group is $c > 0$. It is assumed to be independent of type x. Within each group, consumers are uniformly distributed on the unit interval, $x \in [0,1]$. Firms are located at 0 and 1, respectively.

1. Determine the equilibrium in the simultaneous-move price game in which firms have to set a uniform price to all consumers. Report equilibrium prices, outputs, and profits.
2. Suppose that firms have access to consumer data that allows them to perfectly infer whether a consumer is easy or difficult; no information on x is available. Determine the equilibrium in the simultaneous-move price game in which each firm i sets a price p_i^E to easy consumers and p_i^D to difficult consumers. Report equilibrium prices, outputs, and profits.
3. Suppose that only firm 1 has access to consumer data that allows it to perfectly infer whether a consumer is easy or difficult—this is common knowledge among firms. Determine the equilibrium in the simultaneous-move price game in which firm 1 sets prices (p_1^D, p_1^E) and firm 2 a uniform price p_2. Report equilibrium prices and profits.
4. Consider the two-stage game in which firms, in the first stage, firms can acquire the ability to identify whether consumers are easy or difficult at cost C and in which, in the second stage, firms compete in prices. Using your insights from parts 1 to 3, characterize the subgame-perfect equilibria as a function of C.
5. Suppose that a third party controls the personal data about whether a consumer is easy or difficult. What access price to those data would it set at a prior stage? In the corresponding three-stage game, will one or both firms acquire information? Discuss your findings.

Solutions to Exercise #2

1. Linear Hotelling model. Demand for firm i:

$$\frac{1}{2} - \frac{p_i - p_j}{2t}, j \neq i$$

Firm i's maximization problem is

$$\begin{aligned} & \max_{p_i} \frac{1}{2} p_i \left(\frac{1}{2} - \frac{p_i - p_j}{2t} \right) + \frac{1}{2} (p_i - c) \left(\frac{1}{2} - \frac{p_i - p_j}{2t} \right) \\ = & \max_{p_i} \left(p_i - \frac{1}{2} c \right) \left(\frac{1}{2} - \frac{p_i - p_j}{2t} \right). \end{aligned}$$

The first-order condition is

$$\frac{1}{2} - \frac{2 + p_i - p_j - c/2}{2t} = 0,$$

which gives rise to the best response

$$p_i = \frac{t}{2} + \frac{p_j}{2} + \frac{c}{4}.$$

In symmetric equilibrium, $p_1^* = p_2^* = t + c/2$. Equilibrium demand is $1/2$, and equilibrium profits are $\pi_1^* = \pi_2^* = t/2$.

2. The maximization problem now becomes

$$\max_{p_i^D, p_i^E} \frac{1}{2} p_i^E \left(\frac{1}{2} - \frac{p_i^E - p_j^E}{2t} \right) + \frac{1}{2} (p_i^D - c) \left(\frac{1}{2} - \frac{p_i^D - p_j^D}{2t} \right).$$

Thus, in equilibrium, $p_i^{E*} = t$ and $p_i^{D*} = t + c$. Equilibrium demand is $1/4$ for each firm for each consumer group, and equilibrium profits are $\pi_1^* = \pi_2^* = t/2$.

3. For firm 1, the maximization problem is

$$\max_{p_1^D, p_1^E} \frac{1}{2} p_1^E \left(\frac{1}{2} - \frac{p_1^E - p_2}{2t} \right) + \frac{1}{2} \left(p_1^D - c \right) \left(\frac{1}{2} - \frac{p_1^D - p_2}{2t} \right).$$

For firm 2, the problem is

$$\max_{p_2} \frac{1}{2} p_2 \left(\frac{1}{2} - \frac{p_2 - p_1^E}{2t} \right) + \frac{1}{2} \left(p_2 - c \right) \left(\frac{1}{2} - \frac{p_2 - p_1^D}{2t} \right).$$

The system of first-order conditions is then

$$\begin{aligned} \frac{1}{2} - \frac{2p_1^A - p_2}{2t_A} &= 0 \\ \frac{1}{2} - \frac{2p_1^B - p_2}{2t_B} &= 0 \\ \frac{1}{2} - \frac{2p_2 - p_1^A}{4t_A} - \frac{2p_2 - p_1^B}{4t_B} &= 0 \end{aligned}$$

Solving the first-order conditions, we obtain $p_1^{E*} = t + c/4$, $p_1^{D*} = t + 3c/4$, and $p_2^* = t + c/2$. Equilibrium profits are

$$\begin{aligned} \pi_1^* &= \frac{t}{2} + \frac{c^2}{32t} \\ \pi_2^* &= \frac{t}{2} - \frac{c^2}{16t} \end{aligned}$$

4. For $C < \frac{c^2}{32t}$, each firm has a strict incentive to aquire data access independent of the decision of the other firm. Thus, aquiring data access is a dominant strategy for each firm. The game at the first stage features a prisoner's dilemma. For $C > \frac{c^2}{32t}$, no firm aquires data access. For $C = \frac{c^2}{32t}$, there are two pure-strategy equilibria: either both firms aquire data access or none does.

5. The third party sells data access at a price equal to $c^2/(32t)$. In this simple model, consumer welfare is not affected by the availability of personal data. The sale of personal data by a third party shifts rents from firms to this third pary. Difficult consumers are worse off when the sale of personal data is allowed, while easy consumers are better off.

Exercise 8 *Personalized pricing*

Consider a monopoly internet retailer who sells a single (digital) product at zero marginal costs. Consumers have unit demand and heterogeneous valuations u for this product; the value of the outside option is equal to zero. Valuations u are distributed on the interval $[0, 1]$, according to some continuous cumulative distribution function. Unless a consumer protects her personal data at cost $\varepsilon > 0$, the monopoly internet retailer can infer the consumer valuation perfectly and offer a personalized price; by assumption, arbitrage among consumers is not possible. If a consumer protects her personal data, the monopolist does not learn the valuation of this consumer.

Consider the following timing: First, each consumer decides whether to protect her personal data; second, the monopolist sets a uniform price to all consumers who protect their personal data and a personalized price for each consumer whose valuation is known to the monopolist; third, consumers make purchase decisions.

1. Characterize the subgame-perfect equilibrium in this setting. In particular, what are the privacy choices of consumers (their decision whether to protect their data) and what are the prices set by the monopolist? Is there a unique equilibrium? [You may want to start with the uniform distribution and then extend your analysis to non-uniform distributions.]

2. Suppose that u is uniformly distributed on $[0, 1]$. Suppose, furthermore, that, at stage 1, consumers think that the monopolist will charge the unconditional monopoly price; i.e., $p^m = 1/2$. For ε negligibly small, what is the outcome when consumers hold these possibly irrational beliefs at stage 1 about price in case they hide their valuation, but are otherwise rational (in particular, regarding the personalized price they receive if they do not protect their personal data)? In particular, what are the privacy choices of consumers and what are the prices set by the monopolist?

3. Suppose that u is uniformly distributed on $[0, 1]$. Calculate consumer surplus and monopoly profit in the two cases above. For ε small, comment on your findings concerning the comparison of profits and the comparison of consumer surplus.

#3

Solution to Exercise 10

1. $\max_{q_i}(q_i - c)(1 - q_i - \gamma \sum_{j \neq i} q_j)$

   FOC:

$$1 - 2q_i - \gamma \sum_{j \neq i} q_j - c = 0.$$

   Symmetric equilibrium $q^* = q_1^* = ... = q_4^*$:

$$\begin{aligned}
1 - 2q^* - \gamma(n-1)q^* - c &= 0 \\
[2 + \gamma(n-1)]q^* &= 1 - c \\
q^* &= \frac{1-c}{2+\gamma(n-1)} \\
p_i(q^*, ..., q^*) &= 1 - [1 + \gamma(n-1)]q^* \\
&= 1 - \frac{[1+\gamma(n-1)](1-c)}{2+\gamma(n-1)} \\
&= 1 - \frac{[2+\gamma(n-1)](1-c) - (1-c)}{2+\gamma(n-1)} \\
&= c + \frac{1-c}{2+\gamma(n-1)} \\
\pi_i^* &= \frac{(1-c)^2}{[2+\gamma(n-1)]^2}
\end{aligned}$$

2. Merger of firms with products 1 and 2:

$$max_{q_1,q_2}(q_1 - c)(1 - q_1 - \gamma \sum_{j \neq 1} q_j) + (q_2 - c)(1 - q_2 - \gamma \sum_{j \neq 2} q_j)$$

   FOC w.r.t. q_1:

$$1 - c - 2q_1 - 2\gamma q_2 - \gamma q_3 - \gamma q_4 = 0$$

   In equilibrium, $q_1 = q_2 = q^m$ and $q_3 = q_4 = q$:

$$1 - c - 2(1+\gamma)q^m - 2\gamma q = 0$$

   which is equivalent to

$$\begin{aligned}
1 - c - 2\gamma q &= 2(1+\gamma)q^m \\
q^m &= \frac{1 - c - 2\gamma q}{2(1+\gamma)}
\end{aligned}$$

   For $\gamma = 1/2$, we can write

$$q^m = \frac{1 - c - q}{3}$$

   FOC w.r.t. q_3:

$$1 - c - 2q_3 - \gamma q_2 - \gamma q_3 - \gamma q_4 = 0$$

In equilibrium,

$$1 - c - (2+\gamma)q - 2\gamma q^m = 0$$

which is equivalent to

$$\begin{aligned} 1 - c - 2\gamma q^m &= (2+\gamma)q \\ q &= \frac{1 - c - 2\gamma q^m}{2+\gamma} \end{aligned}$$

For $\gamma = 1/2$, we can write

$$q^m = \frac{2}{5}(1 - c - q)$$

Solving this linear system in two equations for $\gamma = 1/2$:

$$\begin{aligned} q &= \frac{2}{5}\left(1 - c - \frac{1-c-q}{3}\right) \\ &= \frac{2}{5}\left(\frac{2}{3}(1-c) + \frac{q}{3}\right) \\ &= \frac{2}{15}q + \frac{4}{15}(1-c) \\ \frac{13}{15}q &= \frac{4}{15}(1-c) \\ q &= \frac{4}{13}(1-c) \end{aligned}$$

$$\begin{aligned} q^m &= \frac{1 - c - \frac{4}{13}(1-c)}{3} \\ &= \frac{1}{3}\frac{9}{13}(1-c) \\ &= \frac{3}{13}(1-c) \end{aligned}$$

Price for product 1: $p_1 = 1 - \frac{6}{26}(1-c) - \frac{4}{26}(1-c) - \frac{4}{26}(1-c) - \frac{3}{26}(1-c) = \frac{9}{26} + \frac{17}{26}c = c + \frac{9}{26}(1-c)$.

Price for product 3: $p_3 = 1 - \frac{8}{26}(1-c) - \frac{4}{26}(1-c) - \frac{3}{26}(1-c) - \frac{3}{26}(1-c) = \frac{8}{26} + \frac{18}{26}c = c + \frac{8}{26}(1-c)$ which is less than p_1.

Profit of merged firm: $2\left(\frac{9}{26}(1-c)\right)\left(\frac{3}{13}(1-c)\right) = \frac{27}{169}(1-c)^2$.

Profit of firms 3 and 4: $\frac{16}{169}(1-c)^2$.

3. We are now in a position to compare profits of the merged firm to those under single-product oligopoly. Profit of each firm in the latter case is $\frac{(1-c)^2}{[2+\gamma(n-1)]^2} = (1-c)^2/(2+3/2)^2 = 4(1-c)^2/49$. Its price is $c + 2(1-c)/7$.

   The merged firm makes lower profit since $\frac{27}{169} \approx 0.1598 < \frac{8}{49} \approx 0.1632$.

4. In contrast to a homogeneous good linear Cournot oligopoly, under product differentiation a two-firm merger keeps its two products. As in the homogeneous good case, it reduces output. When quantities are strategic substitutes (as is the case in the present model) competitors increase their output, which negatively affects the profit of the merged firm. As a result, the two-firm merger is unprofitable (absent cost savings). The strategic response (i.e., the increase of the competitors quantity in response to a reduction in quantity by the merged firm) becomes less pronounced as γ decreases.

5. Prices for all products are higher under a two-firm merger. Hence, consumer surplus would be lower after the two-firm merger. However, the two-firm merger is unprofitable (unless there are some fixed-cost savings). If it is consumed it is anti-competitive (in terms of consumer welfare). Without any cost savings it is also total-surplus decreasing.

Exercise 11 *Cournot mergers and demand*

Consider the following Cournot merger game. In a homogeneous product market each firm i, with $i \in \{1, ..., n\}$ has constant marginal costs of production c and produces quantity q_i. The inverse demand function is of the form

$$P(q) = a - q^\gamma$$

where $a > 0$, $\gamma > 0$, and $q = \sum_{i=1}^n q_i$ is the total quantity.

1. Discuss the shape of the inverse demand function depending on γ.
2. Determine the Cournot equilibrium profits in this set-up. [You can simply use the first-order conditions without checking whether the solution is a maximizer.]
3. Consider a merger among x firms in an n-firm industry. Write down the condition for the merger to be profitable. Check whether a 3-firm merger is profitable in an industry with initially 4 firms for parameter values $\gamma \in \{1, 2, 3\}$.

Solution to Exercise 11

1. For $\gamma = 1$ demand is linear, for $\gamma < 1$ demand is strictly convex and, for $\gamma > 1$, strictly concave. Demand moves northeast when γ is increased.
2. $\max_{q_i} q_i(a - (q_i + q_{-i})^\gamma - c)$

   The first-order condition of profit maximization is

$$\begin{aligned} a - c &= (q_i + q_{-i})^\gamma + \gamma q_i (q_i + q_{-i})^{\gamma-1} \\ &= (q_i + q_{-i})^{\gamma-1}((1+\gamma) q_i + q_{-i}). \end{aligned}$$