EconS 594 - Industrial Organization
Homework #2 - Due date: Thursday, September 26th, in class.

1. **Price discrimination in duopoly.** Consider a duopoly market with two firms and a continuum of consumers. Each firm $i \in \{1, 2\}$ sells its product at price p_i and incurs marginal costs equal to zero. Consumers are of measure 1 and have unit demand. When buying one unit of product i a consumer of type (t, x) obtains utility

$$r - t|x - l_i| - p_i$$

where l_i is the location of firm i and p_i is its price; if she does not buy her utility is set equal to $-\infty$. Half of consumers belong to the group with type t_A and half of consumers to the other group with type t_B, where $t_A \geq t_B$. Within each group, consumers are uniformly distributed on the unit interval, $x \in [0, 1]$. Firms are located at 0 and 1, respectively.

   (a) Suppose that $t_A = t_B$. Determine the demand function faced by the two firms. Determine the equilibrium in the simultaneous-move price game. Report equilibrium prices, outputs, and profits.

   (b) Suppose that $t_A > t_B$. Determine the demand function faced by the two firms. Determine the equilibrium in the simultaneous-move price game. Report equilibrium prices, outputs, and profits.

   (c) Suppose that $t_A > t_B$. Suppose furthermore that both firms observe consumer type t and that they can condition their price on this type; i.e., firm i sets $p_i(t)$. (Consumers are assumed not to be able to trade among each other). Determine the demand function faced by the two firms. Determine the equilibrium in the simultaneous-move price game. Compare your result to the previous setting. Discuss whether firms benefit from regulation that requires them to set uniform prices.

   (d) Suppose that $t_A > t_B$. Suppose now that only firm 1 observes consumer type t and that it can condition its price on this type —i.e., firm 1 set $p_1(t)$— whereas firm 2 has to charge the same price to all consumers. (Consumers are assumed not to be able to trade among each other). Determine the demand function faced by the two firms. Determine the equilibrium in the simultaneous-move price game.

   (e) Suppose that $t_A = 2 > t_B = 1$. Consider the two-stage game in which, in the first stage, firms acquire the ability to identify a consumer's type t at cost C and, in the second stage, they compete in prices. Using your insights from parts 2 to 4, characterize the subgame-perfect equilibria as a function of C.

   (f) Discuss your findings.

2. **Price discrimination in duopoly with product returns.** Consider a duopoly market with two firms and a continuum of consumers. Each firm $i \in \{1, 2\}$ sells its product at price p_i and incurs marginal costs of production equal to zero. Consumers

are of measure 1 and have unit demand. With the purchase of one unit of product i, a consumer of type x obtains utility

$$r - t|x - l_i| - p_i$$

where l_i is the location of firm i and p_i is its price; if she does not buy her utility is set equal to $-\infty$. Half of consumers belong to the group that never returns a product— we call them "easy" consumers— and the other half ask for the replacement of the product with some probability, which firms have to provide— we call those consumers the "difficult" ones. The expected cost of selling to a consumer in this second group is $c > 0$. It is assumed to be independent of type x. Within each group, consumers are uniformly distributed on the unit interval, $x \in [0, 1]$. Firms are located at 0 and 1, respectively.

(a) Determine the equilibrium in the simultaneous-move price game in which firms have to set a uniform price to all consumers. Report equilibrium prices, outputs, and profits.

(b) Suppose that firms have access to consumer data that allows them to perfectly infer whether a consumer is easy or difficult; no information on x is available. Determine the equilibrium in the simultaneous-move price game in which each firm i sets a price p_i^E to easy consumers and p_i^D to difficult consumers. Report equilibrium prices, outputs, and profits.

(c) Suppose that only firm 1 has access to consumer data that allows it to perfectly infer whether a consumer is easy or difficult— this is common knowledge among firms. Determine the equilibrium in the simultaneous-move price game in which firm 1 sets prices (p_1^D, p_1^E) and firm 2 a uniform price p_2. Report equilibrium prices and profits.

(d) Consider the two-stage game in which firms, in the first stage, firms can acquire the ability to identify whether consumers are easy or difficult at cost C and in which, in the second stage, firms compete in prices. Using your insights from parts 1 to 3, characterize the subgame-perfect equilibria as a function of C.

(e) Suppose that a third party controls the personal data about whether a consumer is easy or difficult. What access price to those data would it set at a prior stage? In the corresponding three-stage game, will one or both firms acquire information? Discuss your findings.

3. **Cournot mergers with differentiated products.** Consider an n-firm Cournot oligopoly in which each firm i faces inverse demand

$$P_i(q_1, q_2, ..., q_n) = 1 - q_i - \gamma q_j$$

where $\gamma \in (0, 1)$. Suppose that all firms have constant marginal costs of production $c = 0$.

(a) Determine equilibrium quantities, prices and profits.

(b) Suppose that $n = 4$ and $\gamma = 1/2$. Determine equilibrium quantities, prices and profits in the case of a two-firm merger.

(c) Suppose that $n = 4$ and $\gamma = 1/2$. Determine whether a two-firm merger is profitable.

(d) Describe the forces at play when considering the profitability of the merger. What is the role of parameter γ?

(e) Discuss the welfare effects of the merger.