Can mergers facilitate collusion?[1]

1. **Can mergers facilitate collusion?** Consider an industry with $n \geq 2$ firms producing an homogeneous good at the same constant marginal cost c and competing in prices. For simplicity, assume that demand is perfectly price-inelastic: all consumers have a unit demand and the same reservation price r, where $r > c$. So, for any price below r, firms can sell an aggregate output of q.

   (a) *No merger.* If firms do not merge, find the minimal discount factor sustaining collusion and label it δ_{pre} where the subscript *pre* indicates "pre-merger."

   - *Collusion.* When firms collude, they can charge monopoly price $p_m = r$, earning collusive profit

   $$\pi^{Col} = \left((r-c)\frac{q}{n}\right) + \delta\left((r-c)\frac{q}{n}\right) + ... = \frac{1}{1-\delta}(r-c)\frac{q}{n}$$

     where, in each period, $r - c$ denotes the firm's per-unit margin, and $\frac{q}{n}$ represents its individual sales.

   - *Deviation.* If a firm, instead, unilaterally deviates from this collusive agreement, it charges a deviating price p^{Dev} that slightly undercuts the collusive price, that is, $p^{Dev} = r - \varepsilon$, where $\varepsilon > 0$. Since ε can be be made arbitrarily close to zero, $\varepsilon \to 0$, the deviating price can be considered, for practical reasons, $p^{Dev} = r$, entailing a deviating profit of $\pi^{Dev} = (r-c)\,q$ since the deviating firm captures all the market, selling q units.
   - *Punishment.* After the deviation is detected, all firms punish such defection setting equilibrium prices under the unrepeated version of the game (Bertrand competition), that is, $p = c$, yielding zero profits thereafter.
   - Putting together our above results, we can say that collusion is sustained if

   $$\frac{1}{1-\delta}(r-c)\frac{q}{n} \geq (r-c)q + \frac{\delta}{1-\delta}0 \qquad ((1))$$

     and, after solving for discount factor δ, we find

   $$\delta \geq 1 - \frac{1}{n} \equiv \delta_{pre}.$$

     Cutoff δ_{pre} satisfies $\delta_{pre} > 0$ since $n \geq 2$ by assumption, and $\delta_{pre} < 1$ since $1 - \frac{1}{n} = \frac{n-1}{n} < 1$ simplifies to $n - 1 < n$, which always holds.

   (b) *Merger - Outsiders.* Assume now that k firms merge (where k satisfies $2 \leq k < n$). Because of synergies, merging firms benefit from a lower marginal cost of production, namely $c - x$, where $x \geq 0$ can be interpreted as the cost-reducing effect of the merger. Find the minimal discount factor sustaining collusion for outsiders, label it $\delta_{outsider}$, and compare it against the minimal discount factor before the merger, δ_{pre}.

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- Recall that k firms merge, $n-k$ firms are outsiders, implying that the total number of firms after the merger is $n-k+1$. Outsiders keep their marginal production cost at c (that is, they do not benefit from the cost-reducing effects of the merger).
- *Collusion.* Colluding, every outsider earns discounted profits $\frac{1}{1-\delta}(r-c)\frac{q}{n-k+1}$, since the number of firm after the merger is $n-k+1$. Intuitively, outsiders make the same per-unit margin as before the merger, but sales are distributed among less firms ($n-k+1$ rather than n).
- *Deviation.* If an outsider deviates, it charges a deviating price $p^{Dev}=r$ (as in part a) earning the same deviating profits as in part (a), that is, $\pi^{Dev}=(r-c)\,q$.
- *Punishment.* Finally, during the punishment stage, all firms revert to the Nash equilibrium of the unrepeated Bertrand competition game, $p=c$, yielding zero profits thereafter.
- Combining our above results, outsiders are willing to collude if

$$\frac{1}{1-\delta}(r-c)\frac{q}{n-k+1} \geq (r-c)q + \frac{\delta}{1-\delta}0 \qquad ((2))$$

Comparing expression (2) against (1), we can see that outsiders have stronger incentives to collude, since the left-hand side of (2) is larger than that of (1), while the right-hand sides coincide in both expressions. After solving for discount factor δ, we find

$$\delta \geq 1 - \frac{1}{n-k+1} \equiv \delta_{outsider}$$

where, as predicted, $\delta_{outsider} < \delta_{pre}$. Intuitively, if a firm has incentives to collude before the merger, it must still have incentives to collude after the merger when it is an outsider.

(c) *Merger - Outsiders.* Find now the minimal discount factor sustaining collusion for insiders, label it $\delta_{insider}$, and compare it against the minimal discount factor before the merger, δ_{pre}.

- *Collusion.* Colluding, the merged entity (the combination of k firms) earns discounted profits

$$\frac{1}{1-\delta}\underbrace{[r-(c-x)]}_{\text{per-unit margin}}\frac{q}{n-k+1},$$

since merged firms see their marginal production cost decrease to $c-x$, which increases their per-unit margin from $r-c$ before the merger to $r-(c-x)$ after the merger. In addition, the number of firm after the merger is $n-k+1$, increasing sales relative to pre-merger levels.
- *Deviation.* If the merged entity deviates, it charges a deviating price $p^{Dev}=r$ (as in part a) earning deviating profits $\pi^{Dev}=[r-(c-x)]\,q$.
- *Punishment.* Finally, during the punishment stage, all firms revert to the Nash equilibrium of the unrepeated Bertrand competition game, $p=c$. Such

price profile, however, does not yield zero profits for the merged entity, since its marginal costs are lower, $c-x$. In particular, its profits in each period of the punishment phase are $[c-(c-x)]q = xq$.

- Combining our above results, the merged entity colludes if

$$\frac{1}{1-\delta}[r-(c-x)]\frac{q}{n-k+1} \geq [r-(c-x)]\,q + \frac{\delta}{1-\delta}xq \qquad ((3))$$

Comparing expression (3) against (1), we can see that the profits that the merged entity earns from colluding (left-hand side) are larger after the merger, but the deviation profit and the profit during the punishment phase is also larger after the merger. As a consequence, we cannot unambigously rank whether the merged entity colludes under larger conditions before or after the merger. Simplifying expression (3), we obtain

$$\begin{aligned}[r-(c-x)]\frac{q}{n-k+1} &\geq (1-\delta)[r-(c-x)] + \delta x \\ &= [r-(c-x)] - \delta(r-c)\end{aligned}$$

and solving for discount factor δ, we find

$$\delta \geq \left(\frac{n-k}{n-k+1}\right)\left(\frac{r-c+x}{r-c}\right) \equiv \delta_{insider}$$

- Cutoff $\delta_{insider}$ is positive, $\delta_{insider} > 0$, since $n \geq k$ and $r > c$ by assumption. In addition, cutoff $\delta_{insider}$ satisfies $\delta_{insider} \leq 1$ when

$$(r-c) + x \leq \frac{n-k+1}{n-k}(r-c).$$

Solving for ratio $\frac{x}{r-c}$, as in our previous analysis of mergers with synergies, yields

$$\theta \equiv \frac{x}{r-c} \leq \frac{1}{n-k} \equiv \overline{\theta}.$$

- *Comparing cutoffs.* We can now compare the merged entity's minimal discount factor supporting collusion before the merger, δ_{pre}, and after the merger, $\delta_{insider}$. Since collusion is sustained when $\delta \geq \delta_{pre}$ before the merger and when $\delta \geq \delta_{insider}$ after the merger, we can claim that the merger facilitates collusion if

$$\delta_{insider} \leq \delta_{pre}.$$

Comparing these cutoffs, we obtain

$$\left(\frac{n-k}{n-k+1}\right)\left(\frac{r-c+x}{r-c}\right) \leq 1 - \frac{1}{n}.$$

Solving for ratio $\frac{x}{r-c}$, as in our previous analysis of mergers with synergies, yields

$$\theta \equiv \frac{x}{r-c} \leq \frac{k-1}{n(n-k)} \equiv \widehat{\theta}.$$

In a line representing the cost-reduction effect from the merger, θ, cutoff $\widehat{\theta}$ divides the line into two regions: one to the left of cutoff $\widehat{\theta}$ where mergers facilitate collusion, and one to the right of cutoff $\widehat{\theta}$ where mergers hinder collusion.

- *Remark:* Note that cutoff $\widehat{\theta}$ satisfies $\widehat{\theta} < \overline{\theta}$ since $\frac{k-1}{n(n-k)} < \frac{1}{n-k}$ simplifies to $k < n+1$, which holds by definition given that $n > k$. This implies that the line of θ values is divided in the following segments:
  - When $\theta \leq \widehat{\theta}$, we have that $\delta_{insider} \leq \delta_{pre}$ and $\delta_{insider} \leq 1$.
  - When $\widehat{\theta} < \theta \leq \overline{\theta}$, we have that $\delta_{insider} > \delta_{pre}$ but we still have $\delta_{insider} \leq 1$.
  - When $\overline{\theta} < \theta$, we have that $\delta_{insider} > \delta_{pre}$ and $\delta_{insider} > 1$.
- **Interpretation of cutoff $\widehat{\theta}$:**
  - When the cost-reduction effect, as captured by ratio $\theta \equiv \frac{x}{r-c}$, is relatively small, $\theta \leq \widehat{\theta}$, the merged entity and outsiders remain relatively similar after the merger, facilitating collusion. In terms of expression (3), the merged entity is not too attracted to deviate since its cost advantage relative to outsiders is small. In this case, we say that the merger facilitates collusion.
  - In contrast, when the cost-reduction effect is sufficiently larger, $\theta > \widehat{\theta}$, then the merged entity is more attracted to defect after the merger than before the merger, sustaining collusion under more restrictive conditions after the merger. In this case, we say that the merger hindered collusion.
- **Comparative statics:** We can finally do comparative statics of cutoff $\widehat{\theta}$, as follows

$$\frac{\partial \widehat{\theta}}{\partial k} = \frac{n-1}{n(n-k)} > 0.$$

since $n > k$ by assumption. Therefore, cutoff $\widehat{\theta}$ shifts rightward as more firms merge (higher k for a given n), expanding the range of cost efficiencies for which the merger facilitates collusion. Graphically, the range $\theta \leq \widehat{\theta}$ expands as $\widehat{\theta}$ moves rightward.
In contrast, cutoff $\widehat{\theta}$ shifts leftward as the industry grows (higher n for a given number of firms merging k) since

$$\frac{\partial \widehat{\theta}}{\partial n} = \frac{\overbrace{(1-k)}^{-}\overbrace{(2n-k)}^{+}}{n^2(n-k)^2} < 0.$$

In this context, the range of cost efficiencies for which the merger facilitates collusion, $\theta \leq \widehat{\theta}$, shrinks.