

EconS 594 - Industrial Organization
Homework #1 - Due date: Thursday, September 5th, in class.

1. **Cournot with convex and asymmetric costs.** Consider a duopoly market with inverse demand curve $p(Q) = 85 - \frac{Q}{20}$, where $Q = q_1 + q_2$ denotes aggregate output. Firm 1 faces cost function $c(q_1) = 3,000 + 9q_1 + \frac{q_1^2}{200}$, while firm 2's cost function is $c(q_2) = 3,500 + 8q_2 + \frac{q_2^2}{200}$. Firms compete a la Cournot.
 - (a) Find each firm's best response function. Identify the output levels making each firm shut down. Plot them in a figure.
 - (b) Find the Cournot equilibrium output pair, equilibrium price, consumer surplus, and social welfare.

2. **Salop circle with quadratic transportation costs.** Consider the Salop circle we presented in class, but assume that transportation costs are now τd^2 , where d denotes the distance that the consumer travels to his selected shop.
 - (a) Find the equilibrium price in this setting. How does it differ from that under linear transportation costs?
 - (b) Find the equilibrium number of firms entering the industry, n^e , when entry cost is $e > 0$.
 - (c) Find the socially optimal number of firms entering the industry, n^{SO} , when entry cost is $e > 0$.
 - (d) Compare your results in parts (b) and (c). Interpret.
 - (e) Compare your result in part (d) against the case in which transportation costs are linear (check your class notes). Interpret.

3. **An investment game.** Consider a duopoly market with a continuum of homogeneous consumers of mass 1. Consumers derive utility $v_i \in \{v^H, v^L\}$ for product i depending on whether the product is of high or low quality. Firms play the following 2-stage game:
 - 1st At stage 1, firms simultaneously invest in quality. The more a firm invests the higher is its probability λ_i of obtaining a high-quality product. The associated investment cost is denoted by $I(\lambda_i)$ and satisfies standard properties that ensure an interior solution: $I(\lambda_i)$ is continuous for $\lambda_i \in [0, 1)$, $I'(\lambda_i) > 0$ and $I''(\lambda_i) > 0$ for $\lambda_i \in (0, 1)$, and the Inada conditions $\lim_{\lambda_i \rightarrow 0^+} I'(\lambda_i)$ and $\lim_{\lambda_i \rightarrow 1^-} I'(\lambda_i) = +\infty$. Before the beginning of stage 2 qualities become publicly observable— i.e., all uncertainty is resolved.
 - 2nd At stage 2, firms simultaneously and independently set prices.
 - a) For any given (λ_1, λ_2) , what are the expected equilibrium profits? In case of multiple equilibria select the (from the view point of the firms) Pareto-dominant equilibrium.

- b) Are investments strategic complements or substitutes? Explain your finding.
- c) Provide the equilibrium condition at the investment stage.
- d) How do equilibrium investments change as valuation v^H increases? How do they change when v^L increases?

4. **Vertical product differentiation and cost of quality.** A consumer with income m who consumes a product of quality s_i and pays p_i obtains the utility $\frac{s_i m}{6} - p_i$. If instead the consumer decides not buy the good, the resulting utility is zero. Consumer income m is uniformly distributed on the interval $[2, 8]$, so its density is $f(m) = \frac{1}{8-2} = \frac{1}{6}$. The total mass of consumers is equal to 1.

There are two firms in the market, 1 and 2, offering qualities s_1 and s_2 , respectively. Assume that $s_1, s_2 \in [1, 2]$. Label firms such that $s_1 \leq s_2$. Suppose that firm i has constant marginal cost equal to $c \times s_i$ where $c < 2$.

- (a) Derive the demand of firms 1 and 2, and calculate the best-response functions of the two firms presuming that first-order conditions hold with equality. Distinguish between full and partial market coverage.
- (b) Calculate the Nash Equilibrium in prices and find the equilibrium profits as a function of s_1 and s_2 . Distinguish between full and partial market coverage.
- (c) What are the equilibrium quality choices of the two firms? (Again distinguish between the full and market coverage cases using first-order conditions.)
- (d) Which firm is more profitable? Consider the two cases mentioned above.
- (e) How does in the partial coverage equilibrium an increase in cost c affect the profits of the two firms?