Part II. Market power

Chapter 3. Static imperfect competition
Oligopolies

- Industries in which a few firms compete
- Market power is collectively shared.
- Firms can’t ignore their competitors’ behaviour.
- **Strategic interaction** → Game theory

Oligopoly theories

- *Cournot* (1838) → quantity competition
- *Bertrand* (1883) → price competition
- Not competing but **complementary theories**
  - Relevant for different industries or circumstances
Organization of Part II

• Chapter 3
  • Simple settings: unique decision at single point in time
  • How does the nature of strategic variable (price or quantity) affect
    • strategic interaction?
    • extent of market power?

• Chapter 4
  • Incorporates time dimension: sequential decisions
  • Effects on strategic interaction?
  • What happens before and after strategic interaction takes place?
Case. DVD-by-mail industry

• Facts
  • < 2004: *Netflix* almost only active firm
  • 2004: entry by *Wal-Mart* and *Blockbuster* (and later *Amazon*), not correctly foreseen by Netflix

• Sequential decisions
  • Leader: *Netflix*
  • Followers: *Wal-Mart, Blockbuster, Amazon*

• Price competition
  • *Wal-Mart* and *Blockbuster* undercut *Netflix*
  • *Netflix* reacts by reducing its prices too.

• Quantity competition?
  • Need to store more copies of latest movies
Chapter 3. Learning objectives

• Get (re)acquainted with basic models of oligopoly theory
  • Price competition: Bertrand model
  • Quantity competition: Cournot model

• Be able to compare the two models
  • Quantity competition may be mimicked by a two-stage model (capacity-then-price competition)
  • Unified model to analyze price & quantity competition

• Understand the notions of strategic complements and strategic substitutes

• See how to measure market power empirically
The standard Bertrand model

• 2 firms
  • Homogeneous products
  • Identical constant marginal cost: $c$
  • Set price simultaneously to maximize profits

• Consumers
  • Firm with lower price attracts all demand, $Q(p)$
  • At equal prices, market splits at $\alpha_1$ and $\alpha_2=1-\alpha_1$

$\rightarrow$ Firm $i$ faces demand

$$Q_i(p_i) = \begin{cases} 
Q(p_i) & \text{if } p_i < p_j \\
\alpha_i Q(p_i) & \text{if } p_i = p_j \\
0 & \text{if } p_i > p_j 
\end{cases}$$
The standard Bertrand model (cont’d)

• Unique Nash equilibrium
  • Both firms set price = marginal cost: \( p_1 = p_2 = c \)
  • Proof
    • For any other \((p_1, p_2)\), a profitable deviation exists.
    • Or: unique intersection of firms’ best-response functions
The standard Bertrand model (cont’d)

• ‘Bertrand Paradox’
  • Only 2 firms \textbf{but} perfectly competitive outcome
  • Message: there exist circumstances under which duopoly competitive pressure can be very strong

\textbf{Lesson}: In a homogeneous product Bertrand duopoly with identical and constant marginal costs, the equilibrium is such that
  • firms set price equal to marginal costs;
  • firms do not enjoy any market power.
The standard Bertrand model (cont’d)

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  • firms do not enjoy any market power.

• Cost asymmetries: \( n \) firms, \( c_i < c_{i+1} \)
  • Equilibrium: any price \( p_i = p_j = p \in [c_1, c_2] \)
  • Select \( p^* = c_2 \)
Bertrand competition with uncertain costs

• Each firm has private information about its costs
  • Trade-off between margins and likelihood of winning the competition
  • See particular model in the book.

• Lesson: In the price competition model with homogeneous products and private information about marginal costs, at equilibrium,
  • firms set price above marginal costs;
  • firms make strictly positive expected profits;
  • more firms $\rightarrow$ price-cost margins↓, output↑, profits↓;
  • number of firms explodes $\rightarrow$ competitive limit.
Price competition with differentiated products

- Firms may avoid intense competition by offering products that are imperfect substitutes.
- **Hotelling model (1929)**

\[
\tau(x - l_1) \quad \tau(l_2 - x)
\]

Mass 1 of consumers, uniformly distributed
Hotelling model (cont’d)

• Suppose location at the extreme points

\[ p_2 + \tau(1-x) \]

\[ p_1 + \tau x \]

\[ \hat{x} = \frac{1}{2} + \frac{p_2 - p_1}{2\tau} \]

Indifferent consumer
Hotelling model (cont’d)

• Resolution
  • Firm’s problem:
    \[
    \max_{p_i} (p_i - c) \left( \frac{1}{2} + \frac{p_j - p_i}{2\tau} \right)
    \]
  • From FOC, best-response function:
    \[
    p_i = \frac{1}{2} (p_j + c + \tau)
    \]
  • Equilibrium prices:
    \[
    p_i = p_j = c + \tau
    \]

• Lesson: If products are more differentiated, firms enjoy more market power.

• Extensions
  1. Localized competition with \(n\) firms: \textbf{Salop} (circle) model
  2. Asymmetric competition with differentiated products
Extension 1: Salop model

- Setting
  - Firms equidistantly located on circle with circumference 1
  - Consumers uniformly distributed on circle
  - They buy at most one unit, from firm with lowest ‘generalized price’
  - Unit transportation cost, \( \tau \)

\[
\begin{align*}
  r - \tau \left( \hat{x}_{i,i+1} - \frac{i}{n} \right) - p_i &= r - \tau \left( \frac{i + 1}{n} - \hat{x}_{i,i+1} \right) - p_{i+1} \\
  \iff \hat{x}_{i,i+1} &= \frac{2i + 1}{2n} + \frac{p_{i+1} - p_i}{2\tau}
\end{align*}
\]
Extension 1: Salop model (cont’d)

- Focus on symmetric equilibrium
- Firm \( i \)'s problem:

\[
\max_{p_i} (p_i - c)Q(p_i, p) = (p_i - c) \left( \frac{1}{n} + \frac{p - p_i}{\tau} \right)
\]

- FOC: \[
1/n + (p - 2p_i + c) / \tau = 0
\]

- Setting \( p_i = p \) yields: \[
p^* = c + \tau / n
\]
  - \( n \uparrow \rightarrow \) closer substitutes on the circle
  - \( \rightarrow \) competitive pressure \( \uparrow \rightarrow p^* \downarrow \)
  - If \( nn \rightarrow \infty \), then \( p^* \rightarrow c \) (perfect competition)
Extension 2: Asymmetric competition with differentiated products

• Same setting as Hotelling model
• Only difference: product 1 is of superior quality
  • Consumer’s indirect utility:

\[
\begin{align*}
\text{if buy 1:} & \quad r_1 - \tau x - p_1 \\
\text{if buy 2:} & \quad r_2 - \tau (1-x) - p_2
\end{align*}
\]

\[\hat{x} = \frac{1}{2} + \frac{(r_1 - r_2) - (p_1 - p_2)}{2\tau} = Q_1(p_1, p_2)\]

• Assume: \( r_2 + \tau > r_1 \) → product 2 more attractive for some consumers
• Indifferent consumer
Extension 2: Asymmetric competition with differentiated products (cont’d)

- Firm 1 chooses $p_1$ to maximize $(p_1 - c)Q_1(p_1, p_2)$
- Similarly for firm 2.
- Solving for the two FOCs:

\[
\begin{align*}
    p_1^* &= c + \tau + \frac{1}{3}(r_1 - r_2) \\
    p_2^* &= c + \tau - \frac{1}{3}(r_1 - r_2)
\end{align*}
\]

\[
Q_1(p_1^*, p_2^*) = \frac{1}{2} + \frac{r_1 - r_2}{6\tau}
\]

- High-quality firm sets a higher price and sells more.
Extension 2: Asymmetric competition with differentiated products (cont’d)

• Welfare maximization $\rightarrow$ sell at marginal cost

\[ Q_1(c, c) = \frac{1}{2} + \frac{r_1 - r_2}{2\tau} > Q_1(p_1^*, p_2^*) = \frac{1}{2} + \frac{r_1 - r_2}{6\tau} \]

• Firm 1’s equilibrium demand is too low from a social point of view.

• Same analysis if $r_1 = r_2 = r$, but $c_1 < c_2$

• **Lesson**: Under imperfect competition, the firm with higher quality or lower marginal cost sells too few units from a welfare perspective.
The linear Cournot model

- **Model**
  - Homogeneous product market with \( n \) firms
  - Firm \( i \) sets quantity \( q_i \)
  - Total output: \( q = q_1 + q_2 + \ldots + q_n \)
  - Market price given by \( P(q) = a - bq \)
  - Linear cost functions: \( C_i(q_i) = c_i q_i \)
  - Notation: \( q_{-i} = q - q_i \)

- **Residual demand**

\[
P(q) = (a - bq_{-i}) - bq_i = d(q_i; q_{-i})
\]
The linear Cournot model (cont’d)

• Firm’s problem
  • Cournot conjecture: rivals don’t modify their quantity
  • Firm $i$ acts as a monopolist on its residual demand:
    \[ \text{max}_{q_i} (P(q) - c_i)q_i \]
    \[ a - c_i - 2bq_i - bq_i - i = 0 \]
  • Best-response function: \[ q_i(q_{-i}) = \frac{1}{2b} (a - c_i - bq_{-i}) \]

• Nash equilibrium in the duopoly case
  • Assume: \[ c_1 \leq c_2 \text{ and } c_2 \leq (a + c_1) / 2 \]
  • Then, \[ q_1^* = \frac{1}{3b} (a - 2c_1 + c_2) \text{ and } q_2^* = \frac{1}{3b} (a - 2c_2 + c_1) \]
    \[ q_1^* \geq q_2^* \Rightarrow \pi_1^* \geq \pi_2^* \]
**The linear Cournot model (cont’d)**

- **Duopoly**

  ![Diagram](image.png)

  \[ \frac{a - c_1}{2b}, \frac{a - c_2}{b}, \frac{a - c_2'}{b}, c_2' > c_2 > c_1 \]

  **Lesson:** In the linear Cournot model with homogeneous products, a firm’s equilibrium profit increases when the firm becomes relatively more efficient than its rivals.
Symmetric Cournot oligopoly

- Suppose that \( c_i = c \) for all \( i = 1 \) to \( n \)
- Then

\[
q^*(n) = \frac{a - c}{b(n+1)} \quad \rightarrow \quad L(n) = \frac{p^*(n) - c}{p^*(n)} = \frac{a - c}{a + nc}
\]

- If \( n \uparrow \) \( \rightarrow \) individual quantity \( \downarrow \), total quantity \( \uparrow \), market price \( \downarrow \), markup \( \downarrow \)
- If \( n \rightarrow \infty \), then markup \( \rightarrow 0 \)

**Lesson**: The (symmetric linear) Cournot model converges to perfect competition as the number of firms increases.
Implications of Cournot competition

- General demand and cost functions
- Cournot pricing formula (details see next slide)

\[ \frac{P(q) - C'_i(q_i)}{P(q)} = \frac{\alpha_i}{\eta} \text{ with } \alpha_i = \frac{q_i}{q} \]

**Lesson**: In the Cournot model, the markup of firm \( i \) is larger the larger is the market share of firm \( i \) and the less elastic is market demand.

- If marginal costs are constant

\[ \frac{p - \sum_{i=1}^{n} \alpha_i c_i}{p} = \frac{I_H}{\eta} \text{ with } I_H = \sum_{i=1}^{n} \alpha_i^2, \text{ Herfindahl index} \]
Details: Cournot pricing formula

- F.O.C. of profit maximization for Cournot firm

\[ P'(q)q_i + P(q) - C'_i(q_i) = 0 \iff \]
\[ P(q) - C'_i(q_i) = -P'(q)q_i \iff \]
\[ \frac{P(q) - C'_i(q_i)}{P(q)} = \frac{-P'(q)q}{P(q)} q_i = \frac{1}{\eta} \alpha_i \]

- Suppose constant marginal costs: \( C_i(q_i) = c_i q_i \)

\[ \frac{p - c_i}{p} = \frac{\alpha_i}{\eta} \to \sum_{i=1}^{n} \pi_i = \sum_{i=1}^{n} (p - c_i)\alpha_i q = \left\{ \begin{array}{l} (p - \sum_{i=1}^{n} \alpha_i c_i)q \\ \frac{pq}{\eta} \sum_{i=1}^{n} \alpha_i^2 \end{array} \right. \]

\[ \Rightarrow \frac{p - \sum_{i=1}^{n} \alpha_i c_i}{p} = \frac{\sum_{i=1}^{n} \alpha_i^2}{\eta} = \frac{I_H}{\eta} \]

→ Lerner index (weighted by market shares) is proportional to Herfindahl index
Price versus quantity competition

• Comparison of previous results
  • Let \( Q(p) = a - p, \ c_1 = c_2 = c \)
  • Bertrand: \( p_1 = p_2 = c, \ q_1 = q_2 = (a-c)/2, \ \pi_1 = \pi_2 = 0 \)
  • Cournot: \( q_1 = q_2 = (a-c)/3, \ p = (a+2c)/3, \ \pi_1 = \pi_2 = (a-c)^2/9 \)

• **Lesson**: Homogeneous product case \( \rightarrow \) higher price, lower quantity, higher profits under quantity than under price competition.

• To refine the comparison
  • Limited capacities of production
  • Direct comparison within a unified model
  • Identify characteristics of price or quantity competition
Limited capacity and price competition

• Edgeworth’s critique (1897)
  • Bertrand model: no capacity constraint
  • But capacity may be limited in the short run.

• Examples
  • Retailers order supplies well in advance
  • DVD-by-mail industry
    • Larger demand for latest movies → need to hold extra stock of copies → higher costs and stock may well be insufficient
  • Flights more expensive around Xmas

• To account for this: two-stage model
  1. Firms precommit to capacity of production
  2. Price competition
Capacity-then-price model (Kreps & Scheinkman)

• Setting
  • Stage 1: firms set capacities $\bar{q}_i$ and incur cost of capacity, $c$
  • Stage 2: firms set prices $p_i$; cost of production is 0 up to capacity (and infinite beyond capacity); demand is $Q(p) = a - p$.
  • Subgame-perfect equilibrium: firms know that capacity choices may affect equilibrium prices

• Rationing
  • If quantity demanded to firm $i$ exceeds its supply...
  • ... some consumers have to be rationed...
  • ... and possibly buy from more expensive firm $j$.
  • Crucial question: Who will be served at the low price?
Capacity-then-price model (cont’d)

• Efficient rationing
  • First served: consumers with higher willingness to pay.
  • Justification: queuing system, secondary markets

Consumers with unit demand, ranked by decreasing willingness to pay

Consumers with highest willingness to pay are served at firm 1’s low price

There is a positive residual demand for firm 2

Excess demand for firm 1
Capacity-then-price model (cont’d)

- **Equilibrium** (details next slides)
  - **Stage 2.** If \( p_1 < p_2 \) and excess demand for firm 1, then demand for 2 is:
    \[
    Q(p_2) = \begin{cases} 
    Q(p_2) - \bar{q}_1 & \text{if } Q(p_2) - \bar{q}_1 \geq 0 \\
    0 & \text{else}
    \end{cases}
    \]
  - **Claim:** if \( c < a < (4/3)c \), then both firms set the market-clearing price: \( p_1 = p_2 = p^* = a - \bar{q}_1 - \bar{q}_2 \)
  - **Stage 1.** Same reduced profit functions as in Cournot:
    \[
    \pi_1(\bar{q}_1, \bar{q}_2) = (a - \bar{q}_1 - \bar{q}_2)\bar{q}_1 - c\bar{q}_1
    \]

- **Lesson:** In the capacity-then-price game with efficient consumer rationing (and with linear demand and constant marginal costs), the chosen capacities are equal to those in a standard Cournot market.
Details: Capacity-then-price model

• Setting
  • Stage 1: firms set capacities $\bar{q}_i$ and incur cost of capacity, $c$
  • Stage 2: firms set prices $p_i$; cost of production is 0 up to capacity (and infinite beyond capacity); demand is $Q(p) = a - p$.
  • Subgame-perfect equilibrium: firms know that capacity choices may affect equilibrium prices
  • Efficient rationing

• Upper bound on capacity at stage 1

$$c\bar{q}_i \leq \max_q (a - q)q = a^2 / 4 \iff \bar{q}_i \leq a^2 / (4c)$$
Details: Capacity-then-price model (cont’d)

• Claim: if \( c < a < (4/3)c \), then both firms set the market-clearing price: \( p_1 = p_2 = p^* = a - q_1 - q_2 \)

• Proof

• Let \( p_1 = p^* \) and show that 2’s best-response is \( p_2 = p^* \).

• \( p_2 < p^* \) doesn’t pay: same quantity (because firm 2 sells all its capacity) sold at lower price

• \( p_2 > p^* \) could pay as firm 1 is capacity constrained...

For this, revenues should be increasing at \( p^* \)...

• Firm 2’s revenues:

\[
p_2 Q(p_2) = \begin{cases} 
    p_2(a - p_2 - q_1) & \text{if } a - p_2 \geq q_1, \\
    0 & \text{else}
\end{cases}
\]
Details: Capacity-then-price model (cont’d)

- **Proof (cont’d)**
  - Max reached at $\bar{p}_2 = (a - \bar{q}_1) / 2$
  - Revenues are decreasing at $p^*$ if

\[
p^* > \bar{p}_2 \iff a - \bar{q}_1 - \bar{q}_2 > \frac{a - \bar{q}_1}{2} \iff a > \bar{q}_1 + 2\bar{q}_2
\]

Since $\bar{q}_1, \bar{q}_2 \leq a^2/(4c)$, $\bar{q}_1 + 2\bar{q}_2 \leq (3/4)(a^2/c)$

Assumption $a < (4/3)c \iff (3/4)(a/c) < 1$

- Hence, not profitable to set $p_2 > p^*$. QED
Differentiated products: Cournot vs. Bertrand

• Setting
  • Duopoly, substitutable products \((b > d > 0)\)
  • Consumers maximize linear-quadratic utility function
    \[
    U(q_0, q_1, q_2) = aq_1 + aq_2 - (bq_1^2 + 2dq_1q_2 + bq_2) / 2 + q_0
    \]
    under budget constraint
  • Inverse demand functions
    \[
    \begin{align*}
    P_1(q_1, q_2) &= a - bq_1 - dq_2 \\
    P_2(q_1, q_2) &= a - bq_2 - dq_1
    \end{align*}
    \]

• Demand functions
  \[
  \begin{align*}
  Q_1(p_1, p_2) &= \bar{a} - \bar{b}p_1 + \bar{d}p_2 \\
  Q_2(p_1, p_2) &= \bar{a} - \bar{b}p_2 + \bar{d}p_1
  \end{align*}
  \]
  with
  \[
  \begin{align*}
  \bar{a} &= a / (b + d), & \bar{b} &= b / (b^2 - d^2), \\
  \bar{d} &= d / (b^2 - d^2)
  \end{align*}
  \]
Differentiated products

\[
\max_{p_i} (p_i - c_i)(\bar{a} - b p_i + \bar{d} p_j)
\]

- Maximization program
  - Cournot: \[
  \max_{q_i} (a - bq_i + dq_j - c_i)q_i
  \]
  - Bertrand: \[
  \max_{p_i} (p_i - c_i)(\bar{a} - b p_i + \bar{d} p_j)
  \]

- Best-response functions
  - Cournot: \[
  q_i(q_j) = \frac{(a - dp_j - c_i)}{(2\bar{b})}
  \]
  Downward-sloping \(\rightarrow\) Strategic substitutes
  - Bertrand: \[
  p_i(p_j) = \frac{(a + dp_j + bc_i)}{(2\bar{b})}
  \]
  Upward-sloping \(\rightarrow\) Strategic complements

- Comparison of equilibria
  - Lesson: Price as the strategic variable gives rise to a more competitive outcome than quantity as the strategic variable.
Appropriate modelling choice: price or quantity?

- **Monopoly**: it doesn’t matter.
- **Oligopoly**: price and quantity competitions lead to different residual demands
  - **Price competition**
    - \( p_j \) fixed → rival willing to serve any demand at \( p_j \)
    - \( i \)'s residual demand: market demand at \( p_i < p_j \); zero at \( p_i > p_j \)
    - So, residual demand is very sensitive to price changes.
  - **Quantity competition**
    - \( q_j \) fixed → irrespective of price obtained, rival sells \( q_j \)
    - \( i \)'s residual demand: “what’s left” (i.e., market demand – \( q_j \))
    - So, residual demand is less sensitive to price changes.
Appropriate modelling choice (cont’d)

- How do firms behave in the market place?
  - Stick to a price and sell any quantity at this price?
    → **price competition**
    → appropriate choice when
      - Unlimited capacity
      - Prices more difficult to adjust in the short run than quantities
      - **Example**: mail-order business
  - Stick to a quantity and sell this quantity at any price?
    → **quantity competition**
    → appropriate choice when
      - Limited capacity (even if firms are price-setters)
      - Quantities more difficult to adjust in the short run than prices
      - **Example**: package holiday industry
- Influence of technology (e.g. Print-on-demand vs. batch printing)
Strategic substitutes and complements

- How does a firm react to the rivals’ actions?
- Look at the slope of reaction functions.
  - **Upward sloping**: competitor ↑ its action → marginal profitability of my own action ↑
    → variables are strategic **complements**
    - Example: price competition (with substitutable products);
      See Bertrand and Hotelling models
  - **Downward sloping**: competitor ↑ its action → marginal profitability of my own action ↓
    → variables are strategic **substitutes**
    - Example: quantity competition (with substitutable products);
      see Cournot model
Strategic substitutes and complements (cont’d)

- Linear demand model of product differentiation
  (with $d$ measuring the degree of product substitutability)
Estimating market power

• Setting
  • Symmetric firms producing homogeneous product
  • Demand equation: \( p = P(q,x) \) (1)
    • \( q \): total quantity in the market
    • \( x \): vector of exogenous variables affecting demand (not cost)
  • Marginal costs: \( c(q,w) \)
    • \( w \): vector of exogenous variables affecting (variable) costs

• Interpretation 1. Nest various market structures in a single model

\[
MR(\lambda) = p + \lambda \frac{\partial P(q,x)}{\partial q}q
\]

\( \lambda = 0 \) competitive market
\( \lambda = 1 \) monopoly
\( \lambda = 1/n \) \( n \)-firm Cournot

Firm’s conjecture as to how strongly price reacts to its change in output
Estimating market power (cont’d)

• Interpretation 1 (cont’d)
  • Basic model to be estimated non-parametrically: demand equation (1) + equilibrium condition (2)

\[
MR(\lambda) = p + \lambda \frac{\partial P(q, x)}{\partial q} q = c(q, w)
\]

• Interpretation 2. Be agnostic about precise game being played
  • From equilibrium condition (2), Lerner index is

\[
L = \frac{p - c(q, w)}{p} = -\lambda \frac{\partial P(q, x)}{\partial q} \frac{q}{p} = \frac{\lambda}{\eta}
\]

• (2) is identified if single \(c(q, w)\) and single \(\lambda\) satisfy it
Review questions

• How does product differentiation relax price competition? Illustrate with examples.
• How does the number of firms in the industry affect the equilibrium of quantity competition?
• When firms choose first their capacity of production and next, the price of their product, this two-stage competition sometimes looks like (one-stage) Cournot competition. Under which conditions?
• Using a unified model of horizontal product differentiation, one comes to the conclusion that price competition is fiercer than quantity competition. Explain the intuition behind this result.
Review questions (cont’d)

• Define the concepts of strategic complements and strategic substitutes. Illustrate with examples.

• What characteristics of a specific industry will you look for to determine whether this industry is better represented by price competition or by quantity competition? Discuss.