EconS 424 – Strategy and Game Theory

Quiz #5 – *Answer key*

Cheap talk game

Figure 1 depicts a cheap talk game. In particular, the sender's payoff coincides when he sends message m_1 or m_2, and only depends on the receiver's response (either *a*, *b* or *c*) and the nature's type. You can interpret this strategic setting as a lobbyist (sender) informing a Congressman (receiver) about the situation of the industry he represents: messages "good situation" or "bad situation" are equally costly for him, but the reaction of the politician to these message (and the actual state of the industry) determine the lobbyist payoff. A similar argument applies for the payoffs of the receiver (Congressman), which do not depend on the particular message he receives in his conversation with the lobbyist, but are only a function of the specific state of the industry (something he cannot observe) and the action he chooses (e.g., the policy that he designs for the industry after his conversation with the lobbyist is over). For instance, when the sender is type t_1, in the left-hand side of the tree, payoff pairs only depend on the receiver's response (*a*, *b* or *c*) but do not depend on the sender's message, e.g., when the receiver responds with *a* players obtain (4,3), both when the original message was m_1 and m_2.

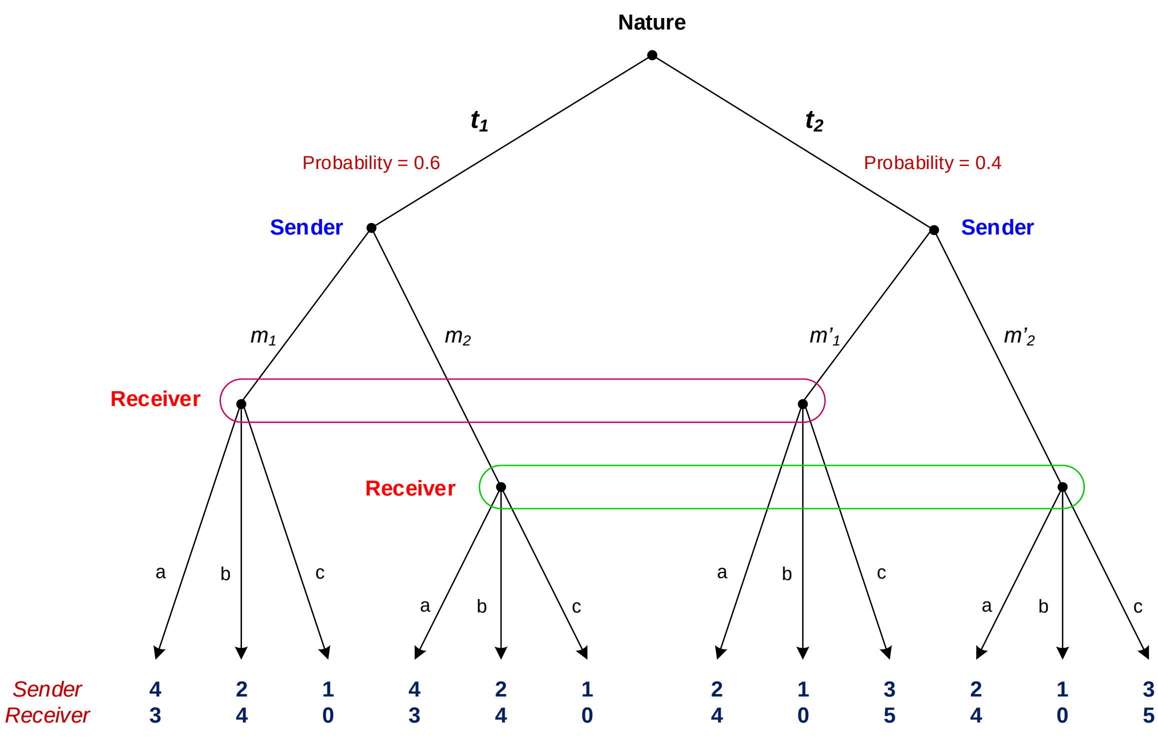

Figure 1. Cheap talk game

This game can alternatively be represented as follows:

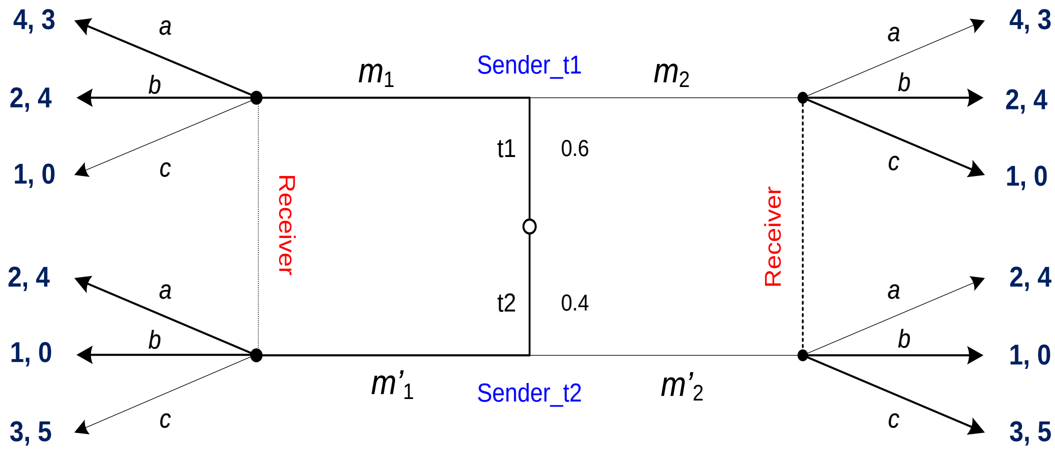

Figure 2. An alternative representation of the cheap talk game

a. Check if a separating strategy profile in which only the sender with type t_1 sends message m_1 can be sustained as a PBE. Figure 3 depicts the separating strategy profile (m_1, m'_2), where message m_1 only originates from a sender of type t_1.

- [*Hint*: For all parts of the exercise, start identifying the receiver's beliefs, then determine the receiver's optimal response given those beliefs, and finally check if each type of sender has incentives to behave as depicted in the figure (given the optimal response of the receiver you just found) or instead has incentives to deviate.]

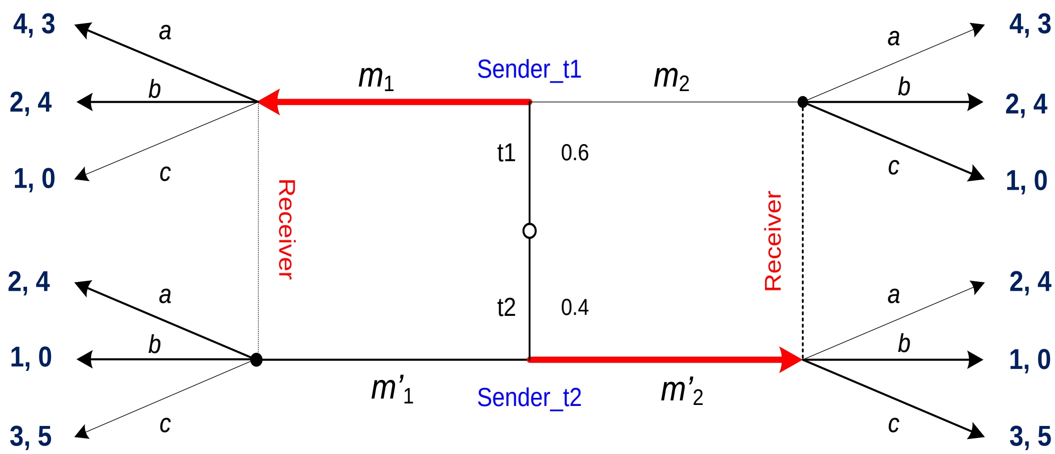

Figure 3. Separating strategy profile

Answer:

<u>Receiver's beliefs:</u>

Let $\mu(t_i|m_j)$ represent the conditional probability that the receiver assigns to the sender being of type t_i given that message m_j is observed. Thus, after observing message m_1, the receiver assigns full probability to such a message originating only from t_1-type of sender,

$$\mu(t_1|m_1) = 1, \text{ and } \mu(t_2|m_1) = 0$$

while after observing message m_2, the receiver infers that it must originate from t_2-type of sender,

$$\mu(t_1|m_2) = 0, \text{ and } \mu(t_2|m_2) = 1$$

Receiver's optimal response:

- After observing m_1, the receiver believes that such a message can only originate from a t_1-type of sender. Graphically, the receiver is convinced to be in the upper left-hand corner of figure 3. In this setting, the receiver's optimal response is b given that it yields a payoff of 4 (higher than what he gets from a, 3, and from c, 0.)

- After observing message m_2, the receiver believes that such a message can only originate from a t_2-type of sender. That is, he is convinced to be located in the lower right-hand corner of figure 3. Hence, his optimal response is c, which yields a payoff of 5, rather than a, which only provides a payoff of 4, or b, which entails a zero payoff.

Figure 3.1 summarizes these optimal responses of the receiver.

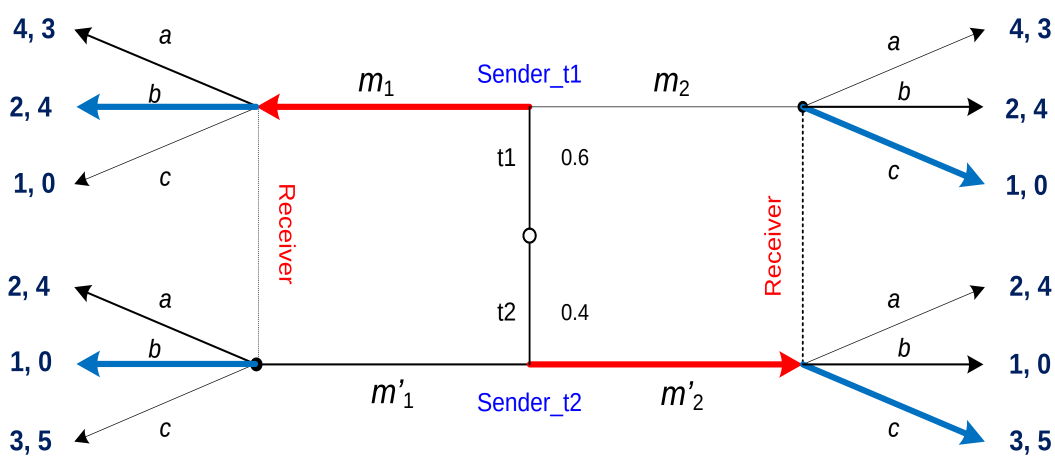

Figure 3.1. Optimal responses in the separating strategy profile.

Sender's optimal messages:

- If his type is t_1, by sending m_1 he obtains a payoff of 2 (since m_1 is responded with b), but a lower payoff of 1 if he deviates towards message m_2 (since such a message is responded with c). Hence, the sender doesn't want to deviate from m_1.

- If his type is t_2, he obtains a payoff of 3 by sending message m_2 (which is responded with c) and a payoff of 1 if he deviates to message m_1 (which is responded with b). Hence, he doesn't have incentives to deviate from m_2.

Hence, the initially prescribed separating strategy profile *can* be supported as a PBE. As a curiosity, note that the opposite separating strategy profile (m_2, m_1') can also sustained as a PBE (you can check that as a practice).

b. Check if a pooling strategy profile in which both types of the sender choose message m_1 can be supported as a PBE. Figure 4 below depicts the separating strategy profile (m_1, m'_1), where both types of senders send the same message m_1.

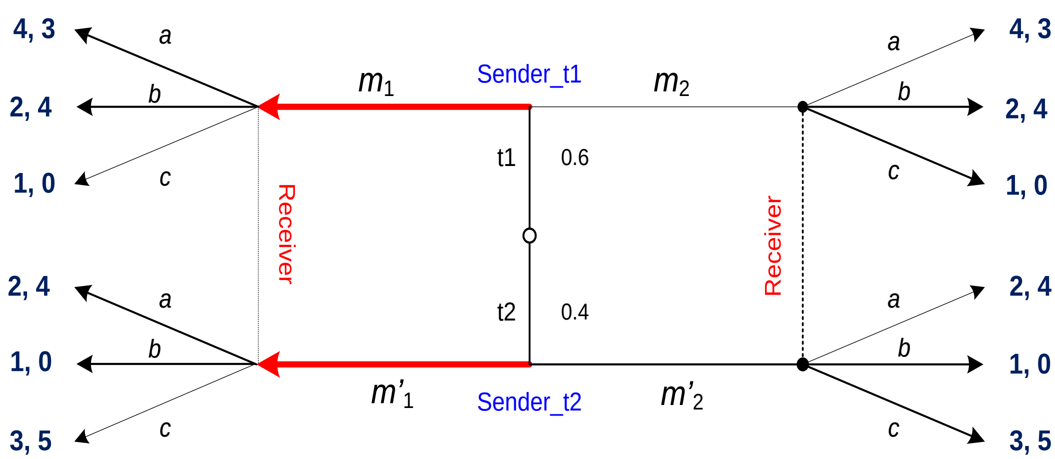

Figure 4. Pooling strategy profile

Receiver's beliefs:

After observing message m_1 (which occurs in equilibrium), beliefs coincide with the prior probability distribution over types. Indeed, applying Bayes' rule we find that

$$\mu(t_1|m_1) = \frac{p(t_1) * p(m_1|t_1)}{p(m_1)} = \frac{0.6 * 1}{0.6 * 1 + 0.4 * 1} = 0.6$$

$$\mu(t_2|m_1) = \frac{p(t_2) * p(m_1|t_2)}{p(m_1)} = \frac{0.4 * 1}{0.6 * 1 + 0.4 * 1} = 0.4$$

After receiving message m_2 (what happens off-the-equilibrium path), beliefs cannot be updated using Bayes' rule since

$$\mu(t_1|m_2) = \frac{p(t_1) * p(m_2|t_1)}{p(m_2)} = \frac{0.6 * 0}{0.6 * 0 + 0.4 * 0} = \frac{0}{0}$$

and hence off-the-equilibrium beliefs must be arbitrarily specified, i.e. $\mu = \mu(t_1|m_2) \in [0,1]$.

<u>Receiver's optimal response:</u>

- After receiving message m_1 (in equilibrium), the receiver's expected utility from responding with actions a, b, and c is, respectively

  Action *a*: $0.6 \times 3 + 0.4 \times 4 = 3.4$,

  Action *b*: $0.6 \times 4 + 0.4 \times 0 = 2.4$, and

  Action *c*: $0.6 \times 0 + 0.4 \times 5 = 2.0$

  Hence, the receiver's optimal strategy is to choose action *a* in response to the equilibrium message of m_1.

- After receiving message m_2 (off-the-equilibrium), the receiver's expected utility from each of his three possible responses are

  $$EU_{Receiver}(a|m_2) = \mu * 3 + (1-\mu) * 4 = 4 - \mu,$$
  $$EU_{Receiver}(b|m_2) = \mu * 4 + (1-\mu) * 0 = 4\mu, \text{ and}$$
  $$EU_{Receiver}(c|m_2) = \mu * 0 + (1-\mu) * 5 = 5 - 5\mu$$

  where the receiver's response critically depends on the particular value of his off-the-equilibrium belief μ, i.e., $4\mu > 4-\mu$ if and only if $\mu > \frac{4}{5}$, $4-\mu > 5-5\mu$ if and only if $\mu > \frac{1}{4}$, and $4\mu > 5-5\mu$ if and only if $\mu > \frac{5}{9}$. Hence, if off-the-equilibrium beliefs lie within the interval $\mu \in \left[0, \frac{1}{4}\right]$ $5-5\mu$ is the highest expected payoff, thus inducing the receiver to respond with *c*; if they lie on the interval $\mu \in (\frac{1}{4}, \frac{4}{5}]$

4-μ becomes the highest expected payoff and the receiver chooses a; finally, if off-the-equilibrium beliefs lie on the interval $\mu \in (\frac{4}{5}, 1]$ 4μ is the highest expected payoff and the receiver responds with b. For simplicity, we only focus on the case in which off-the-equilibrium beliefs satisfy μ=1 (and thus the responder chooses b upon observing message m_2).

Figure 4.1 summarizes the optimal responses of the receiver (a after m_1, and b after m_2) in this pooling strategy profile.

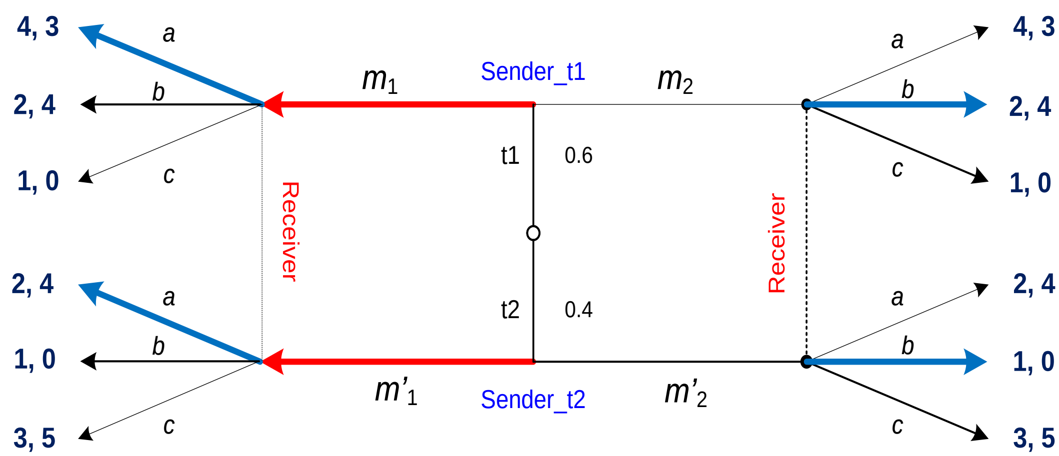

Figure 4.1. Optimal responses in the pooling strategy profile

<u>Sender's optimal message:</u>

- If his type is t_1, the sender obtains a payoff of 4 from sending m_1 (since it is responded with a), but a payoff of only 2 when deviating towards m_2 (since it is responded with b). Hence, he doesn't have incentives to deviate from m_1.
- If his type is t_2 , the sender obtains a payoff of 2 by sending m_1 (since it is responded with a), but a payoff of only 1 by deviating towards m_2 (since it is responded with b). Hence, he doesn't have incentives to deviate from m_1.

Therefore, the initially prescribed pooling strategy profile where both types of sender select m_1 can be sustained as a PBE of the game. Importantly, this equilibrium can be supported for any off-the-equilibrium beliefs, $\mu(t_1|m_2)$, the receiver sustains upon observing message m_2.