

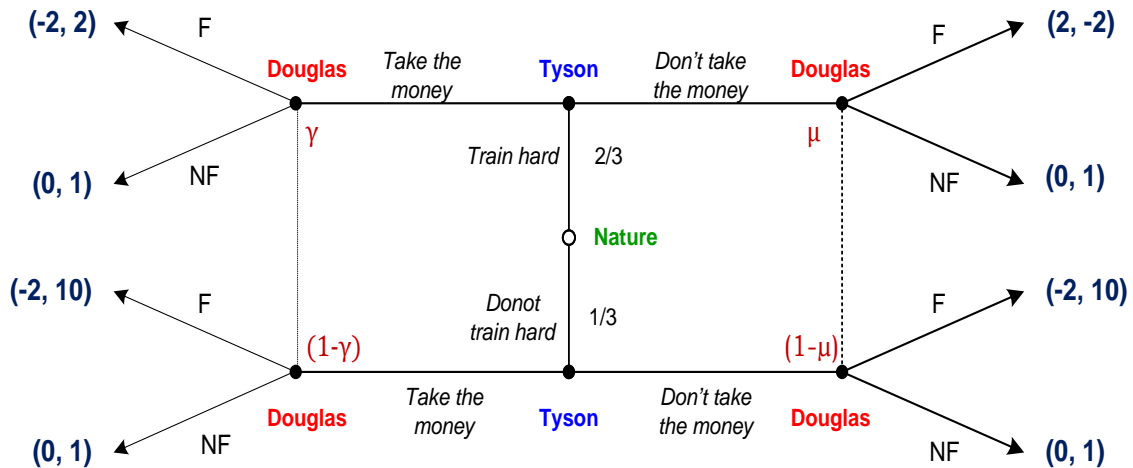
ECONS 424 - STRATEGY AND GAME THEORY
QUIZ #4 (SIGNALING GAMES) – ANSWER KEY

Signaling game – Mike Tyson vs. Buster Douglas

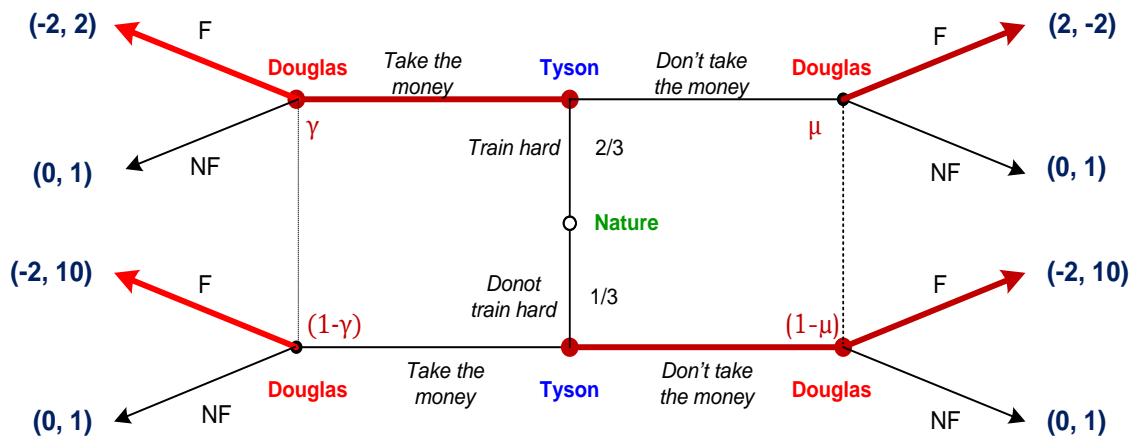
Consider the following sequential move game with incomplete information. The first player to move is Mike Tyson, who privately knows whether he trained hard, or he didn't. Let us assume that the type of training that Mike Tyson receives before a fight is not something he can strategically decide, but instead, it depends on his state of mind between the time he signed up the contract for a fight and the time of the fight. In particular, we will assume that the probability that Mike Tyson trains hard is given by Nature, and it is $2/3$, as can be seen in the figure.

Knowing what kind of physical training he had, Mike Tyson decides whether to offer player 2 (Buster Douglas) \$1 million dollars if he gives up his right to fight Mike Tyson. We will denote Mike Tyson's strategies as "Take the money" (that is, offering the bribe) or "Don't take the money" (don't offering any bribe to Douglas).

After observing whether Mike Tyson has offered him any money, Buster Douglas must decide whether to Fight (F) or Not Fight (NF), without knowing whether Mike Tyson has previously trained hard or not.



a) Show that there is no Separating PBE where Mike Tyson makes the offer “Take the money” when he has trained hard, but does not make such an offer (he chooses “Don’t take the money”) when he has not trained hard. To show this, follow the usual steps for finding PBE.



1. Find Buster Douglas’ beliefs in this Separating PBE (use Bayes’ rule).

After observing the offer “Take the money” from Mike Tyson, Buster Douglas’ beliefs are

$$\gamma = \frac{0.6 \times 1}{0.6 \times 1 + 0.4 \times 0} = 1 \rightarrow \gamma = 1$$

- Graphically, Buster Douglas believes that if he observes “Take the money,” he must be in the node at the upper left-hand corner of the game tree.

After observing “Don’t take the money” from Mike Tyson, Buster Douglas’ beliefs are

$$\mu = \frac{0.6 \times 0}{0.6 \times 0 + 0.4 \times 1} = 0 \rightarrow \mu = 0$$

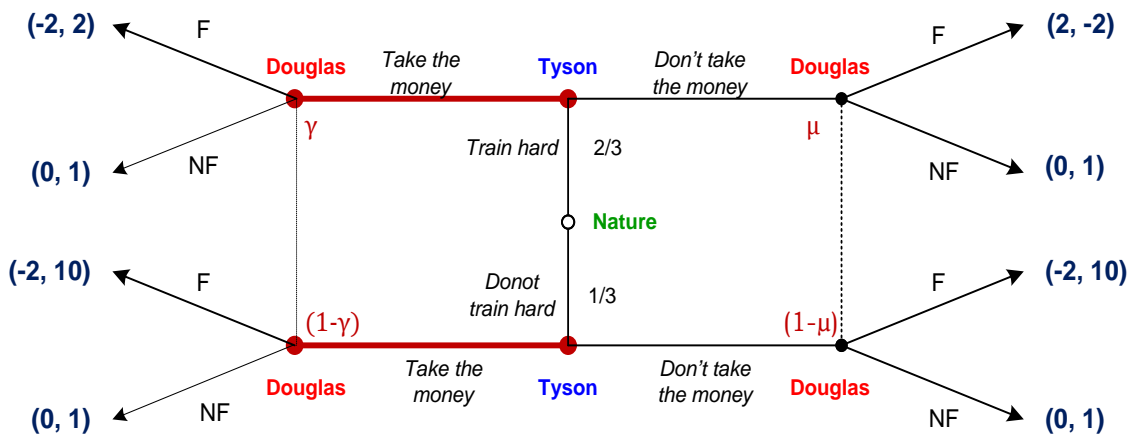
- Graphically, Buster Douglas believes that if he observes “Don’t take the money,” he must be in the node at the lower right-hand corner of the game tree.

2. Find Buster Douglas’ optimal action (whether to Fight or Not Fight) after observing that Mike Tyson offers him “Take the money”. In addition, find Buster Douglas’ optimal action (whether to Fight or Not Fight) after observing that Mike Tyson does not offers him any bribe (Douglas observes the action “Don’t take the money”).

- After observing that Mike Tyson offers “Take the money,” Buster Douglas responds with F, since he believes to be in the node at the upper left-hand corner of the game and $2 > 1$.

- After observing “Don’t take the money,” Buster Douglas responds with F, since he believes to be in the node at the lower right-hand corner of the game and $10 > 1$.
3. Find Mike Tyson’s optimal action when he has trained hard, and when he has not trained hard.
- If he trained hard, Mike Tyson prefers to deviate towards “Don’t take the money” than selecting “Take the money” (as prescribed in this strategy profile), since $2 > -2$.
 - We don’t even need to check whether Mike Tyson chooses “Don’t take the money” when he didn’t trained hard (as prescribed in the separating strategy profile we are testing), since the above argument already shows that this strategy profile cannot be sustained as a PBE.
4. Can this separating PBE be supported from your answer in c)? Obviously, you should obtain that it cannot be supported, but you have to show why from your answers in part c).
- No, since Mike Tyson prefers to deviate towards “Don’t take the money” when he trained hard.

b) Find a Pooling PBE where Mike Tyson makes the offer “Take the money” when he has trained hard, and he also makes this offer “Take the money” when he has not trained hard. To show this, follow the usual steps for finding PBE.



1. Find Buster Douglas’ beliefs in this Pooling PBE (use Bayes’ rule).

- After observing that Mike Tyson offers “Take the money” (in equilibrium), Buster Douglas’ beliefs cannot be updated, and simply coincide with the prior probability distribution, that is

$$\gamma = \frac{\frac{2}{3} \times p^{TH}}{\frac{2}{3} \times p^{TH} + \frac{1}{3} \times p^{NTH}}$$

where p^{TH} denotes the probability that Mike Tyson makes the offer “Take the money” after training hard (TH), and similarly p^{NTH} represents the probability that he makes this offer when he didn’t train hard (NTH). Since in this pooling strategy profile $p^{TH} = p^{NTH} = 1$, Buster Douglas’ beliefs become

$$\gamma = \frac{\frac{2}{3} \times 1}{\frac{2}{3} \times 1 + \frac{1}{3} \times 1} = \frac{2}{3}$$

Intuitively, Buster Douglas cannot infer any additional information from Mike Tyson’s type after observing that he offers “Take the money.”

- After observing that Mike Tyson chooses “Don’t take the money” (which occurs off-the-equilibrium path), Buster Douglas’ beliefs are

$$\mu = \frac{\frac{2}{3} \times 0}{\frac{2}{3} \times 0 + \frac{1}{3} \times 0} = \frac{0}{0}$$

and thus must be left undefined, i.e., $\mu \in [0, 1]$.

2. Find Buster Douglas’ optimal action (whether to Fight or Not Fight) after observing that Mike Tyson offers him “Take the money”. In addition, find Buster Douglas’ optimal action (whether to Fight or Not Fight) after observing that Mike Tyson does not offers him any bribe (Douglas observes the action “Don’t take the money”).

- When Mike Tyson offers “Take the money”, Douglas’ expected payoff if fighting is:

$$EU_B(F|TM) = 2 \times \frac{2}{3} + 10 \times \frac{1}{3} = \frac{14}{3}$$

and if not fighting:

$$EU_B(NF|TM) = 1 \times \frac{2}{3} + 1 \times \frac{1}{3} = 1$$

Thus, Douglas will fight since:

$$EU_B(F|TM) > EU_B(NF|TM), \quad i. e., \frac{14}{3} > 1$$

- When Mike Tyson offers “Don’t take the money”, Douglas’ expected payoff if fighting is:

$$EU_B(F|DTM) = -2\mu + 10(1 - \mu) = 10 - 12\mu$$

and if not fighting:

$$EU_B(NF|DTM) = \mu + (1 - \mu) = 1$$

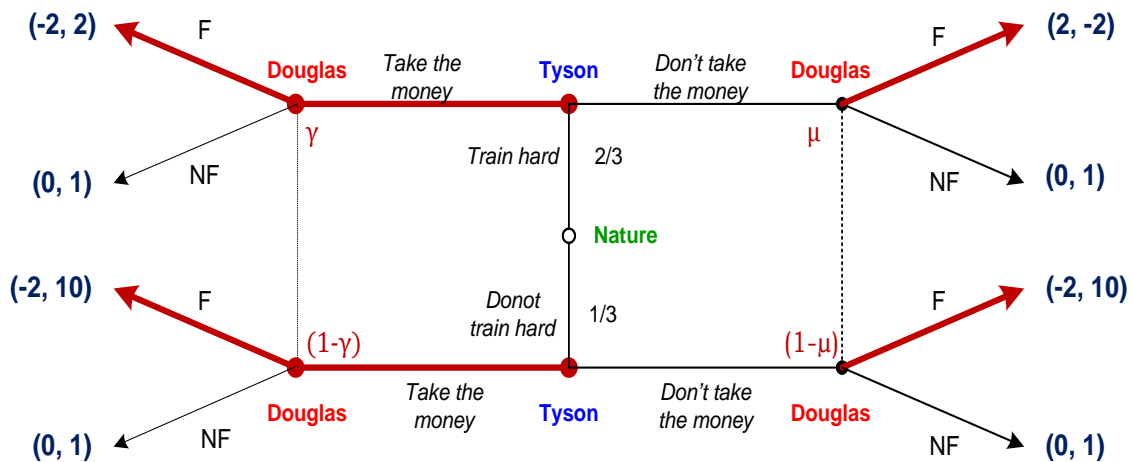
Thus, Douglas will fight only if:

$$EU_B(F|TM) > EU_B(NF|TM) \leftrightarrow 10 - 12\mu \geq 1 \rightarrow \mu \leq \frac{3}{4}$$

We then need to divide our following analysis into two cases:

- Case 1:** $\mu \leq \frac{3}{4}$, and Buster Douglas chooses to fight after “Don’t take the money.”
- Case 2:** $\mu > \frac{3}{4}$, and Buster Douglas chooses *not* to fight after “Don’t take the money.”

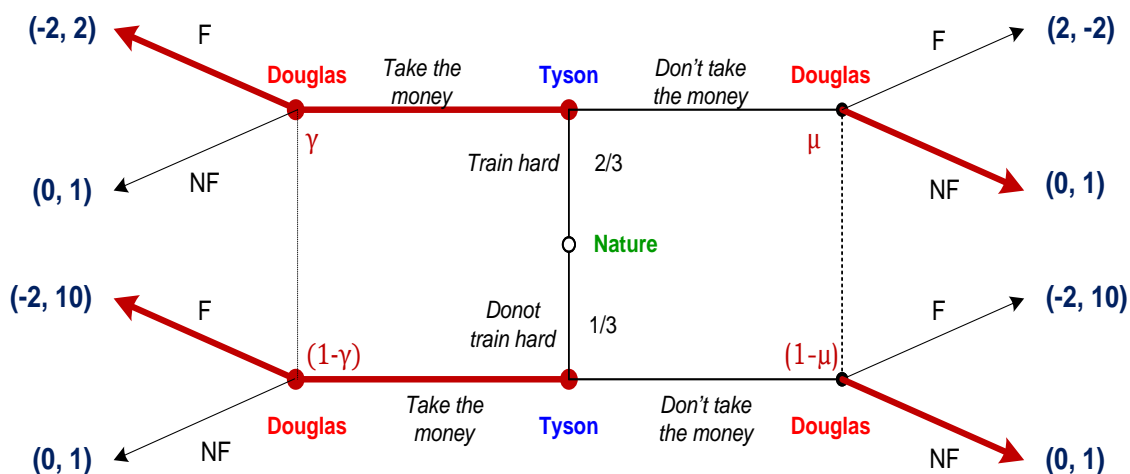
CASE 1: $\mu \leq \frac{3}{4}$. (Buster Douglas chooses to fight after observing that Mike Tyson chooses “Don’t take the money.”)



Let us now check if this pooling strategy profile can be sustained as a PBE in this case ($\mu \leq \frac{3}{4}$):

- If Mike Tyson has trained hard, then he prefers to deviate towards “Don’t take the money,” where he obtains a payoff of 2, than selecting “Take the money” as prescribed in this pooling strategy profile, which only yields a payoff of -2.
- We don’t even need to check whether Mike Tyson chooses “Take the money” when he didn’t trained hard (as prescribed in the pooling strategy profile we are testing), since the above argument already shows that this strategy profile cannot be sustained as a PBE when $\mu \leq \frac{3}{4}$.

CASE 2: $\mu > \frac{3}{4}$. (Buster Douglas chooses not to fight after observing that Mike Tyson chooses “Don’t take the money.”)



Let us now check if this pooling strategy profile can be sustained as a PBE in this case ($\mu > \frac{3}{4}$):

- If Mike Tyson has trained hard, then he prefers to select “Take the money” (as prescribed in this strategy profile), where he obtains a payoff of 2, than deviating towards “Don’t take the money”, which only yields a payoff of 0.
- If Mike Tyson has not trained hard, then he prefers to deviate towards “Don’t take the money,” where he obtains a payoff of 0, than selecting “Take the money” (as prescribed in this strategy profile), which yields a lower payoff of -2.
- Hence, this pooling strategy profile cannot be sustained as a PBE when $\mu > \frac{3}{4}$.
- Concluding, the pooling strategy profile where Mike Tyson makes the offer “Take the money” cannot be supported as a PBE of the game, regardless of Buster Douglas’ off-the-equilibrium beliefs, i.e., regardless of the precise value of μ .