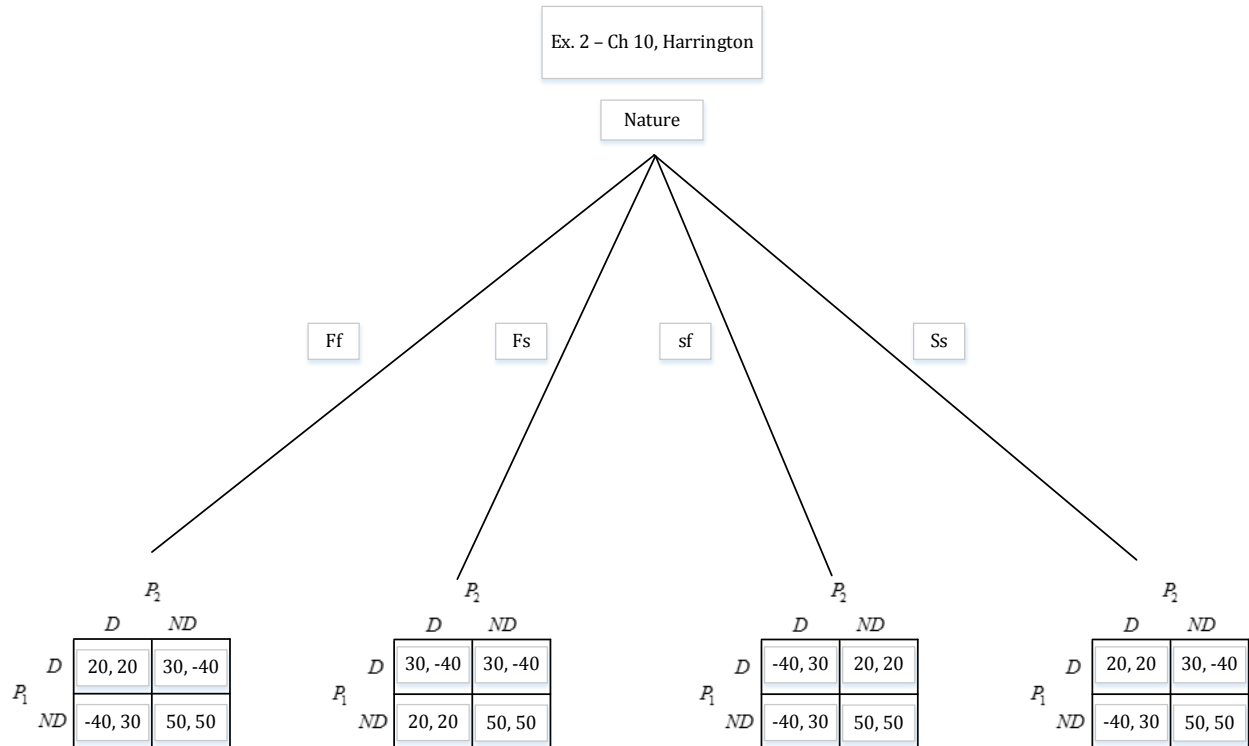


# ECONS 424 – STRATEGY AND GAME THEORY

## HOMEWORK #6 – ANSWER KEY

### HARRINGTON CHAPTER 10 – EXERCISE 2



Where F/S denotes Fast/Slow for player 1 (Bat)

f/s denotes fast/slow for player 2 (Curly Bill)

Where D represents Draw

ND represents Not Draw

fast, then each has a payoff of 20. If at least one chooses draw, then there is a gunfight.

- a. Is it consistent with Bayes–Nash equilibrium for there to be a gunfight for sure? (That is, both gunfighters draw, regardless of their type.)

**ANSWER:** Yes, as it is an equilibrium for each gunfighter to use a strategy that has him draw regardless of his type. The equilibrium conditions for Bat are

$$\text{Fast Type (Draw): } .6 \times 20 + .4 \times 30 \geq .6 \times (-40) + .4 \times 20 \Rightarrow 24 \geq -16$$

$$\text{Slow Type (Draw): } .6 \times (-40) + .4 \times 20 \geq .6 \times (-40) + .4 \times (-40) \Rightarrow \\ -32 \geq -40.$$

The equilibrium conditions for Curly Bill are

$$\text{Fast Type (Draw): } .65 \times 20 + .35 \times 30 \geq .65 \times (-40) + .35 \times 20 \Rightarrow 23.5 \geq -19$$

$$\text{Slow Type (Draw): } .65 \times (-40) + .35 \times 20 \geq .65 \times (-40) + .35 \times (-40) \Rightarrow \\ -19 \geq -40.$$

- b. Is it consistent with Bayes–Nash equilibrium for there to be no gunfight for sure? (That is, both gunfighters wait, regardless of their type.)

**ANSWER:** Yes, as it is an equilibrium for each gunfighter to use a strategy that has him wait regardless of his type. Doing so realizes a payoff of 50—as a gunfight is avoided—and all other outcomes yield a lower payoff, so the expected payoff from any drawing must be less. More explicitly, the equilibrium conditions for Bat are

$$\text{Fast Type (Wait): } .6 \times 50 + .4 \times 50 \geq .6 \times 30 + .4 \times 30 \Rightarrow 50 \geq 30$$

$$\text{Slow Type ( Wait): } .6 \times 50 + .4 \times 50 \geq .6 \times 20 + .4 \times 30 \Rightarrow 50 \geq 24.$$

The equilibrium conditions for Curly Bill are

$$\text{Fast Type (Draw): } .65 \times 50 + .35 \times 50 \geq .65 \times 30 + .35 \times 30 \Rightarrow 50 \geq 30$$

$$\text{Slow Type ( Draw): } .65 \times 50 + .35 \times 50 \geq .65 \times 20 + .35 \times 30 \Rightarrow 50 \geq 23.5.$$

c. Is it consistent with Bayes-Nash equilibrium for a gunfighter to draw only if he is slow?

**ANSWER:** Yes. Consider a strategy profile in which each draws when slow and waits when fast. The equilibrium conditions for Bat are

$$\text{Fast Type (Wait): } .6 \times 50 + .4 \times 20 \geq .6 \times 30 + .4 \times 30 \Rightarrow 38 \geq 30$$

$$\text{Slow Type (Draw): } .6 \times 20 + .4 \times 20 \geq .6 \times 50 + .4 \times (-40) \Rightarrow 20 \geq 14.$$

The equilibrium conditions for Curly Bill are

$$\text{Fast Type (Wait): } .65 \times 50 + .35 \times 20 \geq .65 \times 30 + .35 \times 30 \Leftrightarrow 39.5 \geq 30$$

$$\text{Slow Type (Draw): } .65 \times 20 + .35 \times 20 \geq .65 \times 50 + .35 \times (-40) \Leftrightarrow 20 \geq 18.5.$$

**HARRINGTON CHAPTER 10 – EXERCISE 5**

Player 2 has only two strategies  $S_2 = \{a, b\}$

Player 1 has four strategies  $S_1 = \{xx', xy', yx', yy'\}$  where the first component of every strategy pair denotes what player 1 chooses when his type is H and the second component is what he selects when his type is L.

Here, the Bayesian normal form representation of the game is:

		<u>Player 2</u>	
		a	b
<u>Player 1</u>	xx'		
	xy'		
	yx'		
	yy'		

Let's find the EU from strategy profile  $(xx', a)$ ,

$$EU_1 = p * 3 + (1 - p) * 2 = 2 + p$$

$$EU_2 = p * 1 + (1 - p) * 3 = 3 - 2 * p$$

$$\rightarrow (2 + p, 3 - 2p)$$

Similarly, for strategy profile  $(xy', a)$ ,

$$EU_1 = p * 3 + (1 - p) * 3 = 3$$

$$EU_2 = p * 1 + (1 - p) * 1 = 1$$

$$\rightarrow (3, 1)$$

Proceeding in this fashion, we find the complete normal form game to be:

Player 2

Player 1

\	a	b
xx'	2+p, 3-2p	1, 2+p
xy'	3, 1	4-3p, 3p
yx'	2, 3-2p	1+4p, 2
yy'	3-p, 1	4+p, 2p

A) When  $p = 0.75$  the above matrix becomes:

		<u>Player 2</u>	
		a	b
<u>Player 1</u>	xx'	2.75, 1.5	1, <u>2.75</u>
	xy'	<u>3</u> , 1	1.75, <u>2.25</u>
	yx'	2, 1.5	4, <u>2</u>
	yy'	2.25, 1	<u>4.75</u> , <u>1.5</u>

Doing the usual underlining to find best responses for each player, we find that there is a unique BNE: (yy', b).

B) Player 1:

When player 2 selects a, he prefers:  $3 > 2 + p$  for all  $p$

$$3 > 2$$

$$3 > 3 - p \quad \text{Hence, he selects } xx'$$

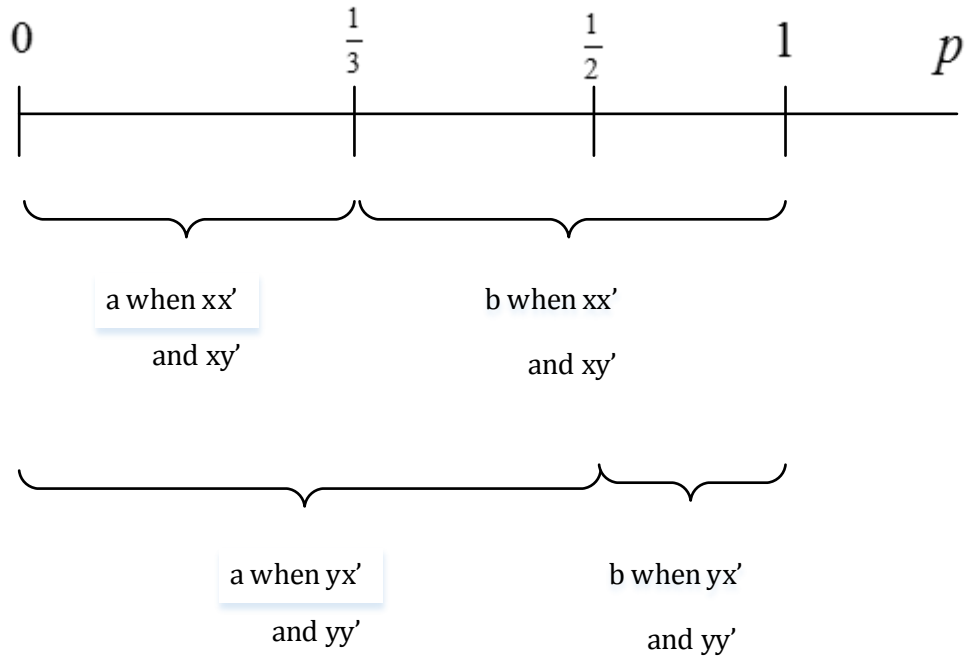
Player 2:

When player 1 selects xx', he prefers a if  $3 - 2p > 2 + p \rightarrow 1 > 3p \rightarrow \frac{1}{3} > p$

When player 1 selects xy', he prefers a if  $1 > 3p \rightarrow \frac{1}{3} > p$  (otherwise he prefers b)

When player 1 selects yx', he prefers a if  $3 - 2p > 2 \rightarrow 1 > 2p \rightarrow \frac{1}{2} > p$

When player 1 selects yy', he prefers a if  $1 > 2p \rightarrow \frac{1}{2} > p$  (otherwise he prefers b)



We can then divide our analysis into three different matrices

- One matrix for  $p < \frac{1}{3}$
- Another matrix for  $p \in \left[\frac{1}{3}, \frac{1}{2}\right]$
- Another matrix for  $p > \frac{1}{2}$

First Case:  $p < \frac{1}{3}$

Player 2

		a	b
Player 1	xx'	$2+p, \underline{3-2p}$	$1, \underline{2+p}$
	xy'	$\underline{3}, \underline{1}$	$4-3p, 3p$
	yx'	$2, \underline{3-2p}$	$1+4p, 2$
	yy'	$3-p, \underline{1}$	$\underline{4+p}, 2p$

Unique BNE: (xy', a)

Second Case:  $p \in \left[ \frac{1}{3}, \frac{1}{2} \right]$

Player 2

		a	b
Player 1	xx'	$2+p, 3-2p$	$1, \underline{2+p}$
	xy'	$\underline{3}, 1$	$4-3p, \underline{3p}$
	yx'	$2, \underline{3-2p}$	$1+4p, 2$
	yy'	$3-p, \underline{1}$	$\underline{4+p}, 2p$



Third Case:  $p > \frac{1}{2}$

Player 2

a

b

Player 1

xx'	$2+p, 3-2p$	$1, \underline{2+p}$
xy'	$\underline{3}, 1$	$4-3p, \underline{3p}$
yx'	$2, 3-2p$	$1+4p, \underline{2}$
yy'	$3-p, 1$	$\underline{4+p}, \underline{2p}$

Unique BNE:  $(yy', b)$

(This BNE is consistent with part (a) of this exercise, where  $p = 0.75 > \frac{1}{2}$ ).