

EconS 424 - Strategy and Game Theory

Quiz #3

Instructions: This is a take-home exam. You are encouraged to work in groups, but each of you must submit your own exam. The due date of the exam is Friday, April 19th, at 11:10am. Late submission will be subject to grade reduction, and no exams will be accepted after the answer key is posted on the course website. Good luck!!

Let us consider a Cournot oligopoly game where two firms compete in quantities. Both firms have the same marginal costs, $MC = \$2$, but they are asymmetrically informed about the actual state of market demand. In particular, Firm 2 does not know what is the actual state of demand, but knows that it is distributed with the following probability distribution

$$p(Q) = \begin{cases} 20 - Q & \text{with probability } \frac{2}{3} \\ 8 - Q & \text{with probability } \frac{1}{3} \end{cases}$$

where $Q = q_1 + q_2$ represents aggregate output. On the other hand, firm 1 knows the actual state of market demand, and firm 2 knows that firm 1 knows this information (i.e., it is common knowledge among the players).

1. Let us first focus on Firm 1, the *informed* player in this game, as we usually do when solving for the Bayesian Nash Equilibrium (BNE) in games of incomplete information.
 - (a) Find firm 1's best response function when the firm operates in a high-demand market. Denote it as $q_1^H(q_2)$.
 - (b) Find firm 1's best response function when the firm operates in a low-demand market. Denote it as $q_1^L(q_2)$.
2. Let us now turn to Firm 2, the *uninformed* player in this game.
 - (a) Write the *expected* profits of this firm, taking into account the above probabilities of operating in a high or low-demand market.
 - (b) Find firm 2's best response function. Denote it as $q_2(q_1^H, q_1^L)$. [Recall that its best response function is only one, since firm 2 does not know whether the market is in high or low demand.]
3. Insert $q_1^H(q_2)$ from exercise 1(a) and $q_1^L(q_2)$ from 1(b) into $q_2(q_1^H, q_1^L)$ from 2(b). Then solve for q_2^* in order to find firm 2's equilibrium production level (this production should be a number).
4. Insert the value of q_2^* you found in part (3) into the expression of $q_1^H(q_2)$ you obtained in exercise 1(a) and in $q_1^L(q_2)$ from 1(b). Summarize your results (you have just found the BNE of this game of incomplete information!).