EconS 424 - Strategy and Game Theory
Quiz #2 - Answer key

1. Consider two firms competing a la Bertrand in a market with demand $Q = 1 - p$. For simplicity, both firms face no production costs, i.e., $c = 0$. Recall the rules of Bertrand competition: if firm i sets a lower price than firm j, $p_i < p_j$, firm i obtains all the market Q and makes profits of

$$p_i \underbrace{(1 - p_i)}_{Q}$$

while firm j gets zero sales (and zero profits). If both firms set the same price, $p_i = p_j$, then both firms obtain half of the above profits, $\frac{1}{2}p_i(1 - p_i)$.

   (a) Considering that firms only interact once (playing an unrepeated Bertrand game), find the equilibrium price for every firm, and the equilibrium profits for every firm.

   - As discussed in class, both firms have incentives to undercut each other's price until they both converge on a common price that coincides with their marginal cost, $c = 0$. Such a profile of equilibrium prices $p_i^B = p_j^B = c$ yields zero profits, $\pi_i^B = 0$, for every firm i where the superscript B denotes Bertrand competition.

   (b) Now assume that they could form a cartel. Which is the price that every firm should set in order to maximize the profits of the cartel? Find the profits that every firm would make in the cartel.

   - In order to find which common price $p_i = p_j = p$ would maximize firms' joint profits, we can solve

$$\max_{p \geq 0} \quad \pi = p(1 - p) = p - p^2$$

     We can now take first-order conditions with respect to p, to obtain $1 - 2p = 0$, and solving for p yields a profit-maximizing price for the cartel of

$$p^C = \frac{1}{2}.$$

   - Given the above price, cartel profits become

$$p^C(1 - p^C) = \frac{1}{2}\left(1 - \frac{1}{2}\right) = \frac{1}{4},$$

     implying that every firm i earns half of that $(\frac{1}{8})$. This cartel profit is higher than what firms make when competing in the unrepeated version of the game (zero profits) as found in part (a).

(c) Let us now analyze if the cartel agreement can be supported as the (cooperative) outcome of the infintitely repeated game. For simplicity, let us use the following grim-trigger strategy: first, firms start cooperating (choosing the cartel price you found in part *b*), and they continue to do so as long as all firms choose this price. If some firm deviates, however, all firms revert to the Bertrand price of part *a*. Assume that both firms assign the same weight to future payoffs (i.e., they both have the same discount factor δ). For which discount factors δ can the cartel agreement be supported in this infinitely repeated game?

- After a history of cooperation, every firm i must decide whether to keep cooperating or to defect. Let's analyze its stream of profits in each case:
  - *Cooperation.* If firm i cooperates, it obtains a stream of cartel profits, 1/8, that is
$$\frac{1}{8}+\delta\frac{1}{8}+\delta^2\frac{1}{8}+...=\frac{1}{1-\delta}\frac{1}{8}$$
  - *Defection.* If, in contrast, firm i defects, it sets a price slightly lower than its rival's cooperative price of $p_j=\frac{1}{2}$, which allows firm i to capture the entire market and maintain the highest mark-up. [Note that the price that firm i sets is slightly lower than 1/2, i.e., $p_i=1/2-\varepsilon$, but we can make such a price sufficiently close to 1/2 by $\varepsilon\to 0$.] Profits then become
$$\frac{1}{2}\left(1-\frac{1}{2}\right)=\frac{1}{4}.$$
    However, such a defection triggers an infinite punishment given the grim-trigger strategy, whereby both firms set a price profile $p_i^B=p_j^B=c$ (which coincides with that in the unrepeated version of the game), thus entailing zero profits thereafter.
  - *Comparison.* Comparing the above two stream of payoffs, yields that every firm i prefers to cooperate as long as
$$\underbrace{\frac{1}{1-\delta}\frac{1}{8}}_{\text{Cooperation}}\geq\underbrace{\frac{1}{4}}_{\text{Deviating period}}+\underbrace{\delta 0+\delta^2 0+...}_{\text{Infinite punishment}}$$
    which simplifies to
$$\frac{1}{1-\delta}\frac{1}{8}\geq\frac{1}{4}$$
    Multiplying both sides by $(1-\delta)$, we obtain
$$\frac{1}{8}\geq\frac{1}{4}(1-\delta)$$
    And rearranging, yields $1\geq 2(1-\delta)$, or $1\geq 2-2\delta$. Finally, solving for discount factor δ, we find
$$\delta\geq\frac{1}{2}$$
    Hence, we need firms to assign a sufficiently high value to future payoffs (in particular, $\delta\geq\frac{1}{2}$) for the cartel agreement to be sustainable.