this cannot be part of a mixed strategy NE.

(ii) If player 2 mixes with the combination ⟨LL,R⟩, we must have $p = \frac{1}{2}$, which gives utilities $u_2(\text{LL}) = u_2(\text{R}) = -49$, and $u_2(\text{M}) = 0$, so this cannot be part of a mixed strategy NE.

(iii) If player 2 mixes with one of the combinations ⟨L,M⟩, ⟨L,R⟩, ⟨M,R⟩, ⟨M,L,R⟩, then player 1 will have D as a strict best response, which in turn has R as player 2's strict best response.

(iv) If player 2 mixes with one of the combinations ⟨LL,M,R⟩, ⟨LL,L,R⟩, or ⟨LL,L,M,R⟩, then the analysis of **(ii)** above implies that this cannot be part of a mixed strategy NE.

(v) If player 2 mixes with the combination ⟨LL,L,M⟩ then the analysis of **(i)** above implies that this cannot be part of a mixed strategy NE.

(c) The choice in part **a)** is not part of any NE described above. It is easy to see that strategy M is rationalizable: If plyer 1 plays $p = \frac{1}{2}$ then M is the unique best response of player 2.

(d) If preplay communication is possible, the players can agree to play one of the pure strategy NE, which are payoff equivalent and Pareto dominant for both players. Therefore, player 2 will play either LL or R depending on the agreed upon equilibrium.

MWG

8.E.1 There are four pure strategies contingent on the type of player:

AA: Attack if either weak or strong type,

AN: Attack if strong and Not Attack if weak,

NA: Not Attack if strong and Attack if weak,

NN: Never attack.

The expected payoff of each pair of strategies can be easily computed and are given in Figure 8.E.1:

Player 2

Player 1	AA	AN	NA	NN
AA	$\frac{M}{4}-\frac{s+w}{2}$, $\frac{M}{4}-\frac{s+w}{2}$	$\frac{M}{2}-\frac{s+w}{4}$, $\frac{M}{4}-\frac{s}{2}$	$\frac{3M}{4}-\frac{s+w}{4}$, $\frac{-w}{2}$	M , 0
AN	$\frac{M}{4}-\frac{s}{2}$, $\frac{M}{2}-\frac{s+w}{4}$	$\frac{M-s}{4}$, $\frac{M-s}{4}$	$\frac{M}{2}-\frac{s}{4}$, $\frac{M-w}{4}$	$\frac{M}{2}$, 0
NA	$\frac{-w}{2}$, $\frac{3M}{4}-\frac{s+w}{4}$	$\frac{M-w}{4}$, $\frac{M}{2}-\frac{s}{4}$	$\frac{M-w}{4}$, $\frac{M-w}{4}$	$\frac{M}{2}$, 0
NN	0 , M	0 , $\frac{M}{2}$	0 , $\frac{M}{2}$	0 , 0

Figure 8.E.1

Any NE of this normal form game is a Baysian NE of the original game.

Case 1: M > w > s, and w > M/2 > s

From the above payoffs we can see that (AA,AN) and (AN,AA) are both pure strategy Bayesian Nash equilibria.

Case 2: M > w > s, and M/2 < s

From the above payoffs we can see that (AA,NN) and (NN,AA) are both pure strategy Bayesian Nash equilibria.

Case 3: w > M > s, and M/2 < s

From the above payoffs we can see that (AN,AN), (AA,NN) and (NN,AA) are pure strategy Bayesian Nash equilibria.

Case 4: w > M > s, and M/2 > s

From the above payoffs we can see that (AA,AN), (AN,AA) and (AN,AN) are pure strategy Bayesian Nash equilibria.

	(B,B)	(B,S)	(S,B)	(S,S)
B	0	1	1	2
S	1	$\frac{1}{2}$	$\frac{1}{2}$	0

Type n_1 of player 1

	(B,B)	(B,S)	(S,B)	(S,S)
B	1	$\frac{2}{3}$	$\frac{1}{3}$	0
S	0	$\frac{2}{3}$	$\frac{4}{3}$	2

Type y_2 of player 2

	(B,B)	(B,S)	(S,B)	(S,S)
B	0	$\frac{1}{3}$	$\frac{2}{3}$	1
S	2	$\frac{4}{3}$	$\frac{2}{3}$	0

Type n_2 of player 2

Figure 54.1 The expected payoffs of type n_1 of player 1 and types y_2 and n_2 of player 2 in Example 276.2.

An exchange game

The following Bayesian game models the situation.

Players The two individuals.

States The set of all pairs (s_1, s_2), where s_i is the number on player i's ticket (an integer from 1 to m).

Actions The set of actions of each player is $\{Exchange, Don't\ exchange\}$.

Signals The signal function of each player i is defined by $\tau_i(s_1, s_2) = s_i$ (each player observes her own ticket, but not that of the other player)

Beliefs Type s_i of player i assigns the probability $\Pr_j(s_j)$ to the state (s_1, s_2), where j is the other player and $\Pr_j(s_j)$ is the probability with which player j receives a ticket with the prize s_j on it.

Payoffs Player i's Bernoulli payoff function is given by $u_i((X, Y), \omega) = \omega_j$ if $X = Y = Exchange$ and $u_i((X, Y), \omega) = \omega_i$ otherwise.

Let M_i be the highest type of player i that chooses *Exchange*. If $M_i > 1$ then type 1 of player j optimally chooses *Exchange*: by exchanging her ticket, she cannot obtain a smaller prize, and may receive a bigger one. Thus if $M_i \geq M_j$ and $M_i > 1$, type M_i of player i optimally chooses *Don't exchange*, because the expected value of the prizes of the types of player j that choose *Exchange* is less than M_i. Thus in any possible Nash equilibrium $M_i = M_j = 1$: the only prizes that may be exchanged are the smallest.

ex. #3 PUBLIC GOOD

(a) We look for an equilibrium where each player contributes if his cost is less than or equal to c^*.

For player i to prefer contributing iff $c_i \le c^*$ the player with cost c* must be exactly indifferent. We must have

$$\begin{aligned} c^* &= \text{Prob(No other player contributes)} \\ &= (1 - P(c^*))^{I-1} \end{aligned}$$

Comparing the values of the left and right sides at $\underline{\theta}$ and 1, we see that there is at least one solution in $(\underline{\theta},1)$. Such a c^* is a Bayesian equilibrium.

(b) Suppose $K \ge 2$, and suppose that all players other than player i play the strategy of never contributing. Regardless of player i's action, the project will not be completed. Thus, player i's payoffs are 0 and $-c_i$ to not contributing and contributing. Not contributing is a best response.

We can also find a symmetric Bayesian equilibrium where player i contributes if and only if $c_i \le c^*$. Again we solve for the c^* which makes a player with cost c^* exactly indifferent, i.e. his cost c^* must be the benefit from the added probability of having the project completed. Player i benefits from contributing if and only if exactly k-1 others have cost below c^*, i.e.

$$c^* = \binom{I-1}{k-1} P(c^*)^{k-1}(1-P(c^*))^{I-k}$$

where $\binom{I-1}{k-1}$ denotes the binomial coefficient. The equation above need not always have a solution.

are $q_2^L = 1/2$ and $q_2^H = 3/8$. Note that player 2 would never produce more than these amounts. To the quantities $q_2^L = 1/2$ and $q_2^H = 3/8$, player 1's best response is $q_1 = 5/16$. Thus, player 1 will never produce more than $q_1 = 5/16$. We conclude that each type of player 2 will never produce more than her best response to 5/16. Thus, q_2^L will never exceed 11/32, and q_2^H will never exceed 7/32. Repeating this logic, we find that the rationalizable set is the single strategy profile that simultaneously satisfies the best response functions, which is the Bayesian Nash equilibrium.

5.

(a) $u_1(p_1, p_2) = 42p_1 + p_1p_2 - 2p_1^2 - 220 - 10p_2$ and $u_2(p_1, p_2) = (22 + 2c)p_2 + p_1p_2 - 2p_2^2 - 22c - cp_1$.

(b) $BR_1(p_2) = \frac{42+p_2}{4}$ and $BR_2(p_1) = \frac{22+2cp_1}{4}$.

(c) $p_1^* = 14$ and $p_2^* = 14$.

(d) $p_1^* = 14$, $p_{2,c=14}^* = 16$, and $p_{2,c=6}^* = 12$.

6.

(LL′, U).

7.

(a)

1 \ 2	X	Y
AA′	0, 1	1, 0
AB′	1/3, 2/3	2/3, 1/3
BA′	2/3, 1/3	5/3, 2/3
BB′	1, 0	4/3, 1

(b) (BA′, Y)

EX. 4, POKER GAME

It is easy to see that, whatever is the strategy of player j, player i's best response has a "cutoff" form in which player i bids if and only if his draw is above some number α_i. This is because the probability of winning when

i bids is increasing in i's type. Let α_j be player j's cutoff. Then, by bidding, player i of type x_i obtains an expected payoff of

$$b(x_i, \alpha_j) = \begin{cases} 1 \cdot \alpha_j + (1 - \alpha_j)(-2) & \text{if } x_i \leq \alpha_j \\ 1 \cdot \alpha_j + (x_i - \alpha_j)(2) + (1 - x_i)(-2) & \text{if } x_i > \alpha_j \end{cases}$$

Note that, as a function of x_i, $b(\cdot, \alpha_j)$ is the constant $3\alpha_j - 2$ up to α_j and then rises with a slope of 4. Player i's best response is to fold if $b(x_i, \alpha_j) < -1$ and bid if $b(x_i, \alpha_j) > -1$. Note that if $\alpha_j > 1/3$ then player i optimally bids regardless of his type (meaning that $\alpha_i = 0$), if $\alpha_j < 1/3$ then player i's optimal cutoff is $\alpha_i = (1 + \alpha_j)/4$, and if $\alpha_j = 1/3$ then player i's optimal cutoff is any number in the interval $[0, 1/3]$. Examining this description of i's best-response, we see that there is a single Nash equilibrium and it has $\alpha_1 = \alpha_2 = 1/3$.

9.

The unique Nash equilibrium is (Bf,B). That is player 1 bids when he has the Ace and folds when he has the King, and player 2 always bids.

27 Lemons, Auctions, and Information Aggregation

2.

Your optimal bidding strategy is $b = v/3$. You should bid $b(3/5) = 1/5$.

4.

(a) Colin wins and pays 82.

(b) Colin wins and pays 82 (or 82 plus a very small number).

(c) The seller should set the reserve price at 92. Colin wins and pays 92.

6.

The equilibrium bidding strategy for player i is $b_i(v_i) = v_i^2/2$.

7.

Let $v_i = 20$. Suppose player i believes that the other players' bids are 10 and 25. If player i bids 20, then she loses and obtains a payoff of 0. However, if player i bids 25, then she wins and obtains a payoff of $20 - 10 = 10$. Thus, bidding 25 is a best response, but bidding 20 is not.

8. EXERCISE #5, INVESTMENT

(a) Clearly, if $p < 200$, then John would never trade, so neither player will trade in equilibrium. Consider two cases for p between 200 and 1,000.

First, suppose $600 \leq p \leq 1,000$. In this case, Jessica will not trade if her signal is $x_2 = 200$, because she then knows that 600 is the most the stock could be worth. John therefore knows that Jessica would only be willing to trade if her signal is 1,000. However, if John's signal is 1,000 and he offers to trade, then the trade could occur only when $v = 1,000$, in which case he would have been better off not trading. Realizing this, Jessica deduces that John would only be willing to trade if $x_1 = 200$, but then she never has an interest in trading. Thus, the only equilibrium has both players choosing "not," regardless of their types.

Similar reasoning establishes that trade never occurs in the case of $p < 600$ either. Thus, trade never occurs in equilibrium. Notably, we reached this conclusion by tracing the implications of common knowledge of rationality (rationalizability), so the result does not rely on equilibrium.

(b) It is not possible for trade to occur in equilibrium with positive probability. This may seem strange compared to what we observe about real stock markets, where trade is usually vigorous. In the real world, players may lack common knowledge of the fundamentals or each other's rationality, trade may occur due to liquidity needs, and there may be differences in owners' abilities to run firms.

(c) Intuitively, the equilibrium strategies can be represented by numbers $\underline{x}_1$ and $\underline{x}_2$, where John trades if and only if $x_1 \leq \underline{x}_1$ and Jessica trades if and only if $x_2 \geq \underline{x}_2$. For John, trade yields an expected payoff of

$$\int_{100}^{\underline{x}_2}(1/2)(x_1 + x_2)F_2(x_2)dx_2 + \int_{\underline{x}_2}^{1000} pF_2(x_2)dx_2 - 1.$$

Not trading yields

$$\int_{100}^{1000}(1/2)(x_1 + x_2)F_2(x_2)dx_2.$$

Simplifying, we see that John's trade payoff is greater than is his no-trade payoff when

$$\int_{\underline{x}_2}^{1000}[p - (1/2)(x_1 + x_2)]F_2(x_2)dx_2 \geq 1. \;(*)$$

For Jessica, trade implies an expected payoff of

$$\int_{100}^{\underline{x}_1}[(1/2)(x_1 + x_2) - p]F_1(x_1)dx_1 - 1.$$

No trade gives her a payoff of zero. Simplifying, she prefers trade when

$$\int_{100}^{\underline{x}_1}[(1/2)(x_1 + x_2) - p]F_1(x_1)dx_1 \geq 1. \;(**)$$

By the definitions of $\underline{x}_1$ and $\underline{x}_2$, (*) holds for all $x_1 \leq \underline{x}_1$ and (**) holds for all $x_2 \geq \underline{x}_2$. Integrating (*) over $x_1 < \underline{x}_1$ yields

$$\int_{100}^{\underline{x}_1}\int_{\underline{x}_2}^{1000}[p - (1/2)(x_1 + x_2)]F_2(x_2)F_1(x_1)dx_2dx_1 \geq \int_{100}^{\underline{x}_1}F_1(x_1)dx_1.$$

Integrating (**) over $x_2 > \underline{x}_2$ yields

$$\int_{100}^{\underline{x}_1}\int_{\underline{x}_2}^{1000}[(1/2)(x_1 + x_2) - p]F_2(x_2)F_1(x_1)dx_2dx_1 \geq \int_{\underline{x}_2}^{1000}F_2(x_2)dx_2.$$

These inequalities cannot be satisfied simultaneously, unless trade never occurs in equilibrium—so that $\underline{x}_1$ is less than 100 and $\underline{x}_2$ exceeds $1,000$, implying that all of the integrals in these expressions equal zero.