EconS 503 - Microeconomic Theory II
Homework #6 - Due date: March 8th, 2019

1. **Exercise from MWG:** 8.E.1

2. **An exchange game.** Each of the two individuals receives a ticket on which there is an integer from 1 to m indicating the size of a prize she may receive. The individuals' tickets are assigned randomly and independently; the probability of an individual's receiving each possible number is positive. Each individual is given the option of exchanging her prize for the other individual's prize; the individuals are given this option simultaneously. If both individuals wish to exchange, then the prizes are exchanged; otherwise each individual receives her own prize. Each individual's objective is to maximize her expected monetary payoff.

   Model this situation as a Bayesian game and show that in any Bayesian Nash equilibrium (BNE) the highest prize that either individual is willing to exchange is the smallest possible prize.

3. **Public good game under incomplete information.** Suppose that there are $N > 2$ players and that a public good is supplied (with benefit 1 for all players) only if at least $K \geq 1$ players contribute. Every player i's cost of contributing is denoted as θ_i, and is independently drawn from the cummulative distribution function $F(\cdot)$ on $[\underline{\theta}, \overline{\theta}]$ where the lower and upper bounds satisfy $\underline{\theta} < 1 < \overline{\theta}$.

   (a) Find a BNE when $K = 1$, that is, the public good is provided if at least one player contributes.

   (b) Assuming that $\underline{\theta} > 0$ and that the public good is only provided if $K \geq 2$, show that there always exists a trivial BNE in which nobody contributes.

   (c) In the setting of part (b), find another BNE (hopefully more interesting than the one you found in part b!).

4. **Incomplete information in a poker game.** Consider the following scenario in a poker game:

   - First, nature selects numbers x_1 and x_2, where $x_i \sim U[0, 1]$ for all $i = \{1, 2\}$.
   - Every player i privately observes x_i, but does not observe x_j where $j \neq i$.
   - Simultaneously and independently, the players choose either to fold or to bid.
   - If both players fold, then they both get a payoff of $-\$1$. If only one player folds, the he obtains $-\$1$ while the other player gets $\$1$. If both players bid, then each player receives $\$2$ if his number is at least as large as the other player's number; otherwise he gets $-\$2$.

   Find the BNE of this game. (*Hint*: Look for a symmetric BNE in which every player bids if and only if his number is greater than some constant α. Your analysis should determine the equilibrium value of α).

5. **Investing in a company under incomplete information.** Suppose John owns a share of stock in Columbus Research, a computer software firm. Jessica is interested in investing in the company. John and Jessica each receive a signal of the stock's value v, which is the dollar amount that the owner will receive in the future. John observes signal x_1, whereas Jessica observes signal x_2. It is common knowledge that x_1 and x_2 are independent random variables with probability distributions F_1 and F_2, respectively. These numbers are between 100 and 1000. The value of the stock is equal to the average of the two signals, that is, $v = \frac{x_1+x_2}{2}$.

   Consider now a game in which John and Jessica may agree to trade the share of stock at a price p, which is exogenously given. (Perhaps some third party or some external market mechanism sets the price.) Simultaneously and independently, John and Jessica select either "trade" or "not trade". A party that chooses "trade" must pay a trading cost of \$1. If they both choose to trade, then the stock is traded at price p; otherwise, the stock is not traded. Therefore:

   - If both choose "trade" then John's payoff is $p-1$ and Jessica's payoff is $v-p-1$.
   - If John chooses "trade" and Jessica chooses "not trade" then John receives $v-1$ and Jessica gets 0.
   - If Jessica chooses trade and John picks "not trade", then John receives v and Jessica receives -1.
   - Finally, if they both say "not trade" John's payoff is v and Jessica's payoff is 0.

   Answer the following questions:

   (a) Suppose that the probability distributions coincide, $F_1 = F_2 = F$, and that they assign positive probability to just two numbers, $x_i = 200$ with probability $\frac{1}{2}$ and $x_i = 1,000$ with probability $\frac{1}{2}$, for all i. In other words, there are two types of player John (one observing a signal of $x_1 = 200$ and another observing a signal of $x_1 = 1,000$) and two types of player Jessica (one observing a signal of $x_2 = 200$ and another observing a signal of $x_2 = 1,000$). Assume that price p satisfies $200 < p < 1,000$. Find the BNE of this trading game.

   (b) Does trade occur in the BNE with positive probability? Do the predictions of this model conform to yoru view of the stock market?

   (c) Allowing for a general probability distributions F_1 and F_2, both on the interval $[200, 1,000]$ and with positive density everywhere in its support, and any price p, find the BNE of the trading game.