EconS 503 - Microeconomic Theory II
Homework #5 - Due date: February 25th, 2019

1. **Collusion with probability of being caught - Harrington (2014).**[1] Consider an industry with N firms. For generality, we do not assume whether they compete in quantities or prices yet, nor the inverse demand function or costs they face. Consider that firms are symmetric and in the Nash equilibrium of the unrepeated game, every firm earns profits π^N, so we label the present value of the noncollusive stream as

$$V^N \equiv \pi^N + \delta\pi^N + ... = \frac{1}{1-\delta}\pi^N.$$

When firms collude, each of them earns profit π^C, where $\pi^C > \pi^N$. When a firm unilaterally deviates from the collusive outcome, it earns a deviating profit of π^D, where $\pi^D > \pi^C$ in that period. Consider a standard Grim-Trigger strategy (GTS) where every firm chooses to collude in period $t = 1$, and continues to do so in subsequent periods $t > 1$ if all firms colluded in previous periods. If one firm did not cooperate in previous periods, however, all firms revert to the Nash equilibrium of the unrepeated game, earning π^N thereafter (permanent punishment scheme). For simplicity, assume that all firms exhibit the same discount factor $\delta \in (0, 1)$.

   (a) Find the minimal discount factor δ that sustains this GTS as a subgame perfect equilibrium of the game.

   (b) For the rest of the exercise, let us assume that the cartel faces a exogeneous probability p of being discovered, prosecuted, and convicted, by a regulatory agency such as the Federal Trade Commission. If caught and convicted in period t, a firm must pay a fine F^t, where $F^t = \beta F^{t-1} + f$. Parameter $1 - \beta$ can be understood as the depreciation rate, which we assume to satisfy $\beta \in (0, 1)$ to guarantee that the penalty is bounded. In addition, assume that $F^0 = 0$, so that $F^1 = f$, $F^2 = \beta F^1 + f$, and similarly for subsequent periods. Find the collusive value $V^C(F)$ given an accumulated penalty F. [*Hint*: Solve for $V^C(F)$ recursively.]

   (c) Write down the condition (inequality) expressing that every firm has incentives to collude, obtaining $V^C(F)$ rather than deviating. For simplicity, you can assume that if the cartel is convicted during the deviation period, it has no chances of being caught during the permanent punishment phase.

   (d) The steady-state penalty is $F = \frac{f}{1-\beta}$, which is found by solving $F = \beta F + f$. Evaluate the collusive value $V^C(F)$ at this penalty, and insert your result in the condition you found in part (c) of the exercise. Rearrange and interpret.

[1] Harrington, Joseph E. Jr. (2014) "Penalties and the Deterrence of Unlawful Collusion," *Economic Letters*, 124, pp. 33-36.

(e) *Bertrand competition.* Assume that firms compete a la Bertrand, selling homogeneous products with inverse demand function $p(Q) = 1 - Q$ where Q denotes aggregate output. All firms face a symmetric marginal cost $c > 0$. In this setting, every firm obtains zero profits in the Nash equilibrium of the unrepeated game, entailing $\pi^N = 0$. If a firm unilaterally deviates from the collusive price (charging a price infinitely close, but below, the collusive price), it captures all industry sales, earning a profit $\pi^D = N\pi^C$ during the deviating period. Evaluate your results from part (d) of the exercise in this context. Then discuss whether collusion becomes easier to sustain when the penalty f increases; and when the number of firms N increases.

2. **Cournot competition when all firms are uninformed - Continuous costs.** Consider two firms competing a la Cournot where every firm i is uninformed about its rival's production costs. Assume that inverse demand function is $p(Q) = 1 - Q$, where Q denotes aggregate output, and that marginal costs are drawn from a continuous, rather than discrete, distribution. In particular, consider that every firm i's marginal cost c_i is drawn from a uniform distribution $c_i \sim U[0, \bar{c}]$ where $\bar{c} > 0$. For simplicity, we assume that the costs of firms i and j, that are, c_i and c_j, respectively, are independently and identically distributed.

   (a) Find the best response function for firm i, $q_i(q_j)$.

   (b) Use your results from part (a) to find the Bayesian Nash Equilibrium (BNE) of the game.

   (c) How do the equilibrium output levels you found in part (b) are affected by changes in c_i, c_j, and $\bar{c}$? Interpret.

3. **Stackelberg game under incomplete information**. Consider a Stackelberg game between a leader (firm 1) and a follower (firm 2). Inverse demand function is given by $p(Q) = 1 - Q$, where Q denotes aggregate output. The leader's marginal cost $MC_1 = \frac{1}{4}$ is common knowledge among the players. Intuitively, the leader is the industry incumbent, and all firms can estimate its costs with relative accuracy. However, the follower's marginal costs are either high $\left(MC_2 = \frac{1}{3}\right)$ or low $\left(MC_2 = \frac{1}{5}\right)$ with probabilities p and $1 - p$ respectively. The entrant is a newcomer, and thus the leader cannot perfectly observe the follower's costs.

   (a) Find the follower's best response function.

   (b) Find the leader's profit-maximizing output, and summarize the subgame perfect Nash equilibrium of the game.

   (c) How do your results in parts (a) and (b) would change if the leader could perfectly observe the follower's marginal costs when these costs are high? What if the follower's costs are low?

4. **Bargaining under incomplete information.** Consider the following bargaining game between a seller and a buyer. The buyer is privately informed about his value for the good, v, drawn from a cumulative distribution function $F_1(v)$ with positive density

in its support. The game starts when the seller makes an offer to the buyer in the first period (a price p_1), which the buyer chooses to accept or reject. If the buyer accepts, the game is over, with payoff p_1 for the seller and $v - p_1$ for the buyer. If he rejects, the seller observes the rejection, and the game proceeds to the second period. In the second period, the seller makes an offer to the buyer (a price p_2), and the buyer chooses to accept or reject it. If the buyer rejects offer p_2, both players' payoff is zero. If the buyer accepts offer p_2, the seller's payoff is δp_2, where $\delta \in [0, 1]$ denotes both players' discount factor; and the buyer's payoff is $\delta (v - p_2)$. The seller's prior distribution in the first period is $F_1(v)$. If the buyer rejects offer p_1, the seller updates his beliefs in the second period to $F_2(v|p_1)$. We next study the PBE of the game by analyzing the second-period game first, and then moving on to the first period.

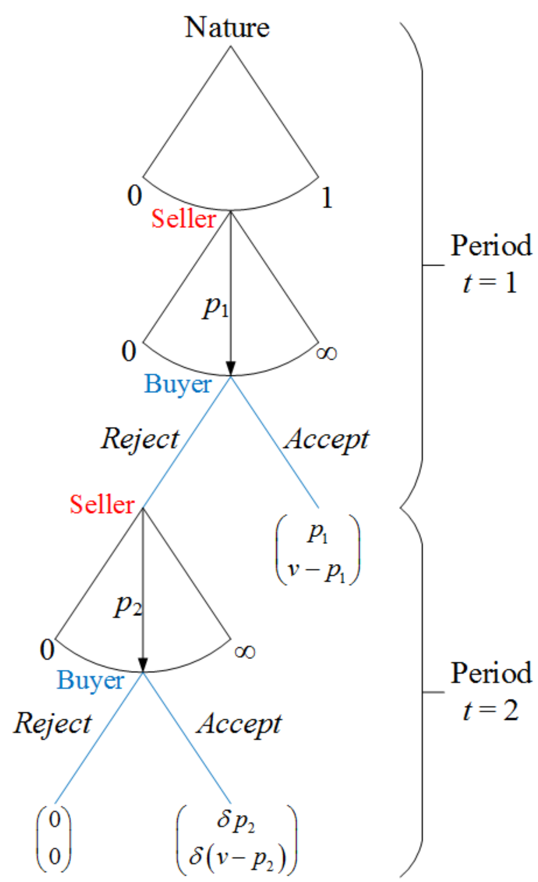

Figure 1. Game tree of the sequential bargaining game.

(a) *Second period.* Find the buyer's acceptance rule in the second period. Anticipating this acceptance rule, find the seller's offer p_2 in this period. [*Hint*: Consider the critical type of buyer, who is indifferent between accepting and rejecting the offer in period 1.]

(b) Assuming a uniformly distributed valuation, where $v \sim F[0, 1]$, what is the seller's offer p_2 in the second period? [*Hint*: It should be an expression in terms of offer p_1.]

(c) *First period.* Find the buyer's acceptance rule in the first period. (In this case, note that the buyer considers his equilibrium payoff in the second period if he rejects p_1 now.) Anticipating the buyer's acceptance rule, find the seller's offer p_1 in this period.

(d) *Numerical Example.* Evaluate the optimal price in period 1, in period 2, and valuation of the critical bidder when $\delta = 0.5$. Then evaluate your results again when $\delta = 0$, and when $\delta = 1$.

5. **First-price auction with entry fees.** Consider a first-price auction with N bidders. Every bidder i's valuation, v_i, is distributed according to a cumulative distribution function $F(v_i)$, with positive support, i.e., $f(v_i) > 0$ for all $v_i \in [0, \overline{v}]$. Consider the following two-stage game: in the first stage, the seller sets an entry fee $E \geq 0$ that every participating bidder must pay, otherwise his bid is ignored; in the second stage, every bidder i independently and simultaneously submit his bid for the object.

   (a) *Second stage.* Starting from the second stage, find the optimal bidding function for bidder i, $b_i(v_i)$ [*Hint*: Assume that there exists a critical bidder whose valuation v_e makes him indifferent between participation or not, given a positive entry fee E].

   (b) How are equilibrium bids affected by an increase in the entry fee E? Do they limit participation in the auction?

   (c) Assume that bidders' valuations are uniformly distributed, that is, $F(v_i) = v_i$ for all $v_i \sim U[0, \overline{v}]$. Evaluate the optimal bidding function found in part (a).

   (d) *First stage.* Anticipating the optimal bidding function $b_i(v_i)$ you found in part (a), what is the optimal entry fee E^* that the seller sets in the first stage to maximize his expected revenue from the auction? For simplicity, assume that the critical bidder, who is indifferent between participation or not, submits a bid, $b_e(v_e) = 0$.

   (e) *Uniformly distributed valuations.* Evaluate the optimal bidding function, $b_i^*(v_i)$, and the optimal entry fee E^* you found in parts (c) and (d) respectively when valuations are uniformly distributed, that is, $F(v_i) = v_i$ for all $v_i \sim U[0, \overline{v}]$. How does the bidding function change with bidder i's valuation, v_i, and the number of bidders, N?