EconS 424 - Strategy and Game Theory
Homework #4 - Due date: February 15th, in class.

1. **Exercises from Watson's textbook, Chapter 16:**

   - Exercises 2, 6 and 8. [See scanned pages at the end of this assignment.]

BONUS EXERCISE:

1. [**Excessive entry in an industry**] Consider a homogenous good industry with perfectly symmetric firms. The firms play the following game. In the first stage, they simultaneously decide whether to enter into the industry. If they enter, they have to incur a fixed set-up cost $F > 0$, and they compete a la Cournot (i.e., firms compete in quantities). To facilitate your calculations, let's assume a marginal cost $c = 0$. Market demand is characterized by $p(Q) = 1 - Q$, where p is the market price and Q the aggregate output.

   (a) Find the number of firms n that would enter at the subgame perfect Nash equilibrium. (For simplicity, consider n as continuous in the whole exercise.)

   (b) Consider now the following game. In the first stage, a social planner decides the number of firms (and pays the fixed costs of firms). The social planner maximizes welfare, as given by the sum of consumer surplus and the producer surplus minus the entry costs every firm has to pay. In the second stage, consider the same quantity competition game. Which is the socially optimal number of firms that the social planner selects? Compare it against the number of firms that would enter at the subgame perfect Nash equilibrium you found in part (a).

(c) Calculating the firms' equilibrium payoffs in part (a), we see that the joint profit is

$$(152.50 - 10) \cdot 4{,}750 = 676{,}875.$$

The manufacturer's profit in part (b) is

$$95 \cdot 9{,}500 = 902{,}500.$$

The difference arises because, in a market in which a monopoly firm must set a single price per unit of output, the monopolist optimally raises the price above marginal cost. This implies an inefficiently low level of trade. In part (a), such a monopoly distortion occurs *twice*—by the manufacturer to the retailer, and again by the retailer to the consumers. But in part (b), distortion occurs just once because the monopolist manufacturer sells directly to consumers.

EXERCISES

1. Consider the model of advertising and Cournot competition analyzed in this chapter. Suppose the two firms could write an externally enforced contract that specifies an advertising level a and a monetary transfer m from firm 2 to firm 1. Would the firms write a contract that specifies $a = 3$? If not, to what level of advertising would they commit?

2. Continuing with the advertising model, suppose the firms compete on price rather than quantity. That is, quantity demanded is given by $Q = a - p$, where p is the price consumers face. After firm 1's selection of the level of advertising, the firms simultaneously and independently select prices p_1 and p_2. The firm with the lowest price obtains all of the market demand at this price. If the firms charge the same price, then the market demand is split equally between them. (To refresh your memory of the price-competition model, review the analysis of Bertrand competition in Chapter 10.) Find the subgame perfect equilibrium of this game and explain why firm 1 advertises at the level that you compute.

3. Consider a variation of the limit-capacity model analyzed in this chapter. Suppose that, instead of the firms' entry decisions occurring sequentially, the firms act simultaneously. After observing each other's entry decisions, market interaction proceeds as in the original model. Find the subgame perfect Nash equilibria of this new model and compare it/them to the subgame perfect equilibrium of the original model.

4. [von Stackelberg duopoly model] Imagine a market in which two firms compete by selecting quantities q_1 and q_2, respectively, with the market price given by $p = 1000 - 3q_1 - 3q_2$. Firm 1 (the incumbent) is already in the market. Firm 2 (the potential entrant) must decide whether or not to enter and, if she enters, how much to produce. First the incumbent commits to its production level, q_1. Then the potential entrant, having seen q_1, decides whether to enter the industry. If firm 2 chooses to enter, then it selects its production level q_2. Both firms have the cost function $c(q_i) = 100q_i + F$, where F is a constant fixed cost. If firm 2 decides not to enter, then it obtains a payoff of 0. Otherwise, it pays the cost of production, including the fixed cost. Note that firm i in the market earns a payoff of $pq_i - c(q_i)$.

   (a) What is firm 2's optimal quantity as a function of q_1, conditional on entry?

   (b) Suppose $F = 0$. Compute the subgame perfect Nash equilibrium of this game. Report equilibrium strategies as well as the outputs, profits, and price realized in equilibrium. This is called the *Stackelberg* or *entry-accommodating outcome*.

   (c) Now suppose $F > 0$. Compute, as a function of F, the level of q_1 that would make entry unprofitable for firm 2. This is called the *limit quantity*.

   (d) Find the incumbent's optimal choice of output and the outcome of the game in the following cases: (i) $F = 18{,}723$, (ii) $F = 8{,}112$, (iii) $F = 1{,}728$, and (iv) $F = 108$. It will be easiest to use your answers from parts (b) and (c) here; in each case, compare firm 1's profit from limiting entry to its profit from accommodating entry.

5. Consider a slight variation of the dynamic monopoly game analyzed in this chapter. Suppose there is only one high-type customer (Hal) and only one low-type customer (Laurie).

   (a) Analyze this game and explain why $p_2 = 200$ is not optimal if Hal does not purchase a monitor in period 1. Find the optimal pricing scheme for Tony. Discuss whether Tony would gain from being able to commit not to sell monitors in period 2.

   (b) Finally, analyze the game with one of each type of customer and ownership benefits given in the following figure. In this case, would Tony gain from being able to commit not to sell monitors in period 2?

	Period 1	Period 2
Benefit to Hal	1200	300
Benefit to Laurie	500	200

6. Imagine a market setting with three firms. Firms 2 and 3 are already operating as monopolists in two different industries (they are not competitors). Firm 1 must decide whether to enter firm 2's industry and thus compete with firm 2, or enter firm 3's industry and thus compete with firm 3. Production in firm 2's industry occurs at zero cost, whereas the cost of production in firm 3's industry is 2 per unit. Demand in firm 2's industry is given by $p = 9 - Q$, whereas demand in firm 3's industry is given by $p' = 14 - Q'$, where p and Q denote the price and total quantity in firm 2's industry and p' and Q' denote the price and total quantity in firm 3's industry.

The game runs as follows: First, firm 1 chooses between E^2 and E^3. (E^2 means "enter firm 2's industry" and E^3 means "enter firm 3's industry.") This choice is observed by firms 2 and 3. Then, if firm 1 chooses E^2, firms 1 and 2 compete as Cournot duopolists, where they select quantities q_1 and q_2 simultaneously. In this case, firm 3 automatically gets the monopoly profit of 36 in its own industry. On the other hand, if firm 1 chooses E^3, then firms 1 and 3 compete as Cournot duopolists, where they select quantities q_1' and q_3' simultaneously; and in this case, firm 2 automatically gets its monopoly profit of $81/4$.

(a) Calculate and report the subgame perfect Nash equilibrium of this game. In the equilibrium, does firm 1 enter firm 2's industry or firm 3's industry?

(b) Is there a Nash equilibrium (not necessarily subgame perfect) in which firm 1 selects E^2? If so, describe it. If not, briefly explain why.

7. Consider the location game (in Chapter 8) with nine possible regions at which vendors may locate. Suppose that, rather than the players moving simultaneously and independently, they move sequentially. First, vendor 1 selects a location. Then, after observing the decision of vendor 1, vendor 2 chooses where to locate. Use backward induction to solve this game (and identify the subgame perfect Nash equilibrium). Remember that you need to specify the second vendor's sequentially optimal strategy (his best move conditional on every different action of vendor 1).

8. Consider the following market game: An incumbent firm, called firm 3, is already in an industry. Two potential entrants, called firms 1 and 2, can each enter the industry by paying the entry cost of 10. First, firm 1 decides whether to enter or not. Then, after observing firm 1's choice, firm 2 decides whether to enter or not. Every firm, including firm 3, observes the choices of firms 1 and 2. After this, all of the firms in the industry (including firm 3) compete in a Cournot oligopoly, where they simultaneously and independently select quantities. The price is determined by the inverse demand curve $p = 12 - Q$, where Q is the total quantity produced in the industry. Assume that the firms produce at no cost in this Cournot game. Thus, if firm i is in the industry and produces q_i,

then it earns a gross profit of $(12 - Q)q_i$ in the Cournot phase. (Remember that firms 1 and 2 have to pay the fixed cost 10 to enter.)

(a) Compute the subgame perfect equilibrium of this market game. Do so by first finding the equilibrium quantities and profits in the Cournot subgames. Show your answer by designating optimal actions on the tree and writing the complete subgame perfect equilibrium strategy profile. [Hint: In an n-firm Cournot oligopoly with demand $p = 12 - Q$ and 0 costs, the Nash equilibrium entails each firm producing the quantity $q = 12/(n + 1)$.]

(b) In the subgame perfect equilibrium, which firms (if any) enter the industry?

9. This exercise will help you think about the relation between inflation and output in the macroeconomy. Suppose that the government of Tritonland can fix the inflation level $\dot{p}$ by an appropriate choice of monetary policy. The rate of nominal wage increase, $\dot{W}$, however, is set not by the government but by an employer–union federation known as the ASE. The ASE would like *real* wages to remain constant. That is, if it could, it would set $\dot{W} = \dot{p}$. Specifically, given $\dot{W}$ and $\dot{p}$, the payoff of the ASE is given by $u(\dot{W}, \dot{p}) = -(\dot{W} - \dot{p})^2$. Real output y in Tritonland is given by the equation $y = 30 + (\dot{p} - \dot{W})$. The government, perhaps representing its electorate, likes output more than it dislikes inflation. Given y and $\dot{p}$, the government's payoff is $v(y, \dot{p}) = y - \dot{p}/2 - 30$. The government and the ASE interact as follows. First, the ASE selects the rate of nominal wage increase. Then the government chooses its monetary policy (and hence sets inflation) after observing the nominal wage increases set by the ASE. Assume that $0 \leq \dot{W} \leq 10$ and $0 \leq \dot{p} \leq 10$.

(a) Use backward induction to find the level of inflation $\dot{p}$, nominal wage growth $\dot{W}$, and output y, that will prevail in Tritonland. If you are familiar with macroeconomics, explain the relationship between backward induction and "rational expectations" here.

(b) Suppose that the government could commit to a particular monetary policy (and hence inflation rate) ahead of time. What inflation rate would the government set? How would the utilities of the government and the ASE compare in this case with that in part (a)?

(c) In the "real world," how have governments attempted to commit to particular monetary policies? What are the risks associated with fixing monetary policy before learning about important events, such as the outcomes of wage negotiations?

10. Regarding the dynamic monopoly game, can you find ownership values for Hal and Laurie that make scheme A optimal? Can you find values that make scheme B optimal?