Strategy and Game Theory: Practice Exercises with Answers,

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Chapter 1 – Dominance Solvable Games

- Page 1, Introduction. At the end of the second paragraph the index "*i*" must be in italics.
- Page 1, Introduction. At the end of the third (last) paragraph, the sixth line from the final "lover" must be changed by "lower".
- Exercise 7, Page 11. Figure 1.14 should delete column *x* rather than *z*. The figure and subsequent text should be changed as follows:

		Player 2	
		y	z
Player 1	b	1,3	0,2
	c	2,1	1,2
	d	0,1	2,4

Figure 1.14. Reduced Normal-form game

"We can now move to player 1 again. For him, strategy *c* strictly dominates *b*, since it provides an unambiguously larger payoff than *b* regardless of the strategy selected by player 2 (regardless of the column). In particular, when player 2 chooses *y* (left-hand column), player 1 obtains a payoff of 2 from selecting strategy *c* but only one from strategy *b*. Similarly, if player 2 chooses *z* (in the right-hand column), player 1 obtains a payoff of one from strategy *c* but a payoff of zero from strategy *b*. As a consequence, strategy *b* is strictly dominated, which allows us to delete strategy *b* from the above matrix, obtaining the reduced matrix in figure 1.15.

		Player 2	
		y	z
Player 1	c	2,1	1,2
	d	0,1	2,4

Figure 1.15. Reduced Normal-form game

At this point, note that returning to player 2 we note that *z* strictly dominates *y*, so we can delete strategy *y* for player 2 and finally, considering player 2 always chooses *z*, for player 1strategy *d* strictly dominates *c*, since the payoff of 2 is higher than one unit derived from playing *c*. Therefore, our most precise equilibrium prediction after using IDSDS are the solely remaining strategy profile (*d,z*), indicating that player 1 will always choose *d*, while player 2 will always select *z*."

- Exercise 8, Page 14. In the third paragraph after x_1=0 change "yields" by "does not yield". Similarly, after x_2=0 change "but the same" by "but lower"
- Exercise 10, Page 17. The reference to "Fig. 1.21" should be changed for "Fig. 1.23."
- Exercise 11, Page 20.
    - In the first displayed equation at the top of the page, the multiplicative sign should have *i*=1 in its subscript below and *I* in the superscript above.
    - In the second displayed equation, its second line should have subscript *i* everywhere instead of *j*.
    - In the third displayed equation, the subscript of the multiplicative sign should be $j \neq i$ instead of $j = 1$.
- Exercise 12, Page 21. Starting the answer change the word "above" for "below".

Chapter 2 – Nash equilibrium and Simultaneous-move games with complete information

- Page 25, Introduction. Nash equilibrium: Change the word "people" for "profile".
- Exercise 4.
    - Page 31. After *c*>0, add "with no fixed costs"
    - Page 32. At the end of the second paragraph of the answer key, "*i*" should be in italics.
    - Page 33. In the figures the best response functions are represented by the acronym *BRF*. To be consistent with the acronym used in the accompanying text, all acronyms in the figures should be *BR*.
- Exercise 8.
    - Page 44. At the end, before the last paragraph starting with "Therefore" change the payoff of $3 for a payoff of $2.
    - Page 46.
        - Second displayed equation should read $2m_x - m_x^2 + 3 \geq 4 - (n - m_x)$
        - The solution to the quadratic equation, at the beginning of the second paragraph, should be $m_x = \frac{-1 \pm \sqrt{1-4(1-n)}}{2}$. The results of the rest of the exercise do not change.
- Exercise 9, Page 48.
    - Add at the end of the first case at the top of the page (immediately after "as depicted in Fig. 2.29"): Similarly, when the location of the three candidates satisfies $x_D^* = x_R^* = x_I^* > 1/2$ each candidate has incentives to deviate towards the left, while if $x_D^* = x_R^* = x_I^* < 1/2$ each candidate has incentives to deviate to the right.
    - Second case. End of the first line. Replace "two candidates chooses" for "two candidates choose"
- Exercise 10, Page 51. The second paragraph in the answer key, the best response function is referred as *BRF*. For consistency, it should be referred to as *BR*.
- Exercise 12.
    - Page 54,. At the end of paragraph 2 instead of "…while the probability of being caught is $\frac{1}{1+xy}$" should be "…while the probability of not being caught is $\frac{1}{1+xy}$."

- Page 55. At the end of part (a) it should read "$BR_G, x(y) = \frac{y}{c^2}$" instead of "$BR_G, x(y) = \frac{y}{c}$". In addition, the next sentence should read "it is convenient to solve for y which yields $y = c^2x$" instead of "it is convenient to solve for y which yields $y = cx$".
- Page 55. Middle of page, makes a reference to "Fig. 2.29" which should be changed to "Fig. 2.32."
- Page 55. In part (b), please replace the sentence "…you can plug in the first expression into the second expression" for "…you can plug in the second expression into the first expression".
- Page 55. The title of Figure 2.32 should be "Incentives and Punishment" instead of "Lobbying-Best response functions and Nash equilibrium".
- Page 56. The title of Figure 2.33 should be "Incentives and Punishment-Comparative Statics" instead of "Lobbying-Comparative Statics".

- Exercise 13.
  - Page 57. The last sentence at the end of part (a) should read "Hence, equilibrium prices are $p_C = 1 - \frac{1-c}{4} - \frac{1-c}{4} - \frac{1-c}{4} = 1 - 3\left(\frac{1-c}{4}\right) = \frac{1+3c}{4}$, and every firm's equilibrium profits are $\pi_C = \left(\frac{1+3c}{4} - c\right)\frac{1-c}{4} = \frac{(1-c)^2}{16}$."
  - Page 58.
    - Last paragraph, fifth line from the bottom, "above this cutoff indicate parameters indicate" must be changed for "above this cutoff indicate".
    - Last paragraph, third line from the bottom, should read "the cost-saving parameter, *e*, is lower than".

Chapter 3 – Mixed strategies, strictly competitive games, and correlated equilibria

- Exercise 2, Page 69. At the msNE listed immediately above part (c) of the exercise, the second parentheses should have 1/2 for both sets of probabilities.
- Exercise 5.
  - Page 76. The first paragraph should say "Similarly, let p_1 represent the probability that player 1 chooses T, p_2 the probability…"
  - Page 76. The second displayed equation in the triplet at the center of the page should read "$EU_1(C) = 1q + 2(1 - q) = 2 - q$".
  - Page 77. The end of the first paragraph, it should read "showing that only some of them can be sustained in equilibrium."
  - Page 77. In the section "*Mixing between T and C alone*" player 1's indifference condition is $EU_1(T) = EU_1(C)$, which in the next line becomes $1 + 2q = 2 + q$. This displayed equation should actually read $1 + 2q = 2 - q$. The text to the right-hand side of the displayed equation should be then changed for "which yields $q = \frac{1}{3}$. The following paragraph (after the displayed equation) should read "Hence, player 1 randomizes between T and C, assigning a probability of $q = \frac{1}{3}$ to T and $1 - q = \frac{2}{3}$ to C. Last, the last displayed equation of the page should read "$2 - q = 2$, which yields $q = 0$."
  - Page 78. The displayed equation at the top of the page should read $1 + 2q = 2 - q = 2$. The line immediately after should then read "Providing us with two equations, $1 + 2q = 2$ and $2 - q = 2$, which cannot simultaneously hold, i.e., $1 + 2q = 2$ entails $q = 1/2$ while $2 - q = 2$ yields $q = 0$.
  - Page 78. Delete the last paragraph of Exercise 5, starting at "Hence, the unique msNE…" and ending at "…probability on each as well."
- Exercise 7. Page 82. In the last paragraph of the page, replace "this result in Fig. 3.19 by noticing that" with "this result in Fig. 3.22 by noticing that"

- Exercise 8.
  - Page 85. The first paragraph of part (c) should end with a parenthesis, "…as illustrated in Fig. 3.27)."
  - Page 86. The last paragraph of Exercise 8 should include the following explanation at the end of the paragraph: "The equation in of the line connecting points $(2,7)$ and $(7,2)$ is $u_2 = 9 - u_1$. To see this, recall that the slope of a line can be found in this context with $m = \frac{2-7}{7-2} = -1$, while the vertical intercept is found by inserting either of the two points on the equation. For instance, using $(2,7)$ we find that $7 = b - 2$ which, solving for b, yields the vertical intercept $b = 9$. It is then easy to check that point $(4.5,4.5)$ lies on this line since $4.5 = 9 - 4.5$ holds with equality."
- Exercise 9.
  - Page 88. The msNE displayed at the center of the page should read $\left\{\left(\frac{2}{3}U, \frac{1}{3}D\right), \left(\frac{1}{3}L, \frac{2}{3}R\right)\right\}$.
  - Page 89. Figure 3.31 should have the labels changed to "(1,2), psNE (D,R)" in the upper left-hand side of the figure, and to "(2,1), psNE (U,L)" in the lower right-hand side of the figure.
  - Page 91. Figure 3.33 should have the labels changed to "(1,5), psNE (D,R)" in the upper left-hand side of the figure, and to "(5,1), psNE (U,L)" in the lower right-hand side of the figure.
- Exercise 11.
  - Page 99. The displayed equation in part (b) should be changed to: "we find that player 1 prefers the latter, i.e.

$$u_1(C, NC) = -10 \ < \ 0 = u_1(NC, C)$$

    while player 2 prefers the former i.e.,

$$u_2(C, NC) = 0 \ > \ -10 = u_2(NC, C)."$$

  - Page 100. The third paragraph starting at "Hence, this game is…" should have $s = (NC, NC)$ rather than $s = (NC; NC)$.
- Exercise 12.
  - Page 102. The last displayed equation of the page should read $3q + 3 = 6 - 6p \Leftrightarrow p = 1/3$.
  - Page 105. The displayed equation at the middle of the page should read $6q = 3q + 6(1 - q) \Leftrightarrow q = 2/3$. (Only the part after the arrow needs to be fixed.)
  - Page 106. The displayed equation at the top of the page should read $\left\{\left(\frac{1}{4}Top, \frac{3}{4}Bottom\right), \left(\frac{2}{3}Left, \frac{1}{3}Right\right)\right\}$. In part (d) of the exercise, the second line of expression EU_1 should read $6\left(\frac{1}{4}\right)\left(\frac{2}{3}\right) + 3\left(\frac{2}{3}\right)\left(\frac{3}{4}\right) + 6\left(\frac{1}{3}\right)\left(\frac{3}{4}\right) = 1 + \frac{3}{2} + \frac{1}{2} = 4$. Similarly, the second line of expression EU_2 should read $6\left(\frac{1}{4}\right)\left(\frac{1}{3}\right) + 2\left(\frac{2}{3}\right)\left(\frac{3}{4}\right) = 1.5$. Last, part (e) of the exercise should read "Player 1's expected utility form playing the msNE of the game, 4, coincides with that from playing his maxmin strategy, 4. A similar argument applies to Player 2, who obtains an expected utility of 1.5 under both strategies."

Chapter 4 – Sequential-move games with complete information

- Exercise 1.
  - Page 108. Two lines before the displayed equation, "his monetary payoff is lower than" should be replaced for "his monetary payoff is higher than"
  - Page 109. The equation $1 - m$, immediately after the first displayed equation, should have a minus sign (not a dash) between the 1 and the m.

- Page 109. At the equilibrium payoffs at the center of the page (displayed equation), the payoff from player 1 should read $\frac{1+\alpha}{1+2\alpha}$. That is, the numerator should be $1+\alpha$ rather than $1-\alpha$.
- Page 109. As a consequence, the derivative two lines below should read $\frac{\partial(1-m^*)}{\partial\alpha} = -\frac{1}{(1+2\alpha)^2}$. This derivate is still negative, so all the subsequent intuition at the end of page 109 remains unaffected.

- Exercise 4. Page 115. In the sixth line, it should read "player 2 has the opportunity to give any, all, or none".
- Exercise 5.
  - Page 117: instead of $Q = q_1 + q_2$ it should be $Q = q_L + q_F$.
  - Page 118. At the bottom of the line, after "which simplifies into" the displayed equation should be $\frac{1}{2}[(1+c_F) - q_L]q_L - c_L q_L$. Hence, the last displayed equation of page 118 should read $\frac{1}{2}(1+c_F) - q_L - c_L = 0$. All subsequent calculations in this exercise are correct.
  - Page 122: in the figure, the Cournot output should be $\frac{2(1-c)}{3}$.
- Exercise 6. Page 125. Both profits at the top of the page should be corrected to $\pi_1 = 5027.7$ and $\pi_2 = 5181.62$, respectively.
- Exercise 7, Page 127. Last displayed equation of the exercise should read, in its second term, $8 - \frac{1}{2}q_1$ rather than $8 - \frac{1}{2}q_2$ since it describes firm 2's best response function $q_2(q_1)$.
- Exercise 12. Page 134. In the seventh line, it should read: "If, instead, the new iPhone is introduced"

Chapter 5 – Applications to industrial organization

- Exercise 5 (Strategic advertising and product differentiation).
  - Page 161. After the sentence "Plugging firm *j*'s best response function into firm *i*'s, we find" the expression should have a d_j in the last numerator on the right-hand side (end of parenthesis) rather than a d_i.
  - Page 161. Expression 5.3 should not have a 2 in the second term of the denominator.
  - Page 162. Expression q_j* in the second line should not have a 2 in the second term of the denominator.
  - Page 162. The derivative of output q_j* with respect to d_i should not have a 2 in the second term of the denominator; neither in the first nor in the second expression.
  - Page 162. The derivative of profit π_j with respect to A_i should not have a 2 in the second term of the denominator; neither in the first nor in the second expression.
  - Page 162. The derivative of d_i with respect to A_i should not have a 2 in the second term of the numerator.
- Exercise 7, page 165. The output $q_i = q_j$ in the second paragraph should not have a 2 in the second term of the denominator.
- Exercise 9, page 173. The price p_1 should go in italics.

Chapter 6 – Repeated games and correlated equilibria

- Exercise 8 (Collusion and Imperfect Monitoring).
  - Page 209. The plus sign in $\frac{d(a+dq_i)}{4b^2}$ (firm i's best response function at the center of page 209) should have a minus sign "-" so it reads $\frac{d(a-dq_i)}{4b^2}$.

- Page 210. The first two "4"s in the denominator of the profits from the collusive agreement, π_i^m (at the center of the page), should be 2s, so the expression reads

$$\pi_i^m = \left(a - b\frac{a}{2(b+d)} - d\frac{a}{2(b+d)}\right)\frac{a}{4(b+d)} = \frac{a^2}{8(b+d)}$$

- Page 211. The $\delta\pi_i^D$ in the right-hand side at the bottom of the page should read $(1+\delta)\,\pi_i^D$.
- Page 212. The second displayed equation should have an 8 in the denominator rather than a discount factor δ; reading $\frac{a^2}{8(b+d)}$ rather than $\frac{a^2}{\delta(b+d)}$.
- Page 215. The previous to the last displayed equation should have a "greater than or equal" $\geq$ sign rather than $=$; reading $\frac{a^2}{2(b+d)} \geq \ldots$

Chapter 7 – Simultaneous-move games with incomplete information

- Exercise 1.
  - Page 220. Table 7.2 should have a payoff pair (1,-1) in the cell corresponding to (*Bf*, *Bet*), located in the second row, left column.
  - Page 220. In the first bullet point, it should read "For player 2, his best response when player 2 bets (in the left-hand column) is to play *Bf* since it yields a higher payoff, i.e., 1, than any other strategy, i.e., $BR_1(Bet)=Bf$.
  - Page 220. In the second bullet point, the second sentence should read "If, instead, player 1 chooses *Bf* (in the second row), player 2's best response is to fold, $BR_2(Bf)=Fold$, since his payoff from folding, -1/3, is larger than from betting, -1.
  - Page 221. Table 7.3 should have a payoff pair (1,-1) in the cell corresponding to (*Bf*, *Bet*), located in the second row, left column. In addition, player 2's underlined payoff in the second row should be -1/3 (that corresponding to Fold in the right-hand column) rather than that of betting.
- Exercise 2, page 223. In the paragraph with title "*Player 2's best responses*", the sixth line should read "i.e., $4+p>1$, $4+p>4-3p$, and $4+p>1+4p$, which hold for all values of p."
- Exercise 5, page 234. The displayed equation after "rearranging yields" in the middle of the page should read $B(p-\beta p) > L(p-\beta p)$. All subsequent results are correct.

Chapter 8 – Auctions

- Exercise 3, page 248. The expressions in the section with title "Direct approach" should not have *N*-1 in the exponents. This applies to the expression of the *prob*(*win*), the $EU_i(v_i)$, and its rearranged representation at the bottom of the page.

Chapter 9- Perfect Bayesian equilibrium and signaling games

- Exercise 1.
  - Page 261. The last sentence of the page should read "with U, which provides him a payoff…"
  - Page 262. Part (b) of the exercise should be preceded by space to separate it from the answer key of part (a).
  - Page 265. Footnote 2. Strategy R' should read R', for consistency with the primes in previous parts of the exercise.

- Exercise 2. Page 274. The paragraph labelled *Case 1* should start as follows "When $q > \frac{2}{5}$, the …" The end of this case (immediately before Case 2) should read "beliefs satisfy $q > \frac{2}{5}$." The paragraph labelled *Case 2* should start as follows "When $q \leq \frac{2}{5}$, the firm…:"
- Exercise 3.
  - Page 277. The first paragraph of part (a), in the last line, should read "only stems from a sender"
  - Page 280. The second line after the three expected utility expressions should read "if and only if $\mu > \frac{4}{5}$, $4 - \mu > 5 - 5\mu$" (Now one of the minus signs looks like a hyphen.)
- Exercise 4.
  - Page 285. The expression of p_2^2 immediately above expression (*B*) should have a 2 in the denominator, rather than a 3. As a consequence, expression (*B*) should become $p_2^2 = 1 + \frac{1}{3} f^{-1}(p_1^1)$.
  - Page 286. The expression of p_2^2 in the paragraph starting with "Firm 1 anticipates…" should be corrected for $p_2^2 = 1 + \frac{1}{3} f^{-1}(p_1^1)$. A similar argument applies to the expression plugged at the end of the big parenthesis in the profit maximization problem immediately after this paragraph. As a consequence, the subsequent calculations should be replaced for the following:
  - "Taking first order condition with respect to p_1^1, we obtain

$$1 - 2p_1^1 + p_2^1 + c_1 + 2\left(1 - \tfrac{1}{3}c_1\right)\tfrac{1}{3} f'\left(f^{-1}(p_1^1)\right) = 0.$$

Simplifying and solving for p_1^1, we find firm 1's best response function in the first period game

$$p_1^1(p_2^1) = \frac{1 + c_1}{2} + \frac{p_2^1}{2} + \frac{(3 - c_1) f'\left(f^{-1}(p_1^1)\right)}{9}$$

Inserting this expression of p_1^1 into $p_2^1 = \frac{1+p_1^1}{2}$, we obtain the optimal second-period price for Firm 1

$$p_2^1 = \frac{1 + c_1}{2} + \frac{2}{3} + \frac{2(3 - c_1) f'\left(f^{-1}(p_1^1)\right)}{27}.$$

Plugging this result into the best response function $p_1^1(p_2^1)$, yields the optimal first-period price for Firm 1

$$p_1^1 = \frac{3 + 2c_1}{3} + \frac{4(3 - c_1) f'\left(f^{-1}(p_1^1)\right)}{27}$$

As suggested in the exercise, let us know…."

  - Page 288. This page should be replaced for the following, starting in the expression at the top of the page:

$$A_0 + A_1 c_1 = \frac{3 + 2c_1}{3} + A_1 \frac{4(3 - c_1)}{27}$$

since A_1 measures the slope of the pricing function (see Fig. 9.30), thus implying $A_1 = f'\left(f^{-1}(p_1^1)\right)$. Rearranging the above expression, we find

$$A_1(31c_1 - 12) = 27 + 18c_1 - 27A_0$$

which, solving for A_1, yields

$$A_1 = \frac{27(1 - A_0) + 18c_1}{31c_1 - 12}$$

In addition, when firm 1's costs are nil, $c_1 = 0$, the above expression becomes

$$27A_0 = 27 + 12A_1$$

or, after solving for A_0,

$$A_0 = 1 + \frac{4}{9}A_1$$

Inserting this result into $A_1 = \frac{27(1-A_0)+18c_1}{31c_1-12}$, yields

$$A_1 = \frac{27\left(1 - \left(1 + \frac{4}{9}A_1\right)\right) + 18c_1}{31c_1 - 12}.$$

Solving for A_1, we obtain $A_1 = \frac{18}{31} \approx 0.58$. Therefore, the intercept of the pricing function, A_0, becomes

$$A_0 = 1 + \frac{4}{9}\frac{18}{31} = \frac{39}{31} \approx 1.26.$$

Hence, the pricing function p_1^1 of firm 1, $p_1^1 = A_0 + A_1 c_1$, becomes

$$p_1^1 = \frac{18}{31} + \frac{39}{31}c_1.$$

- Exercise 6, page 296. The two maximization problems in the middle of the page should have $2c$ in the last term, rather than c. In addition, the first-order condition with respect to p_T should have the parenthesis corrected, as follows

$$\lambda - (1 - \lambda)\frac{\theta_T - \theta_B}{\theta_B} = 0$$

Chapter 10 – More advanced signaling games

- Exercise 1.
  - Page 307. The labels below each term at the center of the page should read $P_2's$, with the apostrophe after the P_2.
  - Footnote 1, page 307. It reads "alpha" when it should read α.
- Exercise 3. Delete part (b), both in the question (page 317), and in the answer key (pages 319-322).
- Exercise 4. Page 322, last line should read "they are associated with a higher wage"
- Exercise 5,
  - Page 327. The second line of the answer key of part (a) should read "while a_E is the benefit".
  - Page 328. Figure 10.8 should have all subscripts in capital letters, such as f_I, f_E, a_I, and a_E. In addition, all notations in this exercise should go in italics.
- Exercise 6, page 340. Figure 10.27 should have branch *Acc* shaded in the lower part of the game tree, since the figure represents the separating strategy profile (*Fight*, *Acc*).