EconS 503 - Microeconomic Theory II
Homework #2 - Due date: Friday, January 25th, in class.

1. **Strict Nash equilibrium.** Consider the following definition: A strategy profile $s^* \equiv (s_1^*, ..., s_N^*)$ is a *strict Nash equilibrium* (SNE) if it satisfies

$$u_i(s_i^*, s_{-i}^*) > u_i(s_i, s_{-i}^*) \text{ for every player } i\text{, and every } s_i \in S_i.$$

   You probably noticed that this definition is almost identical to the definition of Nash equilibrium (NE) strategy profiles, except for using a strict, rather than weak, inequality. In this exercise we connect both solution concepts, but first examine the relationship between a strict Nash equilibrium and IDSDS.

   (a) Show that if a strategy s_i^* is a SNE it must also survive IDSDS.

   (b) Show the opposite relationship to that in part (a), that is, if strategy s_i^* survives IDSDS then s_i^* must be a SNE.

   (c) Show that every player has a unique best response to his rivals' strategies.

   (d) Show that if strategy profile s^* is SNE, it must also be a NE.

   (e) Show that if strategy profile s^* is a NE, it doesn't need to be a SNE. An example suffices.

   (f) Show that if strategy profile s^* is a SNE, the game has no mixed strategy NE.

2. **Contests with N players.** In this exercise we examine equilibrium investment in a contest, where $N \geq 2$ players compete to earn a prize of common value V, and the probability of winning the prize is a function of a player's own investment relative to the aggregate investment of all the players. Hence, contests are often used to model promotions within a firm (where every worker invests time and effort into being selected for a promotion), political campaigns (where candidates invest money and resources to capture a larger share of votes), and R&D races (where firms invest resources into discovering a new product, such as a drug). We consider player i's probability of winning the prize is given by

$$p_i = \frac{x_i^r}{\sum_{j=1}^N x_j^r},$$

   where x_i denotes his investment and the parameter, $r \geq 1$, represents the effectiveness of his investment. For simplicity, we normalize the cost of every unit of investment to one dollar.

   (a) Setup and solve the utility maximization program for player i.

   (b) Verify that the equilibrium investment maximizes the expected utility of player i, that is, check second-order conditions.

   (c) *Comparative statics.* How does the equilibrium investment of player i change with V, r, and N?

(d) Find the range of the effectiveness parameter r in which all the players participate.

(e) Define Rent Dissipation as

$$D \equiv V - \sum_{i=1}^{N} x_i^*,$$

which can be understood as how many resources the society is left with all the players are completed with their investments. Find the Rent Dissipation D and show that it is positive for all the admissible values of r.

(f) Is Rent Dissipation increasing or decreasing in the common value that players assign to the prize, V; in the effectiveness parameter r; and in the number of players N?

(g) What happens when all players coordinate their investment decisions?

3. **Exercises from Tadelis:**

(a) Chapter 5: Exercises 5.11 and 5.16.