Mergers between Green and Brown Firms

Output shifts, merging incentives, and welfare effects

Pak-Sing Choi\textsuperscript{a}, Ana Espínola-Arredondo\textsuperscript{b} and Félix Muñoz-García\textsuperscript{c}

School of Economic Sciences, Washington State University, Pullman, WA 99164-6210, USA

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Abstract

This paper considers incentives of firms to merge under duopoly, where we allow for product differentiation, cost asymmetries, and pollution intensities (green and brown goods). We first analyze firms’ mergers under no regulation, and then evaluate mergers after the introduction of environmental regulation. We find that, in the absence of environmental policy, mergers produce an output shift towards the lowest cost firm. When emission fees are introduced, however, firms also consider their relative pollution intensities, potentially reverting the above output shift. We show that firms have stronger incentives to merge when goods are more differentiated, costs are more symmetric, and firms generate similar environmental damages.

Keywords: Horizontal Mergers, Product Differentiation, Pollution Intensity, Emission Fees, Grim-Trigger Strategy

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\textsuperscript{a}Corresponding author. Address: 207E Hulbert Hall, School of Economic Sciences, Washington State University, Pullman WA 99164-6210, USA. E-mail: paksing.choi@wsu.edu

\textsuperscript{b}Address: 111CHulbertHall,SchoolofEconomicSciences,WashingtonStateUniversity,PullmanWA 99164-6210, USA. E-mail: anaespinola@wsu.edu

\textsuperscript{c}Address: 103HHulbertHall,SchoolofEconomicSciences,WashingtonStateUniversity,PullmanWA 99164-6210, USA. E-mail: fmunoz@wsu.edu
1. Introduction

Many industries nowadays include firms with heterogenous pollution intensities, such as extremely polluting companies competing against relatively clean rivals. However, the literature on environmental regulation mostly overlooks this feature by considering markets where firms are either all brown or all green, rather than a combination of both. As we show in this paper, if regulators incorrectly consider that all firms are brown, they would predict a relatively large output (and pollution) reduction from an industry merger, above the actual output reduction that takes place when regulatory agencies consider the heterogeneous characteristics of firms in the industry. Importantly, these considerations could lead regulators to more likely deem mergers as beneficial when they incorrectly assume that firms are homogeneous than when they correctly account for their heterogeneity, allowing mergers in settings where they should have been prohibited.

Our model considers an industry with a brown and a green firm where, for completeness, we allow for cost asymmetries and product differentiation. Under no regulation, merger produces an output shift from the least to the most cost-efficient firm, disregarding their relative pollution intensities since environmental policy is absent. We then consider environmental regulation, examining how this output shift is affected, since now the merged firm shifts output towards the green firm to save emission fees. For example, if the green firm faces a relatively higher cost than its brown rival, but the latter is significantly more polluting than the green firm, post-merger output may not substantially differ from that before the merger. If, however, the green and brown firms are relatively symmetric in costs, the merger will induce an increase in the green firm’s output and a decrease in its rival’s production. When firms are relatively symmetric in costs, we may observe large post-merger output shift when regulation is in place, but would not otherwise. Understanding it can then help us predict how output shares respond after a merger, anticipating if the merger

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1For example, Corbion, a Dutch biochemical company, acquired Terravia, a green firm producing food, aquaculture, and personal care products with microalgae, in 2017 (Brickley, 2017, September 12). In 2012, Enbridge, a Canadian oil-pipeline operator, acquired the 50-megawatt Silver State North photovoltaic project from First Solar Inc., a solar power plant in Nevada in order to honor its neutral footprint commitment (Martin, 2012, March 23). For other mergers in the energy sector, such as mergers between green (renewable energy) and brown (non-renewable energy) firms induced by the cap-and-trade program of Regional Greenhouse Gas Initiative, see Creti and Sanin (2017, Table 1).
induces more/less pollution.\textsuperscript{2}

We also investigate how these output shifts can make the merger more or less attractive when regulation is present. That is, we seek to understand if the introduction of environmental regulation facilitates the emergence of collusion in equilibrium, expanding the range of parameter values for which collusive behavior can be sustained between the brown and green firm. We then examine under which conditions collusion entails a welfare loss, as in standard models of market power, and in which cases it can yield a welfare gain. To understand this point, consider a setting without regulation where firms do not have incentives to merge, say, goods are relatively homogeneous or costs are asymmetric. Regulation in this context could provide firms with the right incentives to merge, as it shifts output from the brown to the green firm, yielding an unambiguous increase in welfare. Specifically, the reduction in the brown firm’s output, and hence consumer surplus, is more than offset by the increase in green good’s production, thereby reducing overall pollution. In other words, not only the emission fee that the regulator sets, but the stronger incentives to merge that this policy unfolds, contribute to a welfare gain. On the other hand, when firms are relatively homogenous and pollution is less severe, mergers entail a reduction in output and increase in price, such that welfare decreases as in standard merger models. To the best of the authors’ knowledge, this is the first paper that evaluates mergers between firms with (i) differentiated goods, (ii) asymmetric costs, and (iii) heterogeneous pollution intensities under endogenous emission fees.

\textit{Literature Review.} A large body of the literature analyzes horizontal mergers (Canton et al., 2012; Lambertini and Tampieri, 2012; Fikru and Gautier, 2016, 2017) in oligopolistic markets that generate environmental externalities. Canton et al. (2012) considers two separate industries, the eco-industry and the polluting industry, where the former supplies abatement goods to the latter, showing that a merger is profitable for homogeneous firms competing à la Cournot in the eco-industry if there is a sufficient number of firms merging together (Salant et al., 1983) under cost convexities (Perry and Porter, 1985).\textsuperscript{3} However, an increase in exogenous emission fees

\textsuperscript{2}For instance, Fikru and Gautier (2016) report that, among a sample of food processors which engaged in mergers and acquisitions during 2001-2012, there was a 17.6\% increase in the average number of process modifications targeting reduction of toxic pollutant releases a year after the deal was announced, that is, a reduction in the output share, and hence pollution, of the brown firm.

\textsuperscript{3}Some recent examples are (1) in the pollution abatement industry, the acquisition of Auburn FilterSense LLC, a US provider of particulate emissions monitors and intelligent controls for industrial particulate/dust filtration systems,
makes mergers less profitable. In particular, a merger between two firms is profitable if market concentration is relatively high so that firms can gain market power upon merger, and also improves welfare under intermediate values of emission fees, that is, fees are neither too high (low) to yield insufficient output (abatement). In contrast, our model deals with endogenous emission fees, and studies how firms react by changing output levels and making merger decisions.

Lambertini and Tampieri (2012) seek to understand incentives for two firms to merge in a Cournot triopoly, and show that socially efficient mergers arise if the reduction in industry output, which makes merger profitable, is more than offset by the parallel reduction in pollution; an observation that conforms with the efficiency defence argument (Salant et al., 1983; Perry and Porter, 1985; Farrell and Shapiro, 1990; Gaudet and Salant, 1991, 1992, among others) but with externalities. The same argument holds in our context without regulation, as in Lambertini and Tampieri (2012); but regulation expands the range of parameter values for which mergers improve social welfare.

To date, the closest articles to our paper are Fikru and Gautier (2016, 2017), who examine mergers in Cournot markets with product differentiation and emission fees. Contrary to Fikru and Gautier (2016), who consider an exogenously imposed change in production technology or product modification that lowers pollution intensity upon merger; our model characterizes output shifts from the brown firm, with a given pollution intensity, to the less polluting green firm. We then evaluate how welfare changes with different pollution intensities and firm specificities. Our research complements Fikru and Gautier (2017), since we also find that regulation should be more stringent in markets with differentiated goods and high pollution intensities. We show, in addition, that stringent regulation is required when firms are relatively symmetric in their production costs, so that output substitution becomes less costly; and when discount rates are relatively high so that firms are more likely to merge. To this end, we contribute to the literature of mergers under environmental regulation by allowing for (i) cost asymmetries, and (ii) repeated interactions; in which firms have incentives to merge based on emission fees and firm heterogeneities (i.e., differentiation, costs, and

by Nederman, the Swedish environmental technology company (Filtration + Separation, 2018, April 11); and (2) in the waste management industry, the acquisition of Quantex Environmental Inc., an Ontario based company, by Covanta Environmental Solutions, a New Jersey based environmental services provider (Waste 360, 2018, February 12).

Our analysis focuses on non-cumulative pollutants, such as carbon dioxide, sulphur dioxide, and suspended particulates; see Benchekroun and Ray Chaudhuri (2008) for cumulative pollutants.

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discount rates).\(^5\)

The remainder of this article is organized as follows. Section 2 sets up the model. Section 3 examines the firms’ incentives to merge under (1) the benchmark setting of no regulation, and (2) the setting of environmental regulation. Section 4 analyzes profit gains for mergers to arise and collusion to sustain, followed by Section 5 on an investigation of welfare effects. Finally, Section 6 concludes with policy implications and directions for future research.

2. Model

There are two firms in the market, \(i\) and \(j\), where \(i, j = \{B, G\}\) stands for brown and green firms, respectively. The brown (green) firm has lower (higher) marginal production cost \(c_B\) (\(c_G\)), where \(c_G \geq c_B \geq 0\), and pollutes the environment more (less) than its rival, that is, the pollution per unit of output of the brown firm, \(d_B\), is larger than that of the green firm \(d_G\). For simplicity, we normalize the green firm’s cost as \(c = c_G - c_B\), which can be interpreted as its cost differential relative to the brown firm. Analogously, we normalize the pollution parameter of the brown firm as \(d = d_B - d_G\) that be understood as its pollution differential relative to the green firm, where \(d_B \geq d_G \geq 0\); yielding environmental damages \(Env(q_B) = dq_B^2\) that are increasing and convex in the units of brown good produced.

The inverse demand function that firm \(i\) faces is

\[
p_i(q_i, q_j) = 1 - q_i - \beta q_j
\]

where \(\beta \in [0, 1]\) represents the degree of product differentiation. When \(\beta = 0\), goods are completely differentiated that become independent of each other (that is, not affecting the demand of the other product); but when \(\beta = 1\), goods are homogenous that become perfect substitutes.

The structure of the game is as follows:

1. Firms choose whether to merge or not.
2. Observing if firms merged, the regulator responds by setting an emission fee for each firm, \(t_k^i\), where \(k \in \{M, NM\}\) stand for before and after mergers, respectively.

\(^5\)Lambertini (2013) analyzes the effect of exogenous emission fees on collusive behavior assuming symmetric firms.
(3) Observing the emission fees set by the regulator, every firm \( i \) independently chooses its output level (if not merge) or coordinate output levels (if merged).

For completeness, we first introduce a setting without regulation and solve for the firms’ equilibrium output before and after mergers. Next, we study a setting in which the regulator charges a fee \( t_i^k \) to induce socially optimal output \( q_i^{SO} \) from firm \( i \), investigating this firm’s difference in profits, \( \Delta \pi_i^l \equiv \pi_i^M - \pi_i^{NM} \) where \( l \in \{ R, NR \} \) stands for the setting of regulation and no regulation, respectively, as well as the minimum discount rates for both firms to sustain mergers, \( \delta^l \in [0, 1] \).

3. Equilibrium analysis

3.1 No regulation

The following Lemma and Corollary characterize the firms’ output before and after merger.

**Lemma 1.** If not merged, the green firm’s equilibrium output is \( q_G^{NM} = \frac{2-\beta-2c}{4-\beta^2} \) and the brown firm’s equilibrium output is \( q_B^{NM} = \frac{2-\beta+\beta c}{4-\beta^2} \) when \( \beta < \bar{\beta} \). If merged, the green firm’s output becomes \( q_G^M = \frac{1-\beta-c}{2(1-\beta^2)} \) and the brown firm’s output becomes \( q_B^M = \frac{1-\beta+\beta c}{2(1-\beta^2)} \) when \( \beta < \frac{\bar{\beta}}{2} \equiv 1 - c \). Otherwise, the green firm sets \( q_G^k = 0 \) and the brown firm chooses \( q_B^k = \frac{1}{2} \), where \( k = \{ M, NM \} \).

When costs are more asymmetric (higher \( c \)) and goods are more homogeneous (higher \( \beta \)), the green firm is relatively uncompetitive to the brown firm so that this firm does not produce any good, as indicated in the unshaded area of Figure 1a. When firms merge, the production set of the green firm shrinks, as indicated in the shaded area of the figure.

**Corollary 1.** Under merger, the green firm reduces output for all values of \( \beta < \bar{\beta} \). The brown firm also decreases output when \( \beta < \beta_{NR}^B \equiv \frac{3-\sqrt{9-8(1-c)^2}}{2(1-c)} \).

When firms merge, they coordinate output and internalize output externalities that each firm imposes on one another, ameliorating the green firm’s cost asymmetry relative to the brown firm so that the brown firm sells fewer units of output as indicated in the shaded area of Figure 1b. However, when costs are more asymmetric and goods are more homogeneous, the merged firm sells more units of brown good, which is more competitive than green good, as indicated in the
unshaded area of Figure 1b. Finally, when $\beta \geq \bar{\beta}$, the green firm is less differentiated that it stays out of the market before and after merger so that the brown firm’s output does not change.

Figure 1: Output with no regulation

The next Corollary compares the firms’ profits before and after merger.

**Corollary 2.** A merger is profitable for the brown firm under all parameter values, and also for the green firm if $\beta \leq \beta_{G}^{NR}$ where cutoff $\beta_{G}^{NR}$ solves

$$
(1 - \beta) (2 - \beta)^2 - (2 - \beta) (8 + \beta^2) c + (8 + \beta^2) c^2 = 0.
$$

In the unshaded area of Figure 2, the green firm suffers a relatively significant cost asymmetry relative to the brown firm, and both firms sell relatively homogeneous goods. This setting leads to a reduction in the green firm’s output and an increase in the brown firm’s output, which ultimately reduces the green firm’s profits. In contrast, in the shaded area, the green firm exhibits a small cost asymmetry relative to the brown firm and goods are more differentiated, entailing that both firms earn higher profits upon merger. Nonetheless, when $\frac{\bar{\beta}}{2} \leq \beta < \bar{\beta}$, the green firm stops production after merger so that its profits decrease while those of the brown firm increase. Lastly, when $\beta \geq \bar{\beta}$,

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6 In particular, when $c > c_{G}^{NR} \approx 0.293$, the green firm has a significant cost asymmetry that a merger cannot be profit-improving even when goods are completely differentiated.
the green firm produces no output that leaves the brown firm’s profits unchanged before and after merger.

Next, let us examine social welfare in the context of no regulation, for $SW(q_G, q_B) = CS + PS - Env(q_B)$ where $CS$ and $PS$ stand for consumer surplus and producer surplus, respectively, that is summarized in the following Corollary.

**Corollary 3.** Under no regulation, a merger between green and brown firms improves welfare if (i) $d < d^{NR}$ and $\beta > \beta_B^{NR}$, or (ii) $d \geq d^{NR}$ and $\beta \leq \beta_B^{NR}$.

Figures 3a (3b) denotes the case of $d < d^{NR}$ ($d \geq d^{NR}$) when environmental damages $d$ are relatively low (high). On the left panel, when the brown firm reduces (increases) output upon merger, social welfare decreases (increases) because consumers have fewer goods to consume. The opposite is said in the right panel, when the environmental damages from additional units of brown good more than offsets the utility that consumers enjoy. Same as before, when $\beta \geq \bar{\beta}$, the brown firm monopolizes the market before and after merger that yields no change in social welfare.
Having solved for the firms’ equilibrium output and welfare comparison under no regulation, we next study the regulator’s emission fees and the firms’ behavior under regulation.

### 3.2 Regulation

The regulator solves the following social welfare maximization problem,

$$SW(q_G, q_B) = CS + PS + Tax - Env(q_B)$$

where Tax represents emission fees. The next Lemma identifies socially optimal output for different firms.

**Lemma 2.** The socially optimal output for green and brown firms are

\[
\begin{align*}
q^G_{SO} &= \frac{(1+2d)(1-c) \cdot 2\beta}{1+2d-4\beta^2} \quad \text{and} \quad q^B_{SO} = \frac{1-2\beta(1-c)}{1+2d-4\beta^2} \quad \text{if} \quad \beta < \beta^R_G \text{ and } \beta < \beta^R_B \\
q^G_{SO} &= 0 \quad \text{and} \quad q^B_{SO} = \frac{1}{1+2d} \quad \text{if} \quad \beta \geq \beta^R_G \text{ and } \beta < \beta^R_B, \text{ or} \\
q^G_{SO} &= 1-c \quad \text{and} \quad q^B_{SO} = 0 \quad \text{if} \quad \beta < \beta^R_G \text{ and } \beta \geq \beta^R_B, \text{ or} \\
&\quad \beta \geq \beta^R_G \text{ and } \beta \geq \beta^R_B \text{ and } c < c_d
\end{align*}
\]
with cutoffs $\beta^R_G = \frac{(1+2d)(1-c)}{2}$, $\beta^R_B = \frac{1}{2(1-c)}$, and $c_d = 1 - \frac{1}{\sqrt{1+2d}}$.

Figures 4 depict socially optimal output when environmental damages are absent ($d = 0$, as in 4a), mild ($d = \frac{1}{2}$, as in 4b), severe ($d = \frac{3}{2}$, as in 4c), and infinite ($d \to \infty$, as in 4d). When $\beta < \min\{\beta^R_G, \beta^R_B\}$, goods are relatively differentiated so that the regulator assigns output to both firms, yielding $q^{SO}_G, q^{SO}_B > 0$. When $\beta^R_G \leq \beta < \beta^R_B$, goods are relatively homogeneous but cost asymmetry of green good is more significant than environmental damages of brown good so that the regulator only assigns output to the brown firm, and similarly for $\beta \geq \max\{\beta^R_G, \beta^R_B\}$ and $c \geq c_d$, yielding $q^{SO}_B > q^{SO}_G = 0$. In contrast, when $\beta^R_B \leq \beta < \beta^R_G$, cost asymmetry of green good is less significant than environmental damages of brown good so that the regulator only assigns output to the green firm, and similarly for $\beta \geq \max\{\beta^R_G, \beta^R_B\}$ and $c < c_d$, yielding $q^{SO}_G > q^{SO}_B = 0$. 

The next Corollary reports the comparative static results on socially optimal output.

**Corollary 4.** When both goods are produced, $q_{SO}^G$ decreases in $c$, increases in $d$, and decreases in $\beta$ if (i) $c > c_d$, or (ii) $\beta \geq \beta_G^R \equiv \frac{(1+2d)(1-c)-\sqrt{(1+2d)^2(1-c)^2-(1+2d)}}{2}$ and $c \leq c_d$. In contrast, $q_{SO}^B$ decreases in $c$ and increases in $d$. Figure 4: Socially optimal output in different contexts

Figure 4: Socially optimal output in different contexts

(a) No environmental damages, when $d = 0$

(b) Mild environmental damages, when $d = \frac{1}{2}$

(c) Severe environmental damages, when $d = \frac{3}{2}$

(d) Infinite environmental damages, when $d \to \infty$
increases in $c$, decreases in $d$, and increases in $\beta$ if (i) $c < 1 - \frac{1}{1+2d}$, or (ii) $\beta \geq \beta_{d}^{B} \equiv \frac{1-\sqrt{1-(1+2d)(1-c)}}{2}$ and $c \geq 1 - \frac{1}{1+2d}$. When only the green good is produced, $q_{G}^{SO}$ decreases in $c$ but does not change with $d$ or $\beta$; but when only the brown good is produced, $q_{B}^{SO}$ decreases in $d$ but does not change with $c$ or $\beta$.

When the brown firm becomes more polluting (higher $d$), despite stationed cutoff $\beta_{d}^{B}$, cutoff $c_{d}$ shifts to the right and cutoff $\beta_{G}^{B}$ rotates clockwise at the point $(\beta, c) = (0, 1)$, such that the shaded area for which only the brown good is produced shrinks. Intuitively, the regulator substitutes brown good for green good when costs become more asymmetric (higher $c$) and goods become more homogeneous (higher $\beta$). In the case that the merged firm becomes completely brown, that is, only the brown but not green good is produced, the regulator assigns fewer units of brown good if it becomes more polluting, but its levels are unaffected by the costliness or homogeneity of the non-producing green good. In another case that the merged firm becomes completely green, that is, only the green but not brown good is produced, the regulator assigns fewer units of green good if it becomes more costly, but its levels are unaffected by the pollution severity or homogeneity of the non-producing brown good.

The following Proposition studies the regulator’s emission fees (which can be negative in the case of subsidies) to achieve socially optimal output.

**Proposition 1.** The regulator charges emission fees of

\[
\begin{align*}
    t_{G}^{NM} &= 1 - 2q_{G}^{SO} - \beta q_{B}^{SO} - c \\
    t_{B}^{NM} &= 1 - 2q_{B}^{SO} - \beta q_{G}^{SO}
\end{align*}
\]

and

\[
\begin{align*}
    t_{G}^{M} &= 1 - 2q_{G}^{SO} - 2\beta q_{B}^{SO} - c \\
    t_{B}^{M} &= 1 - 2q_{B}^{SO} - 2\beta q_{G}^{SO}
\end{align*}
\]

to induce green and brown firms to produce $q_{G}^{SO}$ and $q_{B}^{SO}$, respectively.

- Under no merger, the green firm’s fee satisfies $t_{G}^{NM} \geq 0$ if either

  (i) \[ \frac{3-\sqrt{9-9(1-c)^2(1+2d)}}{4(1-c)} \equiv \beta_{G}^{NM} \leq \beta < \beta_{d} \equiv \frac{\sqrt{1+2d}}{2} \text{ for } q_{G}^{SO}, q_{B}^{SO} > 0, \text{ or} \]

  (ii) $\beta \geq \beta_{d}$ and $c \geq c_{d}$ for $q_{B}^{SO} > q_{G}^{SO} = 0$.

- Under no merger, the brown firm's fee satisfies $t_{B}^{NM} \geq 0$ if either

  (i) \[ \frac{(3-2d)(1-c)+\sqrt{(3-2d)^2(1-c)^2-8(1-2d)}}{4} \equiv \beta_{B}^{NM} \leq \beta < \beta_{d} \text{ for } q_{B}^{SO}, q_{G}^{SO} > 0, \text{ or} \]

  (ii) $\beta \geq \beta_{d}$ and $c \geq c_{d}$ for $q_{B}^{SO} > q_{G}^{SO} = 0$. 

(ii) $\beta \leq \beta_{NM}^{\text{B}} \equiv \frac{(3-2d)(1-c)-\sqrt{(3-2d)^2(1-c)^2-8(1-2d)}}{4}$ for $q_{SO}^{G}, q_{SO}^{B} > 0$, or

(iii) $\beta \geq \beta_d$ and $c < c_d$ for $q_{SO}^{G} > q_{SO}^{B} = 0$, or

(iv) $d \geq 1$, $\beta \geq \beta_d$, and $c \geq c_d$ for $q_{SO}^{G} > q_{SO}^{B} = 0$.

- Under merger, the green firm’s fee $t_{MG}^M < 0$ always holds, and the brown firm’s fee satisfies $t_{MB}^M \geq 0$ ($< 0$) if $d \geq \frac{1}{2}$ ($d < \frac{1}{2}$).

For illustration purposes, Figure 5 considers $c = 0.5$. In particular, Figures 5a (5b) depicts the tax on the green (brown) firm under no merger. In Figure 5a, when $\beta < \beta_{NM}^{G}$, environmental damages are relatively significant so that the regulator subsidizes the green firm to increase output, yielding $t_{NM}^{NM} < 0$ as indicated in the unshaded area. However, when $\beta_{NM}^{G} \leq \beta < \beta_d$, goods are relatively homogeneous so that the regulator taxes the green firm to reduce output. A similar result emerges when $\beta \geq \beta_d$ so that the green firm is taxed to remain inactive.\textsuperscript{7} In Figure 5b, when $\overline{\beta_{NM}^{B}} < \beta < \overline{\beta_{NM}^{B}}$ (shaded area at the bottom-right corner of the figure), environmental damages are relatively significant so that the regulator taxes the brown firm to decrease output, yielding $t_{NM}^{NM} \geq 0$. However, when $\overline{\beta_{NM}^{B}} \leq \beta < \beta_d$ or $\beta \leq \beta_{NM}^{B}$,\textsuperscript{8} goods are relatively homogeneous so that the regulator subsidizes the brown firm to increase output, yielding $t_{NM}^{NM} < 0$ as indicated in the unshaded area. Lastly, when $d < \frac{1}{2}$ ($d \geq \frac{1}{2}$) and $\beta \geq \beta_d$, only this firm operates and the regulator subsidizes (taxes) it because consumer surplus more than (does not) compensates environmental damages. In comparison, under merger, the green firm always receives a subsidy, that is, $t_{MG}^M < 0$ holds for all parameter values; and the brown firm is taxed (subsidized), that is, $t_{MB}^M \geq 0$ ($< 0$), when its environmental damages are relatively (in)significant, where $d \geq \frac{1}{2}$ ($d < \frac{1}{2}$).

\textsuperscript{7}Check that $c = 0.5 \geq c_d$ for $d \leq \frac{1}{2}$ yielding $t_{NM}^{NM} \geq 0$ as indicated in the shaded area.

\textsuperscript{8}In Figure 5b, cutoff $\overline{\beta_{NM}^{B}}$ intersects cutoff $\overline{\beta_{NM}^{B}}$ at $\beta = \hat{\beta} \approx 0.268$ as depicted with a horizontal dotted line.
4. Profit gains from the merger

In this section, we investigate the firms’ incentives to merge.

**Corollary 5.** Under regulation, a merger is weakly profitable for every firm $i$ for all parameter values.

When the regulator charges emission fees, the merged firms can coordinate output and internalize output externalities on each other, which, in turn, help them save emission fees.\(^9\) As the next Corollary shows, the regulator can use emission fees to induce green and brown firms to merge if firm $i$ can gain larger profits through merger than those without regulation.

**Corollary 6.** The green firm has more incentives to merge under regulation than under no regulation, $\Delta \pi_G^R \geq \Delta \pi_G^{NR}$, except when $\beta_B^R < \beta < \beta_G^{NR}$ or when $\beta > \bar{\beta}$. The brown firm exhibits similar incentives, $\Delta \pi_B^R \geq \Delta \pi_B^{NR}$, if $\beta < \min \{ \beta_B^R, \beta_B^\Lambda \}$, where cutoff $\beta_B^\Lambda$ solves $\Delta \pi_B^R = \Delta \pi_B^{NR}$.

\(^9\)When $q_G^{SO}$ or $q_B^{SO}$ is zero, merging with an inactive firm does not change profits of the operating firm, that is, $\Delta \pi_i^R = 0$. 

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Figure 5: Taxes under no merger
Overall, both firms have more incentives to merge under regulation than not if $\beta$, $c$ and/or $d$ are low.\(^{10}\) When $\beta > \overline{\beta}$ and $d \leq \frac{3}{2}$, the green firm remains inactive before and after merger so that it earns zero profits, that is, $\Delta \pi^R_G = \Delta \pi^0_G = 0$, as indicated in the top-right corner of Figure 6a. However, when $\beta^R_B < \beta < \beta^R_G$, the green firm benefits from merger under no regulation, but does not under regulation because the brown firm stops operation which merger does not yield additional profits, that is, $\Delta \pi^R_G < \Delta \pi^0_G$, as indicated in the unshaded area in the top-left corner. Otherwise, the green firm benefits from regulation because of higher profits when $\beta < \min\{\beta^R_B, \beta^R_G\}$, changes from losses to profits when $\beta^R_G \leq \beta < \beta^R_B$, or stops making losses when (i) $\beta^R_G \leq \beta < \overline{\beta}$, or (ii) $\beta \geq \min\{\beta^R_B, \beta^R_G\}$, as indicated in the shaded area of the figure.\(^{11}\)

When $\beta > \overline{\beta}$ and $d \leq \frac{3}{2}$, the brown firm monopolizes the market before and after merger so

\(^{10}\)In the case that $\beta = 1$, $c = 0$ and $d = 0$, as in the top-left corner of Figures 6a and 6b, the firms are symmetric and they have no incentives to merge under no regulation (Salant et al., 1983) and also under regulation.

\(^{11}\) When environmental damages become more significant, cutoff $\beta^R_G$ rotates clockwise at the point $(\beta, c) = (0, 1)$. In particular, when $d > \frac{3}{2}$, cutoff $\beta^R_G$ is to the right of cutoff $\overline{\beta}$ so that the green firm, which remains inactive under no regulation, starts production under regulation. A similar argument applies when it merges with the brown firm that yields additional profits. Accordingly, the area between cutoffs $\beta^R_G$ and $\overline{\beta}$, which entails $\Delta \pi^R_G > \Delta \pi^0_G$, is shaded.
that its profits do not change, that is, $\Delta \pi_B^R = \Delta \pi_B^{NR} = 0$, as indicated in the top-right corner of Figure 6b. On the other hand, when $\beta \geq \beta_B^R$, the brown firm benefits from merger under no regulation, but does not under regulation because the regulator stops the operation of either (i) the green firm (as in the case of $d = 0$), or (ii) the brown firm (as in the case of $d = \frac{3}{2}$), yielding the merger between an operating firm and an inactive firm unprofitable, that is, $\Delta \pi_B^R < \Delta \pi_B^{NR}$, as indicated in the unshaded area in the top-left corner. Similarly, for $\beta_B^A \leq \beta < \max(\beta_B^R, \beta_B^A)$, the brown firm still makes additional profits from merger under regulation, but falls below those under no regulation, such that the area in the middle of the figure is unshaded. Otherwise, when $\beta < \min(\beta_B^R, \beta_B^A)$, the brown firm is sufficiently competitive than the green firm so that a merger is profit-enhancing under regulation than not, that is, $\Delta \pi_B^R > \Delta \pi_B^{NR}$, as indicated in the shaded area in the bottom-left corner.\footnote{When environmental damages become more significant, cutoff $\beta_B^A$ rotates anti-clockwise at $(\beta, c) = (0, 1)$ such that the shaded area shrinks. Intuitively, as the regulator assigns fewer units of brown good, the brown firm needs to be more competitive than the green firm to still benefit from merger under regulation.}

In sum, both firms benefit from regulation when goods are more differentiated (lower $\beta$) and costs are more symmetric (lower $c$). In the case of $d = 0$, the area in which both firms are shaded is the largest because the regulator subsidizes both firms to expand output absent environmental damages. However, in the extreme case of infinite environmental damages, where $d \to \infty$, $\beta_B^R$ is vertical at $c = 1$ while $\beta_B^A$ is horizontal at $\beta = 0$, so that an operating green firm merging with an inactive brown firm under regulation does not increase profits under all parameter values.

4.1 Cartel Formation

Appendix 1 studies collusion in an infinitely-repeated game, first, under no regulation and then under regulation. For simplicity, we focus on those parameter conditions for which both firms have individual incentives to merge in the one-shot version of the game, that is, $\beta < \beta_G^{NR}$, under no regulation. In addition, we consider a Grim-Trigger strategy where every firm $i$ cooperates in the first period (producing the merger output $q_{iM}$), and continues to do so in every subsequent period as long as every firm cooperated in all the previous periods (that is, cooperate if the output vector was $(q_{iM}, q_{jM})$ in all the previous periods). Otherwise, firm $i$ reverts to Nash equilibrium of the stage game, producing $q_{iNM}$ thereafter. An analogous approach applies to the setting with regulation, where parameter values satisfy $\beta < \min(\beta_B^R, \beta_B^A)$ for the green and brown firms.
Our results show that under no regulation, firms have stronger incentives to merge if the green firm is relatively competitive to the brown firm, that is, when (i) goods are more differentiated ($\beta$ is low), or (ii) costs are more symmetric ($c$ is low). The same result holds for the green firm under regulation; but the brown firm has stronger incentives to merge if it is relatively competitive to its green rival, where (i) goods are more differentiated ($\beta$ is low), or (ii) costs are more asymmetric ($c$ is high), or environmental damages are small ($d$ is low). In comparison, firms are more likely to merge under regulation than not if they are relatively competitive to each other, that is, (i) goods are more differentiated, or (ii) costs or environmental damages are at moderate levels, that is, the former (latter) is neither too high to make the green (brown) firm uncompetitive, nor too low to make the brown (green) firm uncompetitive.

5. Welfare effects from the merger

While Corollary 3 shows that, under no regulation, a merger yields welfare gains under certain conditions, the following result demonstrates that, under regulation, welfare levels coincide before and after the merger. We also investigate how this common welfare level is affected by parameter changes.

**Corollary 7.** The regulator achieves the same levels of social welfare whether firms merged or not. When both goods are produced, social welfare decreases with $\beta$, $c$, and $d$. When only green (brown) good is produced, social welfare decreases with $c$ ($d$) but does not change with $d$ ($c$, respectively) and $\beta$.

As the regulator charges different fees to different firms depending on market structure, he can induce the same social optimal output whether firms merge or not. Specifically, when $\beta < \min\{\beta_B^R, \beta_G^R\}$, both goods are produced, but social welfare decreases if (i) goods become more homogenous, or (ii) cost of green good increases, or (iii) brown good becomes more polluting. From Corollary 4, when (i) $\beta_B^R \leq \beta < \beta_G^R$, or (ii) $\beta \geq \max\{\beta_B^R, \beta_G^R\}$ and $c < c_d$, only green good is produced so its levels, as well as social welfare, decrease in $c$ but do not change with $d$ or $\beta$. In the same vein, when (i) $\beta_G^R \leq \beta < \beta_B^R$, or (ii) $\beta \geq \max\{\beta_B^R, \beta_G^R\}$ and $c \geq c_d$, only brown good is produced so its levels, as well as social welfare, decrease in $d$ but do not change with $c$ or $\beta$. 

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6. Conclusion

Our results demonstrate that under no regulation, a merger can be welfare-improving if the brown firm reduces (increases) output on the basis that environmental damages more than offset (fall below) consumer surplus. Furthermore, if the regulator has perfect information on market structure that enables him to charge emission fees on the brown firm and offer subsidies to the green firm, output shifts from the former to the latter that maximizes social welfare.

While this paper examines per-unit emission fees/subsidies on one brown firm and one green firm, our model can be generalized into settings with different numbers of green and brown firms, and to study the welfare effects of different types of mergers (Fikru and Gautier, 2016, 2017) under incomplete information (Qiu and Zhou, 2002; Espínola-Arredondo and Muñoz-García, 2015). Extending Cunha and Vasconcelos (2015) and Kanjilal and Muñoz-García (2018), we can also allow endogenous equity shares between green and brown firms, in which a merger is considered as a special case of 50% equity acquisition, and investigate how regulation affects the equity shares owned and output levels coordinated by each firm. Lastly, building upon the literature on Stackelberg mergers with asymmetric costs (Huck et al., 2001; Escrihuela-Villar, 2018), it would be interesting to characterize the incentives of leaders and followers to coordinate output under environmental regulation or not; whereby the leader, who has first-mover advantage in production costs, acquires equity shares of the follower, who has second-mover advantage in abatement technologies.

7. Appendices

7.1 Appendix 1 - Cartel Formation

In this appendix, we first consider the setting of no regulation, and then move on to the setting of regulation; studying the firms’ individual incentives to merge in an infinitely repeated game.

**Proposition A1.** Under no regulation, the green firm has incentives to merge if $\delta_{GR}^{NR} \geq \delta_{GR}^{NR}$. Similarly, the brown firm has incentives to merge if $\delta_{BR}^{NR} \geq \delta_{BR}^{NR}$, where

$$\delta_{GR}^{NR} = \frac{(4 - \beta^2)^2 (1 - \beta + \beta c)^2}{[(1 - \beta) (2 - \beta) - (\beta^2 + 2) c] [(1 - \beta) (2 - \beta) (\beta^2 + 8\beta + 8) - (\beta^4 - 14\beta^2 + 16) c]}$$
and
\[
\delta^{NR}_B = \frac{(4 - \beta^2)^2 (1 - \beta - c)^2}{[(1 - \beta) (2 - \beta) + 3 \beta c] [(1 - \beta) (2 - \beta) (\beta^2 + 8 \beta + 8) + \beta (8 - 5 \beta^2) c]}
\]

Cutoff \(\delta^{NR}_G\) (cutoff \(\delta^{NR}_B\)) increases (decreases) in \(\beta\) and \(c\), and satisfies \(\delta^{NR}_G \geq \delta^{NR}_B\).

Both firms have incentives to collude when \(\delta \geq \delta^{NR} \equiv \max\{\delta^{NR}_G, \delta^{NR}_B\}\). As an illustration, Figures A1a and A1b depict the cases of \(c = 0.05\) and \(\beta = 0.5\), respectively, where the solid and dotted lines represent the cutoffs for green and brown firms, above which a merger sustains in the shaded area. Figure A1a illustrates how a merger is affected when goods become more homogeneous (higher \(\beta\)). In this setting, the brown firm becomes more competitive than its green rival, entailing that the minimal discount factor for the green (brown) firm increases (decreases) which also shrinks the shaded region for both firms to sustain collusion. Figure A1b helps us understand how collusion is affected when firms become more asymmetric (higher \(c\)). Since the green firm becomes less competitive as \(c\) increases, its minimal discount factor sustaining collusion increases while that of its rival decreases; ultimately implying that collusion becomes more difficult to sustain. \(^{13}\)

Figure A1: Collusion under no regulation

**Proposition A2.** Under regulation, the green firm has incentives to merge if \(\delta^{R}_G \geq \delta^{R}_G\); and similarly, the brown firm has incentives to merge if \(\delta^{R}_B \geq \delta^{R}_B\), where

\[
\delta^{R}_G = \frac{\beta q^{SO}_G}{4 q^{SO}_G + \beta q^{SO}_B} \quad \text{and} \quad \delta^{R}_B = \frac{\beta q^{SO}_B}{4 q^{SO}_B + \beta q^{SO}_G}.
\]

\(^{13}\)In this example, collusion is unsustainable when \(\beta > \beta^{NR}_G \approx 0.689\) or \(c > c^{NR}_G \approx 0.097\).
Cutoff $\delta_{NR}^G$ (cutoff $\delta_{NR}^B$) increases (decreases) in $c$, decreases (increases) in $d$, and increases (decreases) in $\beta$ if (i) $q_{SO}^G$ decreases in $\beta$, and (ii) $q_{SO}^B$ decreases in $\beta$.

As an illustration, Figures A2a, A2b and, A2c depict the cases of $(c, d) = (0.05, 0.5), (\beta, d) = (0.5, 0.5), \text{ and } (\beta, c) = (0.5, 0.05)$ respectively, where the solid and dotted lines represent the cutoffs for green and brown firms, respectively. These figures exhibit an opposite (similar) interpretation as those under no regulation in Figure A1a (A1b), where collusion can be sustained in the shaded region above the maximum of both cutoffs, that is, $\delta \geq \delta^R \equiv \max\{\delta^R_G, \delta^R_B\}$. First, Figure A2a describes that, as goods become more homogeneous (higher $\beta$), the regulator can substitute green for brown goods. Even though the green good is more costly to produce, it more than offsets environmental externalities of brown good, so that the green (brown) firm needs a lower (higher) discount rate to sustain collusion. Second, Figure A2b indicates that when firms become more asymmetric in costs (higher $c$), the green (brown) firm is less (more) competitive, entailing that collusion is more difficult to sustain. Finally, Figure A2c illustrates a setting which could not arise in the model without regulation since environmental damages were absent. Specifically, the figure indicates that, as pollution becomes more significant (higher $d$), the regulator sets more stringent (lax) fees to the brown (green) firm, which makes this firm less (more) competitive, thus shrinking the region of parameter values sustaining collusion.\footnote{Unlike Figure A1a, however, cutoff $\delta^R_G$ can originate below cutoff $\delta^R_B$ when firms are relatively symmetric in costs; for example, cutoff $\delta^R_B$ crosses cutoff $\delta^R_G$ from above at $\hat{c} = 0.333$. Note that when $c > 0.5$, $\beta > \beta_R^G$, and the regulator does not assign any output to the green firm that yields collusion unsustainable for all parameter values.}
\footnote{Analogously, cutoff $\delta^R_B$ can originate below cutoff $\delta^R_G$, which occurs when environmental damages are relatively low; for example, cutoff $\delta^R_G$ crosses cutoff $\delta^R_B$ from above at $\hat{d} = 0.053$. On one hand, when environmental damages are very low, in this case where $d < 0.027$, the regulator only assigns output to the brown firm yielding collusion unsustainable for all parameter values. On the other hand, when environmental damages become infinite, only green good but not brown good is produced so that cutoff $\lim_{d \to \infty} \delta^R_G = 0$ and cutoff $\lim_{d \to \infty} \delta^R_B = \lim_{d \to \infty} \delta^R_B = 1.$}
To this end, we are interested in the comparison of cutoffs in different regulatory settings. Figure A3 depicts the minimal discount factor sustaining collusion, $\delta^R$ under regulation and $\delta^{NR}$ under no regulation. Overall, Figures A3a to A3c indicate that collusion is more likely to arise under regulation, that is, $\delta^R < \delta^{NR}$ if (a) goods are relatively differentiated, or (b) costs are moderate, or (c) environmental damages are relatively small. Our results indicate that, in this highly competitive setting, firms are more likely to collude under regulation if the brown firm does not produce a highly polluting good; and thus, is subject to a relatively lax emission fee. When this firm produces a very polluting good, its fee is stringent, thus making this firm less likely to collude with the green firm than without regulation. A similar argument applies if firms’ competition is less severe (such as cost symmetries) where firms are more inclined to collude without regulation.
7.2 Appendix 2 - Technical Proofs

7.2.1 Proof of Lemma 1

**No merger.** For \( i = \{ B, G \} \), each firm \( i \) chooses \( q_i \) to solve

\[
\max_{q_i \geq 0} \pi(q_i) = (1 - q_i - \beta q_j) q_i - (c_i + t_i) q_i
\]

Taking the first order condition with respect to \( q_i \), and assuming interior solutions,

\[
1 - 2q_i - \beta q_j - c_i = t_i \quad (A1)
\]

such that the best response function of firm \( i \), in response to firm \( j \)'s output, becomes

\[
q_i(q_j) = \frac{1 - \beta q_j - c_i - t_i}{2}
\]
At a Cournot Nash equilibrium, firms’ output are mutual best response to each other, so that

\[
q_{i}^{NM} = \frac{2 - \beta - 2(c_i + t_i) + \beta(c_j + t_j)}{4 - \beta^2}
\]

Substituting \( q_{i}^{NM} \) into the profit function, equilibrium profits become

\[
\pi_{i}^{NM} = \left( \frac{2 - \beta - 2(c_i + t_i) + \beta(c_j + t_j)}{4 - \beta^2} \right)^2
\]

**Merger.** The profit maximization problem of the merged firm becomes

\[
\max_{q_i, q_j} \pi(q_i, q_j) = (1 - q_i - \beta q_j - c_i - t_i) q_i + (1 - q_j - \beta q_i - c_j - t_j) q_j
\]

Taking the first order condition with respect to \( q_i \), and assuming interior solutions,

\[
1 - 2q_i - 2\beta q_j - c_i = t_i \quad (A2)
\]

Solving \( q_i \) and \( q_j \) simultaneously, we obtain

\[
q_{i}^{M} = \frac{1 - \beta - (c_i + t_i) + \beta(c_j + t_j)}{2(1 - \beta^2)}
\]

Substituting and \( q_{i}^{M} \) into the profit function, equilibrium profits become

\[
\pi_{i}^{M} = \frac{1 - c_i - t_i}{2} \cdot \frac{1 - \beta - (c_i + t_i) + \beta(c_j + t_j)}{2(1 - \beta^2)}
\]

Set \( t_i = 0 \) to find output and profits of the firms before and after merger under no regulation. Since output must be non-negative, \( q_{i}^{NM} > 0 \) if \( \beta < \bar{\beta} \equiv 2 \left( 1 - c \right) \) and \( q_{i}^{M} > 0 \) if \( \beta < \frac{\bar{\beta}}{2} \equiv 1 - c \).

Otherwise, \( q_{G}^{k} = \pi_{G}^{k,NR} = 0 \), where \( k = \{ M, NM \} \), and the brown firm solves expression (A1) before merger or expression (A2) after merger by setting \( q_{B}^{k} (q_{G}^{k} = 0) = \frac{1}{2} \) that yields \( \pi_{B}^{k,NR} = \frac{1}{4} \).

**7.2.2 Proof of Corollary 1**

For the green firm to reduce output upon merger, \( q_{G}^{M} < q_{G}^{NM} \) entails \( \frac{1 - \beta - c}{2(1 - \beta^2)} < \frac{2 - \beta + 2c}{4 - \beta^2} \), which simplifies to \( (1 - \beta)(2 - \beta) + 3\beta c > 0 \) that holds for all values of \( \beta < \bar{\beta} \).

For the brown firm to reduce output upon merger, \( q_{B}^{M} < q_{B}^{NM} \) entails \( \frac{1 - \beta + \beta c}{2(1 - \beta^2)} < \frac{2 - \beta + \beta c}{4 - \beta^2} \), which holds for all values of \( c < \frac{(1 - \beta)(2 - \beta)}{2 + \beta^2} \), or rearranging to yield \( \beta < \bar{\beta}_{B}^{NR} \equiv \frac{3 - \sqrt{9 - 8(1 - c)^2}}{2(1 - c)} \).
7.2.3 Proof of Corollary 2

The green firm needs \( \Delta \pi_{G}^{NR} \geq 0 \) to merge, where

\[
\Delta \pi_{G}^{NR} = \pi_{G}^{M,NR} - \pi_{G}^{N,NM,R} = \frac{\beta^{2} \left[ (1 - \beta) (2 - \beta)^{2} - (2 - \beta) (8 + \beta^{2}) c + (8 + \beta^{2}) c^{2} \right]}{4 (1 - \beta^{2}) (4 - \beta^{2})^{2}}
\]

for \( \beta < \frac{\bar{\beta}}{2} \) that is positive for \( c \leq c_{G}^{NR} = \frac{8 + \beta^{2} - (2 + \beta) \sqrt{8 + \beta^{2}}}{2 (1 - \beta) (2 - \beta)} \), or rearranging to yield \( \beta \leq \beta_{G}^{NR} \) where cutoff \( \beta_{G}^{NR} \) solves \( (1 - \beta)(2 - \beta)^{2} - (2 - \beta)(8 + \beta^{2}) c + (8 + \beta^{2}) c^{2} = 0 \). When \( \frac{\bar{\beta}}{2} \leq \beta < \bar{\beta} \), the green firm stops operation after merger so that its profits decrease. In comparison, when \( \beta \geq \bar{\beta} \), the green firm remains inactive before and after merger that yields zero profits.

The brown firm needs \( \Delta \pi_{B}^{NR} \geq 0 \) to merge, where

\[
\Delta \pi_{B}^{NR} = \pi_{B}^{M,NR} - \pi_{B}^{N,NM,R} = \frac{\beta^{2} \left[ (1 - \beta) (2 - \beta)^{2} + (2 - \beta) (4 + 6 \beta - \beta^{2}) c - 4 (1 - \beta^{2}) c^{2} \right]}{4 (1 - \beta^{2}) (4 - \beta^{2})^{2}}
\]

for \( \beta < \frac{\bar{\beta}}{2} \). Differentiating it with respect to \( c \) yields \( \frac{\partial \left[ \pi_{B}^{M,NR} - \pi_{B}^{N,NM,R} \right]}{\partial c} = (2 - \beta)(4 + 6 \beta - \beta^{2}) - 8(1 - \beta^{2}) c \geq \beta^{3} - 8 \beta^{2} + 8 \beta + 8 - 8(1 - \beta^{2}) = \beta(\beta^{2} + 8) \geq 0 \). Taking \( c = 0 \), its minimum gain becomes

\[
\left[ \pi_{B}^{M,NR} - \pi_{B}^{N,NM,R} \right]_{c=0} = \frac{\beta^{2}}{4(1+\beta)(2+\beta)^{2}}
\]

that is positive for \( 0 \leq \beta \leq 1 \), so that the brown firm gains from a merger for \( \beta < \frac{\bar{\beta}}{2} \). Suppose \( \frac{\bar{\beta}}{2} \leq \beta < \bar{\beta} \), the green firm stops production after merger, rendering the brown firm’s gain of \( \pi_{B}^{M,NR} - \pi_{B}^{N,NM,R} = \frac{\beta(\beta + c - 1)}{4(1 - \beta^{2})} \) that is positive. In comparison, when \( \beta \geq \bar{\beta} \), the brown firm monopolizes the market so that its profits do not change.

7.2.4 Proof of Corollary 3

Consumer surplus is the area bounded by the demand curve and the price paid by consumers, for \( CS(q_{i}) = \int_{p_{i}(q_{i},q_{j})}^{p_{i}(0,q_{j})} p_{i} \left( \hat{q}_{i}, q_{j} \right) d \hat{q}_{i} = \int_{1-q_{i}-\beta q_{j}}^{1-\beta q_{j}} \left( 1 - \hat{q}_{i} - \beta q_{j} \right) d \hat{q}_{i} = \left[ (1 - \beta q_{j}) \hat{q}_{i} - \frac{\hat{q}_{i}^{2}}{2} \right]_{1-q_{i}-\beta q_{j}}^{1-\beta q_{j}} = \frac{q_{i}^{2}}{2} \), yielding social welfare of

\[
SW(q_{G},q_{B}) = \frac{q_{G}^{2} + q_{B}^{2}}{2} + \frac{p_{G}(q_{G},q_{B})q_{G} - c_{G}q_{G} + p_{B}(q_{B},q_{G})q_{B} - \frac{d q_{B}^{2}}{E_{nv}(q_{B})}}{\frac{p_{S}}{2}} = \left( 1 - \frac{1}{2} q_{G} - \beta q_{B} - c \right) q_{G} + \left( 1 - \frac{1 + 2d}{2} q_{B} - \beta q_{G} \right) q_{B}
\]

(A3)

If firms do not merge, \( SW(q_{G}^{NM},q_{B}^{NM}) = \frac{3(q_{G}^{NM})^{2} + (3-2d)(q_{B}^{NM})^{2}}{2} \). However, if firms merge, \( SW(q_{G}^{M},q_{B}^{M}) = \frac{3(q_{G}^{M})^{2} + 4 \beta q_{G} q_{B}^{M} + (3-2d)(q_{B}^{M})^{2}}{2} \).
Differentiating expression (A3) with respect to $q_G$ and $q_B$, and assuming interior solutions, socially optimal output, $q_{SO}^G$ and $q_{SO}^B$, that maximize social welfare satisfy

$$1 - q_G - 2\beta q_B - c = 0$$
$$1 - q_B - 2\beta q_G - 2dq_B = 0$$

Solving the simultaneous equations above, we find

$$q_{SO}^G = \frac{(1 + 2d) (1 - c) - 2\beta}{1 + 2d - 4\beta^2}$$  \hspace{1cm} (A4)
$$q_{SO}^B = \frac{1 - 2\beta (1 - c)}{1 + 2d - 4\beta^2}$$  \hspace{1cm} (A5)

Setting the numerator or denominator of the above expressions zero, we identify three cutoffs:

$$\beta_G^R = \frac{(1 + 2d) (1 - c)}{2}$$
$$\beta_B^R = \frac{1}{2 (1 - c)}$$
$$\beta_d = \frac{\sqrt{1 + 2d}}{2}$$

and it is obvious that $\beta_G^R$ intersects $\beta_B^R$ at $\beta_d$.

Since output cannot be negative, the regulator assigns zero output to the firm which numerator or denominator of expressions (A4) or (A5) have negative signs, and positive output for the other firm that solves the welfare maximization problem by balancing marginal benefit with marginal social costs, where $q_{SO}^G = 1 - c$ and $q_{SO}^B = \frac{1}{1+2d}$ yielding welfare of $W_G^R = \frac{(1-c)^2}{2}$ and $W_B^R = \frac{1}{2 (1+2d)}$, respectively. In this context, when $\beta_B^R \leq \beta < \beta_d$, $W_G^R = \frac{(1-c)^2}{2} \geq \frac{1}{8\beta^2} > \frac{1}{2 (1+2d)} = W_B^R$, such that $q_{SO}^G > 0$ and $q_{SO}^B = 0$. Similarly, when $\beta_d \leq \beta < \beta_G^R$, $W_G^R = \frac{(1-c)^2}{2} > \frac{2\beta^2}{(1+2d)^2} \geq \frac{1}{2 (1+2d)} = W_B^R$, yielding $q_{SO}^G > q_{SO}^B = 0$. In contrast, when $\beta_d \leq \beta < \beta_B^R$, $W_B^R = \frac{1}{2 (1+2d)} \geq \frac{1}{8\beta^2} > \frac{(1-c)^2}{2} = W_G^R$, yielding $q_{SO}^B > q_{SO}^G = 0$. Lastly, when $\beta_G^R \leq \beta < \beta_d$, $W_B^R = \frac{1}{2 (1+2d)} > \frac{2\beta^2}{(1+2d)^2} \geq \frac{(1-c)^2}{2} = W_G^R$, yielding $q_{SO}^B > q_{SO}^G = 0$. Otherwise, when $\beta < \min \{\beta_G^R, \beta_B^R\}$ that also satisfies $\beta < \beta_d$, expressions (A4) and (A5) solve $q_{SO}^G > 0$ and $q_{SO}^B > 0$. Nonetheless, when $\beta \geq \max \{\beta_G^R, \beta_B^R\}$ that also satisfies $\beta \geq \beta_d$, output decisions are based on cutoff for costs, $c_d = 1 - \frac{1}{\sqrt{1+2d}}$; such that when $c \geq c_d$, $W_B^R \geq W_G^R$ yields $q_{SO}^B > q_{SO}^G = 0$, but when $c < c_d$, $W_B^R < W_G^R$ yields $q_{SO}^G > q_{SO}^B = 0$. 

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7.2.6 Proof of Corollary 4

When \( q^SO_G, q^SO_B > 0 \), differentiating expression (A4) with respect to \( c \), \( d \), and \( \beta \) yields

\[
\frac{\partial q^SO_G}{\partial c} = -\frac{1+2d}{(1+2d-4\beta^2)}, \quad \frac{\partial q^SO_G}{\partial d} = \frac{4\beta[1-2\beta(1-c)]}{(1+2d-4\beta^2)^2} > 0, \quad \frac{\partial q^SO_G}{\partial \beta} = \frac{2[(1+2d)(4\beta(1-c)-1)-4\beta^2]}{(1+2d-4\beta^2)^2} < 0 \text{ if (i) } c > c_d \text{, or}
\]

(ii) \( \beta \geq \beta^R_B \equiv \frac{(1+2d)(1-c)-\sqrt{(1+2d)^2(1-c)^2-(1+2d)}}{2} \) and \( c \leq c_d \). Similarly, differentiating expression

(A5) with respect to \( c \), \( d \), and \( \beta \) yields

\[
\frac{\partial q^SO_B}{\partial c} = \frac{2\beta}{(1+2d-4\beta^2)^2} > 0, \quad \frac{\partial q^SO_B}{\partial d} = -\frac{2[1-2\beta(1-c)]}{(1+2d-4\beta^2)^2} < 0, \quad \text{and}
\]

\[
\frac{\partial q^SO_B}{\partial \beta} = \frac{2[\beta_1(1-c)(1+2d+4\beta^2)]}{(1+2d-4\beta^2)^2} > 0 \text{ if (i) } c < 1-\frac{1}{1+2d}, \text{ or (ii) } \beta \geq \beta^R_B \equiv \frac{1-\sqrt{1-(1+2d)(1-c)}}{2} \text{ and } c \geq 1-\frac{1}{1+2d}.
\]

Lastly, differentiating cutoffs \( \beta^*_G, \beta^*_B, \beta^*_d, \) and \( c_d \) with respect to \( c \) and \( d \) yields

\[
\frac{\partial \beta^*_G}{\partial d} = -\frac{1+2d}{2(1-c)}, \quad \frac{\partial \beta^*_B}{\partial c} = -\frac{1}{2(1-c)} > 0, \quad \frac{\partial \beta^*_B}{\partial d} = \frac{1}{2\sqrt{1+2d}} > 0, \quad \text{and} \quad \frac{\partial c_d}{\partial d} = \left(1+2d\right)^{-\frac{1}{2}} > 0.
\]

7.2.7 Proof of Proposition 1

For \( i \in \{B,G\} \) and \( k = \{M,NM\} \), set \( q^SO_i = q^k_i \) in expressions (A1) and (A2) to find \( t^k_i \).

First, for the green firm under no merger, when \( q^SO_G, q^SO_B > 0 \), fee \( t^NM_G = \frac{(1-c)(1+2d+2\beta^2)-3\beta}{1+2d-4\beta^2} \)

is positive if \( \frac{3\sqrt{9-9(1-c)^2(1+2d)}}{4(1-c)} \equiv \beta^NM_G \leq \beta < \beta_d \). When \( q^SO_B > q^SO_G = 0 \), fee \( t^NM_G = \frac{(1-c)(1+2d)-\beta}{1+2d} \)
is always positive even if \( \beta \geq (1-c)(1+2d) \) because the regulator can set fee \( t^NM_G = 0 \) and this firm does not produce any output due to profit losses. When \( q^SO_G > q^SO_B = 0 \), fee \( t^NM_G = -(1-c) \) is always negative.

Second, for the brown firm under no merger, when \( q^SO_G, q^SO_B > 0 \), fee \( t^NM_B = \frac{-2\beta^2-\beta(3-2d)(1-c)-1+2d}{1+2d-4\beta^2} \)
is positive if either (i) \( \left(\frac{3-2d}{1-c}\right)+\sqrt{\left(\frac{3-2d}{1-c}\right)^2-8(1-2d)} \equiv \beta^NM_B \leq \beta < \beta_d \), or

(ii) \( \beta \leq \beta^NM_B \equiv \frac{(3-2d)(1-c)-\sqrt{(3-2d)^2(1-c)^2-8(1-2d)}}{4} \), where cutoffs \( \beta^NM_B \) and \( \beta^NM_B \) intersect at \( \frac{1}{2} \equiv \frac{1+\sqrt{c(2-c)}}{1-c} \). When \( q^SO_B > q^SO_G = 0 \), fee \( t^NM_B = \frac{2d-1}{1+2d} \) is positive (negative) if \( d \geq \frac{1}{2} \) (\( d < \frac{1}{2} \)).

When \( q^SO_G > q^SO_B = 0 \), fee \( t^NM_G = 1 - \beta(1-c) \) is positive for all parameter values. Third, for the green firm under merger, when \( q^SO_G, q^SO_B > 0 \), fee \( t^M_G = \frac{-(1+2d)(1-c)-2\beta}{1+2d-4\beta^2} \) is negative since \( \beta < \beta^R_G \).

When \( q^SO_B > q^SO_G = 0 \), fee \( t^M_G = \frac{-(1+2d)(1-c)-2\beta}{1+2d-4\beta^2} \) is negative since \( \beta \geq \beta^R_G \). When \( q^SO_G > q^SO_B = 0 \), fee \( t^M_B = -(1-c) \) is negative.

Fourth, for the brown firm under merger, when \( q^SO_G, q^SO_B > 0 \), fee

\[
t^M_B = \frac{(2d-1)(1-2\beta(1-c))}{1+2d-4\beta^2}, \quad \text{and} \quad t^M_B = \frac{2d-1}{1+2d} \text{ when } q^SO_B > q^SO_G = 0; \text{ both of which are positive (negative)}
\]

if \( d \geq \frac{1}{2} \) (\( d < \frac{1}{2} \)). When \( q^SO_G > q^SO_B = 0 \), fee \( t^M_B = 1 - 2\beta(1-c) \) is positive since \( \beta < \beta^R_G \).
7.2.8 Proof of Corollary 5

Under regulation, pre- and post-merger firm $i$ generates profits of

$$\pi_{i}^{NM,R} = \left( \frac{(2 - \beta) - 2(c_i + t_i^{NM}) + \beta(c_j + t_j^{NM})}{4 - \beta^2} \right)^2 = \left( q_i^{SO} \right)^2$$

respectively, such that firm $i$ generates more profits from merger under all parameter values.

7.2.9 Proof of Corollary 6

When $\beta < \min \{ \beta_B R, \beta_G^R \}$, $q_G^{SO}, q_B^{SO} > 0$ so that the merging green firm benefits from regulation if $\Delta \pi_G^R \geq \Delta \pi_G^{NR}$, that is, $\frac{1 - 2\beta(1 - c)](1 + 2d)(1 - c) - 2\beta}{\beta(1 + 2d - 4\beta^2)^2} \geq \frac{(1 - \beta)(2 - \beta)^2(2 - \beta)(8 + \beta^2)c + (8 + \beta^2)c^2}{4(1 - \beta^2)(4 - \beta^2)^2}$; which is rearranged to give $g_G(\beta, c, d) \geq 0$, where

$$g_G(\beta, c, d) = -\beta \left[ (1 + 2d - 4\beta^2)^2(8 + \beta^2) + 8(1 - \beta^2)(2 + \beta)^2(2 - \beta)^2(1 + 2d) \right] c^2$$

$$- (2 - \beta) \left[ 4(1 - \beta^2)(2 + \beta)^2(2 - \beta)((1 - \beta^2)^2 + 2d(1 - 4\beta)) - \beta(1 + 2d - 4\beta^2)^2(8 + \beta^2) \right] c$$

$$+ (1 - \beta)(2 - \beta)^2 \left[ 4(1 + \beta)(2 + \beta)^2((1 - \beta^2)(1 - 2\beta + 2d)) - \beta(1 + 2d - 4\beta^2)^2 \right]$$

and differentiating with respect to $c$ yields $-\beta^2(1 + 2d - 4\beta^2)^2(8 + \beta^2) - 4(1 - \beta^2)(2 + \beta)^2(2 - \beta)^2(1 - 2\beta)^2 + 4\beta + 2d) < 0$ for any values of $\beta$ and $d$, so that when $\beta \leq \beta_G^{NR}, g_G(\beta, c, d) \geq g_G(\beta, c_{G}^{NR}, d) > 0$ yielding $\Delta \pi_G^R \geq \Delta \pi_G^{NR} \geq 0$. In contrast, $\beta > \beta_G^{NR}$ yields $\Delta \pi_G^R \geq 0 > \Delta \pi_G^{NR}$.

Similarly, when $\beta < \min \{ \beta_B R, \beta_G^R \}$, the merging brown firm benefits from regulation if $\Delta \pi_B^R \geq \Delta \pi_B^{NR}$, that is, $\frac{1 - 2\beta(1 - c)](1 + 2d)(1 - c) - 2\beta}{\beta(1 + 2d - 4\beta^2)^2} \geq \frac{(1 - \beta)(2 - \beta)^2(2 - \beta)(4 + 6\beta - \beta^2)c + (4 + 6\beta - \beta^2)c^2}{4(1 - \beta^2)(4 - \beta^2)^2}$; which is rearranged to give $g_B(\beta, c, d) \geq 0$, with an associated cutoff $\beta_B^A$ that solves $g_B(\beta, c, d) = 0$, where

$$g_B(\beta, c, d) = 4\beta(1 - \beta^2) \left[ (1 + 2d - 4\beta^2)^2 - 2(2 + \beta)^2(2 - \beta)^2(1 + 2d) \right] c^2 - (2 - \beta)$$

$$\left[ 4(1 - \beta^2)(2 + \beta)^2(2 - \beta)((1 - \beta^2)^2 + 2d(1 - 4\beta)) + \beta(1 + 2d - 4\beta^2)^2(4 + 6\beta - \beta^2) \right] c$$

$$+ (1 - \beta)(2 - \beta)^2 \left[ 4(1 + \beta)(2 + \beta)^2((1 - \beta^2)(1 - 2\beta + 2d)) - \beta(1 + 2d - 4\beta^2)^2 \right]$$

7.2.10 Proof of Corollary 7

When $\beta < \min \{ \beta_B R, \beta_G^R \}$, $SW(q_G^{SO}, q_B^{SO}) = \frac{(1 - c)q_G^{SO} + q_B^{SO}}{2} = \frac{(1 + 2d)(1 - c) - 2\beta}{2(1 + 2d - 4\beta^2)}$ whether firms merged or not. Differentiating it with respect to $c, d$, and $\beta$ yields $\frac{\partial SW}{\partial c} = -\frac{(1 + 2d)(1 - c) - 2\beta}{2(1 + 2d - 4\beta^2)} < 0, \frac{\partial SW}{\partial d} = \ldots$
\[-\frac{1-4\beta(1-c)(1-\beta(1-c))}{(1+2d-4\beta)^2} < 0, \text{ and } \frac{\partial SW}{\partial \beta} = -\frac{2[1-2\beta(1-c)][(1-c)(1+2d)-2\beta]}{(1+2d-4\beta)^2} < 0. \]

When (i) \( \beta_B^R \leq \beta < \beta_G^R \), or (ii) \( \beta \geq \max\{\beta_B^R, \beta_G^R\} \) and \( c < c_d \), \( W_G^R \equiv \frac{(c-1)^2}{2} \) that decreases in \( c \) but does not change with \( d \) or \( \beta \). In the same vein, when (i) \( \beta_B^R \leq \beta < \beta_B^R \), or (ii) \( \beta \geq \max\{\beta_B^R, \beta_G^R\} \) and \( c \geq c_d \), \( W_B^R \equiv \frac{1}{2(1+2d)} \) that decreases in \( d \) but does not change with \( c \) or \( \beta \).

7.2.11 Proof of Proposition A1

Suppose firm \( i \) deviates but firm \( j \) does not, then firm \( i \) inserts \( q_j^M \) into its best response function to obtain \( q_i^D(q_j^M) = \frac{1-\beta q_j^M - c - t_i}{2} = \frac{(1-\beta)(2+\beta) - (2-\beta^2)(c+\beta(t_i))}{4(1-\beta^2)} \), which yields profits of \( \pi_i^D = \left( \frac{(1-\beta)(2+\beta) - (2-\beta^2)(c+\beta(t_i))}{4(1-\beta^2)} \right)^2 \), where the superscript \( D \) stands for deviation. Setting \( t_i = 0 \), output and profits under no regulation are \( q_G^{D, NR} = \frac{(1-\beta)(2+\beta) - (2-\beta^2)c}{4(1-\beta^2)} \) and \( \pi_G^{D, NR} = \left[ \frac{(1-\beta)(2+\beta) - (2-\beta^2)c}{4(1-\beta^2)} \right]^2 \) for the green firm; and similarly \( q_B^{D, NR} = \frac{(1-\beta)(2+\beta) + \beta c}{4(1-\beta^2)} \) and \( \pi_B^{D, NR} = \left[ \frac{(1-\beta)(2+\beta) + \beta c}{4(1-\beta^2)} \right]^2 \) for the brown firm. Using Grim-trigger strategy, the minimal discount rate that sustains collusion is \( \delta_{i}^l = \frac{\pi_i^{D, I} - \pi_i^{M, I}}{\pi_i^{D, M} - \pi_i^{M, I}} \), where \( l \in \{R, NR\} \). It is straightforward to verify that \( \delta_{i}^{NR}(\beta, 0) = \delta_{i}^{NR}(\beta, 0) \) entailing that \( \delta_{i}^{NR} \geq \delta_{i}^{NR} \) for \( \beta < \beta_G^R \).

7.2.12 Proof of Proposition A2

Substituting fee \( t_i^M \) from Proposition 1 into \( q_i^D \) in the proof of Proposition A1, output and profits under regulation are \( q_G^{D, R} = \frac{2q_G^{SO} + \beta q_B^{SO}}{4} \) and \( \pi_G^{D, R} = \left( \frac{2q_G^{SO} + \beta q_B^{SO}}{4} \right)^2 \) for the green firm; and similarly \( q_B^{D, R} = \frac{2q_B^{SO} + \beta q_B^{SO}}{4} \) and \( \pi_B^{D, R} = \left( \frac{2q_B^{SO} + \beta q_B^{SO}}{4} \right)^2 \) for the brown firm. Using Corollary 4 and differentiating cutoff \( \delta_{r}^G \) with respect to \( c \), \( d \), and \( \beta \) yields

\[
\frac{\partial \delta_{r}^G}{\partial c} = \frac{4\beta}{(4q_G^{SO} + \beta q_B^{SO})^2} \left( q_G^{SO} \frac{\partial q_G^{SO}}{\partial c} - q_B^{SO} \frac{\partial q_B^{SO}}{\partial c} \right) < 0, \quad \text{and} \quad \frac{\partial \delta_{r}^G}{\partial \beta} = \frac{4q_G^{SO} q_B^{SO} + 4\beta \left( q_G^{SO} \frac{\partial q_G^{SO}}{\partial \beta} - q_B^{SO} \frac{\partial q_B^{SO}}{\partial \beta} \right)}{(4q_G^{SO} + \beta q_B^{SO})^2} > 0 \text{ if } (i) \frac{\partial q_G^{SO}}{\partial \beta} > 0, \quad \text{and} \quad (ii) \frac{\partial q_B^{SO}}{\partial \beta} < 0.
\]

Similarly, differentiating cutoff \( \delta_{r}^B \) with respect to \( c \), \( d \), and \( \beta \) yields

\[
\frac{\partial \delta_{r}^B}{\partial c} = \frac{4\beta}{(4q_B^{SO} + \beta q_B^{SO})^2} \left( q_B^{SO} \frac{\partial q_B^{SO}}{\partial c} - q_B^{SO} \frac{\partial q_B^{SO}}{\partial c} \right) < 0, \quad \text{and} \quad \frac{\partial \delta_{r}^B}{\partial \beta} = \frac{4q_B^{SO} q_B^{SO} + 4\beta \left( q_B^{SO} \frac{\partial q_B^{SO}}{\partial \beta} - q_B^{SO} \frac{\partial q_B^{SO}}{\partial \beta} \right)}{(4q_B^{SO} + \beta q_B^{SO})^2} > 0 \text{ if } (i) \frac{\partial q_B^{SO}}{\partial \beta} > 0, \quad \text{and} \quad (ii) \frac{\partial q_B^{SO}}{\partial \beta} < 0.
\]

Note that when \( d \to \infty \), \( \lim_{d \to \infty} \delta_{i}^G = \lim_{d \to \infty} \frac{\beta(1-2\beta(1-c))}{(2(1-c))(2(1+2d)-2\beta)[1-\beta(1-c)]} = 0 \) and \( \lim_{d \to \infty} \delta_{i}^B = \lim_{d \to \infty} \frac{\beta(1-2\beta(1-c)-2\beta)}{(2(1+2d)-2\beta)(1-2\beta(1-c))(2\beta-1)} \)
(using L’Hôpital’s rule) for the green and brown firms, respectively.

References


