EconS 503 - Advanced Microeconomics II
Handout on Social Choice

1. MWG - Decisive Subgroups

Recall proposition 21.C.1: (Arrow's Impossibility Theorem) Suppose that the number of alternatives is at least three and that the domain of admissible individual profiles, denoted A, is either $A = R^I$ or $A = P^I$. Then every social welfare functional $F : A \to R$ that is Paretian and satisfies the pairwise independence condition is dictatorial in the following sense:
There is an agent h such that, for any $\{x, y\} \subset X$ and any profile $(\succsim_1, \ldots, \succsim_I) \in A$, we have that x is socially preferred to y, that is, $xF_p(\succsim_1, \ldots, \succsim_I)\, y$, whenever $x \succ_h y$.

(a) Show that if for some $\{x, y\} \subset X$, a subgroup of individuals $S \subset I$ is decisive for x over y, then for any third alternative z, The subgroup S is decisive for z over y.

Answer:

Consider a preference profile $(\succsim_1, \ldots, \succsim_I)$ satisfying

$$\begin{aligned} z &\succ_i x \succ_i y \text{ for all } i \in S, \text{ and} \\ y &\succ_i z \succ_i x \text{ for all } i \in I \backslash S. \end{aligned}$$

Since S **is decisive** for x over y we must have $xF_p(\succsim_1, \ldots, \succsim_I)\, y$. Furthermore, Since $z \succ_i x$ for every i, by the Pareto property we must have $zF_p(\succsim_1, \ldots, \succsim_I)\, x$. Intuitively, since all agents prefer z to x, the social preferences must order z above x. Thus, we can invoke transitivity, obtaining $zF_p(\succsim_1, \ldots, \succsim_I)\, xF_p(\succsim_1, \ldots, \succsim_I)\, y$ which implies $zF_p(\succsim_1, \ldots, \succsim_I)\, y$. Moreover, by pairwise independence this relation should hold regardless of individual ranking of x, which implies that whenever $z \succ_i y$ for all $i \in S$, and $y \succ_i z$ for all $i \in I \backslash S$, we should have $zF_p(\succsim_1, \ldots, \succsim_I)\, y$, i.e., subgroup S is decisive for z over y.

2. MWG 21.B.1

1. [**Majority voting - Some properties**] Consider majority voting between two alternatives x and y, so the preferences of every individual i over these two alternatives can be represented as $\alpha_i = \{1, 0, -1\}$, where $\alpha_i = 1$ indicates that individual i strictly prefers x to y; $\alpha_i = 0$ reflects that he is indifferent between alternatives x and y; and $\alpha_i = -1$ represents that he strictly prefers y to x. Let us check that, in this context, majority voting satisfies the following three properties: (a) symmetry among agents, (b) neutrality between alternatives, and (c) positive responsiveness.

- *Symmetry.* Consider a permutation $\pi : \{1, 2, \cdots, I\} \to \{1, 2, \cdots, I\}$ (i.e., an onto function altering the identities of at least one individual). Then the sum of α_i across all individuals yields

$$\sum_{i=1}^{I} \alpha_i = \sum_{i=1}^{I} \alpha_{\pi(i)},$$

That is, reordering the identities of the individuals just reorders the position of each α_i, but does not alter the sum. This implies that $\text{Sign}\left(\sum_{i=1}^{I} \alpha_i\right) = \text{Sign}\left(\sum_{i=1}^{I} \alpha_{\pi(i)}\right)$, thus yielding the same social welfare functional.

- *Neutrality.* The social choice functional of a given preference profile $(\alpha_1, ..., \alpha_n)$ is $F(\alpha_1, \cdots, \alpha_I) = \text{Sign}\left(\sum_{i=1}^{I} \alpha_i\right)$, which can be rearranged as

$$\text{Sign}\left(-\sum_{i=1}^{I}(-\alpha_i)\right) = -\text{Sign}\left(\sum_{i=1}^{I}(-\alpha_i)\right) = -F(-\alpha_1, \cdots, -\alpha_I).$$

Therefore, $F(\alpha_1, \cdots, \alpha_I) = -F(-\alpha_1, \cdots, -\alpha_I)$; as required by neutrality. In words, the social welfare functional of preference profile $(\alpha_1, ..., \alpha_n)$ coincides with the negative of the preference profile in which the preferences of all individuals have been reversed, i.e., $(-\alpha_1, \cdots, -\alpha_I)$.

- *Positive Responsiveness.* Assume a preference profile $(\alpha_{\rightarrow}, \cdots, \alpha_I)$ for which the social welfare functional yields $F(\alpha_1, \cdots, \alpha_I) \geq 0$. Then, since we are dealing with majority voting with two alternatives, we must have $\text{Sign}\left(\sum_{i=1}^{I} \alpha_i\right) \geq 0$ which, in turn, implies that $\sum_{i=1}^{I} \alpha_i \geq 0$. Let us now take another preference profile $(\alpha_1', \cdots, \alpha_I') \geq (\alpha_1, \cdots, \alpha_I)$ such that $(\alpha_1', \cdots, \alpha_I') \neq (\alpha_1, \cdots, \alpha_I)$, that is, a preference profile in which the consideration of alternative x has increased, $\alpha_i' > \alpha_i$, for at least one individual i. Then, the sum $\sum_{i=1}^{I} \alpha_i'$ must satisfy $\sum_{i=1}^{I} \alpha_i' > 0$. Indeed:

  - If the sum for the original profile of preferences was zero, $\sum_{i=1}^{I} \alpha_i = 0$, then the new preference profile satisfies $\sum_{i=1}^{I} \alpha_i' > 0$ since $\alpha_i' > \alpha_i$ for at least one individual i.
  - If, instead, the sum for the original profile of preferences was positive, $\sum_{i=1}^{I} \alpha_i > 0$, then the new preference profile must also be positive, $\sum_{i=1}^{I} \alpha_i' > 0$, since $\alpha_i' > \alpha_i$.

  Therefore, $\text{Sign}\left(\sum_{i=1}^{I} \alpha_i'\right) > 0$, which in turn implies that $F(\alpha_1', \cdots, \alpha_I') = 1$, as required by positive responsiveness.

3. Based on MWG 21.B.2

1. **[Three examples of social welfare functionals]** In this exercise, we consider a setting with two alternatives x and y, and discuss three specific social welfare functionals

$F(\alpha_1, .., \alpha_I)$ in parts (a)-(c) below. For each functional, determine whether or not it satisfies the three properties of majority voting (symmetry among agents, neutrality between alternatives, and positive responsiveness).

(a) Let us first consider the lexicographic social welfare functional

$$F(\alpha_1, .., \alpha_I) \begin{cases} \alpha_1 \text{ if } \alpha_1 \neq 0 \\ \alpha_2 \text{ if } \alpha_1 = 0 \text{ and } \alpha_2 \neq 0 \\ \alpha_3 \text{ if } \alpha_1 = \alpha_2 = 0 \text{ and } \alpha_3 \neq 0 \\ \cdots \end{cases}$$

Intuitively, society selects the alternative that individual 1 strictly prefers. However, if he is indifferent between alternatives x and y, society follows the strict preferences of individual 2 (if he has a strict preference over x or y). If both individuals 1 and 2 are indifferent between x and y, the strict preferences of individual 3 are considered, and so on.

- *Symmetry among agents*: We can easily find settings in which this property does not hold. In particular, consider a preference profile in which $\alpha_1 > 0 > \alpha_j$, where $j \neq 1$ represents any individual different from 1. In this case, since $\alpha_1 > 0$, the lexicographic swf yields $F(\alpha_1, \alpha_j, \alpha_{-1,j}) > 0$, where $\alpha_{-1,j} = (\alpha_2, \alpha_3, ..., \alpha_{j-1}, \alpha_{j+1}, ..., \alpha_I)$ denotes the preference profile of all individuals other than 1 and j. In this context, if we reorder the identities of the individuals so that individual j now becomes 1, and individual 1 becomes j, the lexicographic swf would select the alternative that individual j (who is now the first) strictly prefers, that is, $F(\alpha_j, \alpha_1, \alpha_{-1,j}) < 0$. Hence, symmetry among agents is violated.
- *Neutrality between alternatives*: Let us first recall that the lexicographic social welfare functional F is equal to α_1 if $\alpha_1 \neq 0$; is equal to α_2 if $\alpha_1 = 0$ but $\alpha_2 \neq 0$; and, similarly, is equal to α_k when $\alpha_1, \alpha_2, ..., \alpha_{k-1} = 0$ but $\alpha_k \neq 0$. [Formally, $F(\alpha_1, ..., \alpha_k, ..., \alpha_I) = \alpha_k$, where α_k is the first non-zero element in the preference profile $(\alpha_1, \alpha_2, ..., \alpha_I)$, that is, $(0, ..., 0, \alpha_k, ..., \alpha_I)$] Hence, the negative of $F(\alpha_1, ..., \alpha_k, ..., \alpha_I)$ is

$$-F(\alpha_1, ..., \alpha_k, ..., \alpha_I) = -\alpha_k,$$

In addition, if we create the profile of individual preferences in which every individual's α_i has been "reversed" to $-\alpha_i$, i.e., $(-\alpha_1, ..., -\alpha_k, ..., -\alpha_I)$, its social welfare functional becomes

$$F(-\alpha_1, ..., -\alpha_k, ..., -\alpha_I) = -\alpha_k$$

Intuitively, since the original preference profile is $(\alpha_1, \alpha_2, ..., \alpha_I) = (0, ..., 0, \alpha_k, ..., \alpha_I)$, the new preference profile is $(-\alpha_1, ..., -\alpha_k, ..., -\alpha_I) = (0, ..., 0, -\alpha_k, ..., -\alpha_I)$, implying that the lexicographic social welfare functional produces $-\alpha_k$. Hence, $F(\alpha_1, ..., \alpha_k, ..., \alpha_I) = -F(-\alpha_1, ..., -\alpha_k, ..., -\alpha_I) = \alpha_k$; as required by neutrality between alternatives.

- *Positive responsiveness.* Again, let α_k be the first non-zero element in the preference profile $(\alpha_1, ..., \alpha_I)$. Consider a preference profile $(\alpha_1, ..., \alpha_k, ..., \alpha_I)$ producing a social welfare functional $F(\alpha_1, ..., \alpha_k, ..., \alpha_I) \geq 0$. Let us now specify another preference profile $(\alpha'_1, ..., \alpha'_k, ..., \alpha'_I)$ in which alternative x increased its importance relative to y for at least one individual, i.e., $(\alpha'_1, ..., \alpha'_k, ..., \alpha'_I) \geq (\alpha_1, ..., \alpha_k, ..., \alpha_I)$ where $(\alpha'_1, ..., \alpha'_k, ..., \alpha'_I) \neq (\alpha_1, ..., \alpha_k, ..., \alpha_I)$. In this new preference profile, let α'_j represent the first non-zero element (being analogous to α_k in the original preference profile). Two cases then arise:
  - If $j < k$, then $F(\alpha'_1, ..., \alpha'_j, ..., \alpha'_k, ..., \alpha'_I) = \alpha'_j > 0$ since $(\alpha'_1, ..., \alpha'_k, ..., \alpha'_I) \geq (\alpha_1, ..., \alpha_k, ..., \alpha_I)$.
  - If $j \geq k$, then $F(\alpha'_1, ..., \alpha'_k, ..., \alpha'_j, ..., \alpha'_I) = \alpha'_k > 0$.
- Hence, for any relation between j and k, the lexicographic social welfare functional is strictly positive, $F(\alpha'_1, ..., \alpha'_k, ..., \alpha'_I) > 0$; as required by positive responsiveness. Intuitively, alternative x is chosen over y by the social welfare functional under the new preference profile. The only difference between the two cases analyzed above is that in the first (second) case alternative x is socially chosen over y because individual j (k, respectively) was the first individual with a strict preference for x over y (as all other previous elements in the profile are zero).

(b) A constant social welfare functional $F(\alpha_1, ..., \alpha_I) = 1$ for all $(\alpha_1, ..., \alpha_I)$, thus representing that society chooses alternative x over y regardless of the profile of individual preferences $(\alpha_1, ..., \alpha_I)$.

- *Symmetry among agents*: If we were to rearrange the identities of at least one individual in preference profile $(\alpha_1, ..., \alpha_I)$, the result would not change since the swf always returns a 1, regardless of the preference profile. Hence, symmetry among agents is satisfied.
- *Neutrality between alternatives*: Since the social welfare functional produces $F(\alpha_1, ..., \alpha_I) = 1$ for all preference profiles $(\alpha_1, ..., \alpha_I)$, then the "reversed" preference profile $(-\alpha_1, ..., -\alpha_I)$ yields $F(-\alpha_1, ..., -\alpha_I) = 1$ as well. Therefore, its negative is $-F(-\alpha_1, ..., -\alpha_I) = -1$, implying that

$$F(\alpha_1, ..., \alpha_I) = 1 \neq -1 = -F(-\alpha_1, ..., -\alpha_I)$$

  and neutrality among alternatives is violated.
- *Positive responsiveness.* Consider a different preference profile $(\alpha'_1, ..., \alpha'_I)$ where alternative x increased its importance relative to y for at least one individual, i.e., $(\alpha'_1, ..., \alpha'_I) \geq (\alpha_1, ..., \alpha_I)$ but $(\alpha'_1, ..., \alpha'_I) \neq (\alpha_1, ..., \alpha_I)$. Since $F(\alpha_1, ..., \alpha_I) = 1$ for all preference profiles $(\alpha_1, ..., \alpha_I)$, then the social welfare functional for the new profile $(\alpha'_1, ..., \alpha'_I)$ is also equal to 1, i.e., $F(\alpha'_1, ..., \alpha'_I) = 1$, thus being positive; as required by positive responsiveness.

(c) A constant social welfare functional $F(\alpha_1, ..., \alpha_I) = 0$ for all $(\alpha_1, ..., \alpha_I)$, thus indicating that society is indifferent between alternatives x and y regardless of the profile of individual preferences $(\alpha_1, ..., \alpha_I)$.

- *Symmetry among agents*: If we were to rearrange the identities of at least one individual in preference profile $(\alpha_1, ..., \alpha_I)$, the result would not change as the swf always is equal to 0 regardless of the preference profile. Thus, symmetry among agents is satisfied.

- *Neutrality between alternatives*: Since the social welfare functional produces $F(\alpha_1, ..., \alpha_I) = 0$ for all prefrence profiles $(\alpha_1, ..., \alpha_I)$, then the "reversed" preference profile $(-\alpha_1, ..., -\alpha_I)$ yields $F(-\alpha_1, ..., -\alpha_I) = 0$ as well. Hence,

$$F(\alpha_1, ..., \alpha_I) = 0 = -F(-\alpha_1, ..., -\alpha_I)$$

  and neutrality among alternatives is satisfied.

- *Positive responsiveness.* Consider a different preference profile $(\alpha'_1, ..., \alpha'_I)$ where alternative x increased its importance relative to y for at least one individual, i.e., $(\alpha'_1, ..., \alpha'_I) \geq (\alpha_1, ..., \alpha_I)$ but $(\alpha'_1, ..., \alpha'_I) \neq (\alpha_1, ..., \alpha_I)$. Since $F(\alpha_1, ..., \alpha_I) = 0$ for all preference profiles $(\alpha_1, ..., \alpha_I)$, the social welfare functional for the new profile $(\alpha'_1, ..., \alpha'_I)$ is also equal to 0, i.e., $F(\alpha'_1, ..., \alpha'_I) = 0$. As a consequence, $F(\alpha'_1, ..., \alpha'_I)$ is not strictly positive, ultimately violating positive responsiveness.

4. Policy Application

Political Coalitions and Public School Finance Policy: In this exercise, we consider some policy issues related to public support for schools — and the coalitions between income groups that might form to determine the political equilibrium.

Part A: Throughout, suppose that individuals vote on only the single dimension of the issue at hand — and consider a population that is modeled on the Hotelling line $[0, 1]$ with income increasing on the line. (Thus, individual 0 has the lowest income and individual 1 has the highest income, with individual 0.5 being the median income individual.)

(a) Consider first the case of public school funding in the absence of the existence of private school alternatives. Do you think the usual median voter theorem might hold in this case — with the public school funding level determined by the ideal point of the median income household?

Answer:

While it depends on the nature of the tax system used to fund the public schools, it is not unlikely that, given the tax system used to fund the schools, demand for public school spending increases with household income. In that case, the ideal points for public spending would be increasing along the Hotelling line — with the median voter's most preferred public spending level representing the Condorcet winning policy.

(b) Next, suppose private schools compete with public schools, with private schools charging tuition and public schools funded by taxes paid by everyone, How does this change the politics of public school funding?

Answer:

Conditional on sending one's children to private schools, one's preferred tax — and thus one's preferred public school spending level — would drop to zero. Thus, the politics of public school funding changes in the sense that those who choose private schools become low demanders of public school quality.

(c) Some have argued that political debates on public school funding are driven by "the ends against the middle". In terms of our model, this means that the households on the ends of the income distribution on the Hotelling line will form a coalition with one another — with households in the middle forming the opposing coalition. What has to be true about who disproportionately demands private schooling in order for this "ends against the middle" scenario to unfold?

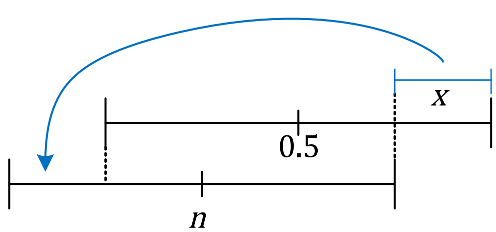
Answer:

For the "ends against the middle" scenario to unfold, it must be that demand for private schools comes disproportionately from high income families who would favor high public school spending in the absence of private schools but now favor low public school spending.

(d) Assume that the set of private school students comes from high income households. What would this model predict about the income level of the new median voter?

Answer:

This is illustrated in below. The first line from 0 to 1 represents the original Hotelling line with income and ideal levels of public school spending rising along the line. As a result, 0.5 is the original median voter whose preferred public school spending level is implemented in the absence of private schools. If then a private school opens and draws the highest x income households away from the public system, that x segment of the Hotelling line now moves to the other side of the original median voter in terms of its ideal point for public good spending. As a result, the new median voter is $n = \frac{1-x}{2}$ which implies the new median voter (in the presence of private schools) has less income than the original median voter (in the absence of private school).



(e) Consider two factors: First, the introduction of private schools causes a change in the income level of the median voter, and second, we now have private school attending households that pay taxes but do not use public schools. In light of this, can you tell whether per pupil public school spending increases or decreases as private school markets attract less than half the population? What if they attract more than half the population?

Answer:

As we have seen, the median voter's income will fall — which, all else being equal, would imply a decrease in public school spending. But fewer kids attending public school and thus every dollar in tax revenue results in a larger increase in per pupil spending. Thus, "all else is not equal" — and the two forces point in opposite directions. This makes it impossible to tell without further information whether per pupil public sending on education rises or falls as more high income individuals go to private schools — at least so long as fewer than 50% do so. If more than 50% attend private schools, the median voter will be someone who sends her child to private schools — which would cause us to predict a sharp drop in public school funding (to zero, if we take the model completely literally). But if private school attendance is less than 50%, you can think of public school attendees actually getting a subsidy from private school attendees — so, although they would vote for less spending if the price remained the same, they might vote for more given the implicit subsidy that allows them to free ride on the tax payments of private school attending households.

(f) So far, we have treated public school financing without reference to the local nature of public schools. In the U.S., public schools have traditionally been funded locally — with low income households often constrained to live in public school districts that provide low quality. How might this explain an "ends against the middle" coalition in favor of private school vouchers (that provide public funds for households to pay private school tuition)?

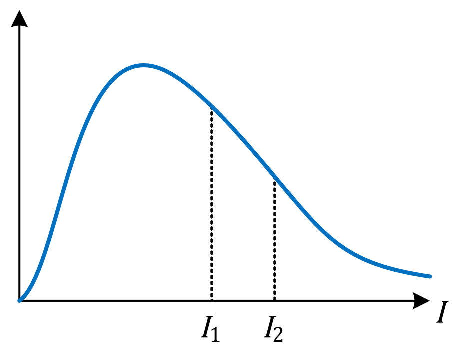
Answer:

In a system with widely varying school quality based on local income levels, it may be the case that low income parents would be the first to switch to private schools if they received a voucher — and would find this preferable to their current public school. High income households already send their children to private schools — so vouchers would be a pure income transfer to them (as they would not have to pay as much of their children's tuition bills anymore). Thus, there are two natural constituencies for private school voucher: the poor who are constrained in the public system to the worst schools, and the rich who already use private schools.

(g) In the 1970's, California switched from local financing of public schools to state-wide (and equalized) financing of its public schools. State-wide school spending appears to have declined as a result. Some have explained this by appealing to the fact that the income distribution is skewed to the left, with the statewide median income below the statewide mean income. Suppose that local financing implies that each public school is funded by roughly identical households (who have self-selected into different districts as the Tiebout model would predict), while state financing implies that the public school spending level is determined by the state median voter. Can you explain how the skewedness of the state income distribution can then explain the decline in state-wide public school spending as the state switched from local to state financing?

Answer:

Such a skewed income distribution is graphed below where I_1 is the median income level and I_2 is the mean income level. In a local system where income types separate into distinct communities, there is broad agreement within the community on how much to spend on the local school — because everyone has the same income. Thus, each community funds its school in relation to the common income level of its residents — with the average per pupil spending level in the state therefore approximately equal to the income level that the average person would have chosen. When per pupil spending is determined in state-wide elections, however, the median income level will determine the spending level — and since the median is below the mean, we would expect the skewedness of the income distribution to result in less overall public spending on education in the statewide system than in the local system.



Part B: Suppose preferences over private consumption x, a public good y and leisure l can be described by the utility function $u(x, y, l) = x^{\alpha} y^{\beta} l^{\gamma}$. Individuals are endowed with the same leisure amount L, share the same preferences but have different wages. Until part (e), taxes are exogenous.

(a) Suppose a proportional wage tax t is used to fund the public good y and a tax rate t results in public good level $y = \delta t$. Calculate the demand function for x and the labor supply function. (Note: Since t is not under the control of individuals, neither t nor y are choice variables at this point.)

Answer:

The consumer then solves the problem

$$\max_{x,l} x^{\alpha} y^{\beta} l^{\gamma} \text{ subject to } x = (1-t)\, w\, (L - l)\,.$$

Solving this in the usual way, we get leisure demand of

$$l = \frac{\gamma L}{\alpha + \gamma}$$

Substracting this from L gives the labor supply l^*, and plugging it into the budget constraint gives us x^* — which are

$$l^* = \frac{\alpha L}{\alpha + \gamma} \quad \text{and } x^* = (1-t)\, w \left(\frac{\alpha L}{\alpha + \gamma}\right)$$

(b) Suppose instead that a per-capita tax T is used to fund the public good; i.e. everyone has to pay an equal amount T. Suppose that a per-capita tax T results in public good level $y = T$. Calculate the demand function for x and the labor supply function.

Answer:

We now solve

$$\max_{x,l} \quad x^\alpha y^\beta l^\gamma$$

$$\text{subject to} \quad x = w\,(L - l) - T$$

Solving this in the usual way, we get leisure demand

$$l = \frac{\gamma\,(wL - T)}{(\alpha + \gamma)\, w}$$

Substracting from L gives labor supply l^* and substituting into the budget constraint gives demand for x — i.e.

$$l^* = \frac{\alpha w L - \gamma T}{(\alpha + \gamma)\, w} \quad \text{and} \quad x^* = \frac{\alpha w L - (2\gamma + \alpha)\, T}{\alpha + \gamma}$$

(c) *True or False:* Since the wage tax does not result in a distortion of the labor supply decision while the per-capita tax does, the former has no deadweight loss while the latter does.

Answer:

This is false. Efficiency losses from taxes happen from substitution effects —which occur when taxes change opportunity costs and not when they do not. The wage tax changes the price of leisure —and thus gives rise to substitution effects which happen to be masked by an offsetting wealth effect in our example. But the substitution effect create deadweight loss. The lump sum tax T, on the other hand, does not give rise to any substitution effects —even though it's wealth effect causes a change in the optimal labor supply decision. But the wealth effect does not cause a deadweight loss.

(d) Calculate the indirect utility function for part (a) (as a function of L, w and t).

Answer:

To get the indirect utility function, we plug l^* and x^* from (a) into the utility function and replace y with δt to get

$$\begin{aligned} V &= \left[(1-t)\, w \left(\frac{\alpha L}{\alpha+\gamma}\right)\right]^{\alpha} (\delta t)^{\beta} \left(\frac{\gamma L}{\alpha+\gamma}\right)^{\gamma} \\ &= (1-t)^{\alpha}\, t^{\beta}\, (\alpha w)^{\alpha}\, \delta^{\beta} \gamma^{\gamma} \left(\frac{L}{\alpha+\gamma}\right)^{\alpha+\gamma}. \end{aligned}$$

(e) Now suppose that a vote is held to determine the wage tax t. What tax rate will be implemented under majority rule? (*Hint*: Use your result from (d) to determine the ideal point for a voter.)

Answer:

To determine a voter's ideal tax rate t^*, all we have to do is maximize the indirect utility function with respect to t. Taking the derivation of V with respect to t and setting it to zero, we can then solve for voter's optimal tax rate as

$$t^* = \frac{\beta}{\alpha+\beta}.$$

Since this optimal tax rate for our voter is not a function of wage (which is the only dimension on which voters differ), all voters agree that this is the optimal tax rate —i.e., all voters have the same ideal point. This is because, although higher income voters demand more y all else equal, they also have to pay more of a tax share — with the latter effect offsetting the former.

(f) Suppose that y is per pupil spending on public education. What does this imply that δ is (in terms of average population income $\bar{I}$, number of taxpayers K and number of kids N in school)?

Answer:

Tax revenue from a tax rate t is then $tK\bar{I}$ —and per pupil spending is revenue divided by N. Thus,

$$\delta = \frac{K\bar{I}}{N} \text{ when } y = \delta t.$$

(g) Now suppose there exists a private school market that offers spending levels demanded by those interested in opting out of public education (and assume that spending is all that matters in people's evaluation of school quality). People attending private school no longer attend public school but still have to pay taxes. Without doing any additional math, what are the possible public school per pupil spending levels y that you think could emerge in a voting equilibrium (assuming that public education is funded through a proportional wage tax)? Who will go to what type of school?

Answer:

As more children go to private school, N — the number of children going to public school — falls. This implies that δ increases as more children go to private school. Our expression of t^* is independent of δ — which implies that those who remain in public schools continue to favor the same tax rate as before, but those choosing private school will now favor a tax rate of zero. So long as the fraction going to private school is less than 0.5, however, t^* remains the majority rule equilibrium — which implies that per pupil public spending increases as private school markets expand. Those who attend private school will be the richest households.

(h) Can you think of necessary and sufficient conditions for the introduction of a private school market to result in Pareto improvement in this model?

Answer:

So long as less than half of the population goes to private school, the introduction of private schools represents a Pareto improvement in this model. This is because per pupil spending rises in public schools while tax rates remain the same —implying that public school attending households are better off than they would be in the absence of private schools. Private school attending households could choose to consume the higher levels of public school spending at the same tax rates as well but choose instead to opt for private school —which implies they are at least as well off as they would be in the public schools that now spend more. Thus, private school attending households must also be better off than they would be in the absence of a private school market. With everyone benefitting from the introduction of private schools, the model therefore predicts a Pareto improvement.

(i) In part (e), you should have concluded that, under the proportional wage tax, everyone unanimously agrees on what the tax rate should be (when there are no private schools). Would the same be true if schools were funded by a per-capita tax T?

Answer:

No. Under a per capita tax T, everyone pays the same amount (rather than the same rate). This implies that ideal points will not all be the same in the absence of private school market —with ideal points for T increasing with household income and the median income household determining the level of T (and thus the level of per pupil spending) in a voting equilibrium. Thus, only the median income voter would get his preferred level of taxation.