EconS 503 – Homework #8

Due date: April 13th, in class.

Exercise #1 - Damaged-good strategy (Menu Pricing)

A firm sells a product in a market where there are two types of consumer, high and low-valuation consumers. There are equally many of the two types of consumer, and the total number of consumers is normalized to 1. The product has value 3 to the high-valuation consumers and value 1 to the low-valuation consumers. All consumers have unit demand, that is, they buy either one unit or do not participate. The product is produced at constant marginal cost equal to 0. The firm considers introducing a damaged version of the product. The damaged version is produced at constant marginal cost equal to 1/10. It results in a utility of 5/10 to the low-valuation consumers and of 6/10 to the high-valuation consumers.

1. Find the profit-maximizing price in the absence of a damaged version. Calculate the firm's profit.
2. Fin the optimal price of the normal and of the damaged version of the product when the two versions are offered. Calculate the firm's profit.
3. Should the firm introduce the damaged version? What are the welfare consequences of the introduction of the damaged version?

Exercise #2 - Nonlinear pricing

A monopolist produces a good with constant marginal cost equal to $c, c < 1$. Assume for now that all consumers have the demand $Q(p) = 1 - p$. The population is of size 1.

1. Suppose that the monopolist cannot discriminate in any way among the consumers and has to charge a uniform price, p^U. Calculate both the price that maximizes profits and the profits that correspond to this price.
2. Suppose now that the monopolist can charge a two-part tariff (m, p), where m is the fixed fee and p is the price per unit. Expenditure then is $m + pq$. Calculate the two-part tariff that maximizes profits and the profits that correspond to this tariff. Compare p^U and m and comment briefly. Compare the situation with a uniform price and a two-part tariff in terms of welfare (a verbal argument is sufficient).
3. Assume now instead that there are two types of consumer. The consumers of type 1 have demand $Q_1(p) = 1 - p$ and the consumers of type 2 have demand $Q_2(p) = 1 - p/2$. The population is of size 1 and there are equally many consumers of the two types. Finally, it is assumed in this question that $c = 1/2$. Calculate the two-part tariff that maximizes the profits of the monopolist. Compare the two-part tariffs found in (2) and (3) for $c = 1/2$ and comment briefly.

Exercise #3 - Moral Hazard

A friend has asked you to sneak a six-pack of beer for him into a concert. Being the kind of friend that you are, you cannot be trusted not to drink the beer yourself just before going into the concert. Unfortunately, if you do not drink the beer, then there is some possibility (1/10) that the beer will be confiscated by security on your way in. Thus, if you show up with no beer, your friend cannot tell whether you drank the beer or it was confiscated. (Remember, this is fiction.) You are going to offer a deal to your friend where the fee your friend pays for your service depends on whether you deliver the beer. Here are the important things you need to know in order to design an optimal contract:

1. Your utility if you get x dollars out of this transaction is
$$u(x) = \begin{cases} -e^{-.2x} & \text{if you don't drink the beer} \\ -e^{-.2(x+5)} & \text{if you drink the beer.} \end{cases}$$
(x is positive if you receive a payment from your friend and negative if you pay money to your friend.) Thus (i) you are risk averse, with constant absolute risk aversion, and (ii) beer and money are perfect substitutes, with the six-pack being equivalent to $5.
2. Your friend's utility if she gets x dollars out of this transaction is
$$v(x) = \begin{cases} x & \text{if she doesn't drink the beer} \\ x+3 & \text{if she drinks the beer before the concert} \\ x+8 & \text{if she drinks the beer in the concert} \end{cases}$$
(x is positive if she receives a payment from you and is negative if she pays money to you.) Thus (i) your friend is risk neutral, and (ii) beer and money are perfect substitutes, with beer being equivalent to $8 if drunk in the concert and $3 if drunk before the concert.

   (a) Suppose your friend will accept any deal you offer her for which her expected utility is at least 3, which is what she would get if she rejected your deal and just drank the beer before the concert.
   Write down the two equations whose solution gives the optimal contract. (OPTIONAL: Solve the two equations numerically.)
   (b) Suppose the bargaining power is shifted to your friend. She makes a take-it-or-leave-it offer, and you accept the deal as long as your expected utility is at least -1 (i.e., $-e^0$), which is what you would get if you didn't make any deal with your friend. Write down the two equations whose solution gives the optimal contract for your friend. (This time, if you write the equations properly, you can solve them easily.) What is the first-best contract, and how do the first-best and second-best contracts compare in terms of the expected utility that you and your friend get?

Exercise #4 – Moral Hazard again

- Exercises 12 and 13 from Bolton and Dewatripont.