EconS 424 - Strategy and Game Theory
Quiz #2 - April 18th, 2018

Instructions: You have 20 minutes to complete this exam. Please read the questions carefully, answer them in a formal and concise manner, but include all your steps, this will allow you to obtain partial credit. Good luck!!

Let us consider a Cournot oligopoly game where two firms compete in quantities. Both firms have the same marginal costs, $MC = \$2$, but they are asymmetrically informed about the actual state of market demand. In particular, Firm 2 does not know what is the actual state of demand, but knows that it is distributed with the following probability distribution

$$p(Q) = \begin{cases} 20 - Q \text{ with probability } \frac{2}{3} \\ 8 - Q \text{ with probability } \frac{1}{3} \end{cases}$$

On the other hand, firm 1 knows the actual state of market demand, and firm 2 knows that firm 1 knows this information (i.e., it is common knowledge among the players).

1. Let us first focus on Firm 1, the *informed* player in this game, as we usually do when solving for the BNE of games of incomplete information.

   (a) Find firm 1's best response function when the firm operates in a high-demand market. Denote it as $q_1^H(q_2)$.

   (b) Find firm 1's best response function when the firm operates in a low-demand market. Denote it as $q_1^L(q_2)$.

2. Let us now turn to Firm 2, the *uninformed* player in this game.

   (a) Write the *expected* profits of this firm, taking into account the above probabilities of operating in a high or low-demand market.

   (b) Find firm 2's best response function. Denote it as $q_2(q_1^H, q_1^L)$. [Recall that its best response function is only one, since firm 2 does not know whether the market is in high or low demand.]

3. Insert $q_1^H(q_2)$ from exercise 1(a) and $q_1^L(q_2)$ from 1(b) into $q_2(q_1^H, q_1^L)$ from 2(b). Then solve for q_2^* in order to find firm 2's equilibrium production level (this production should be a number).

4. Insert the value of q_2^* you found in part (3) into the expression of $q_1^H(q_2)$ you obtained in exercise 1(a) and in $q_1^L(q_2)$ from 1(b). Summarize your results (you have just found the Bayesian Nash Equilibrium of this game of incomplete information!).

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Quiz #2 - *Answer Key*

1) Firm 1 (informed firm):

1a) If *high* demand:

$$\pi_1^H = (20 - q_1^H - q_2) * q_1^H - 2q_1^H$$

Taking FOCs with respect to q_1^H,

$$20 - 2q_1^H - q_2 - 2 = 0 \ \rightarrow 18 - q_2 = 2q_1^H$$

Hence, solving for q_1^H, we obtain firm 1's best response function when facing a high demand

$$BRF_1^H \rightarrow q_1^H(q_2) \rightarrow q_1^H = 9 - \frac{1}{2}q_2$$

1b) If *low* demand:

$$\pi_1^L = (8 - q_1^L - q_2)q_1^L - 2q_1^L$$

Taking FOCs with respect to q_1^L,

$$8 - 2q_1^L - q_2 - 2 = 0 \rightarrow 6 - q_2 = 2q_1^L$$

Hence, solving for q_1^L, we obtain firm 1's best response function when facing a low demand

$$BRF_1^L \rightarrow q_1^L(q_2) \rightarrow q_1^L = 3 - \frac{1}{2}q_2$$

2) Firm 2 (uninformed firm):

2a) Expected profit for Firm 2

$$\mathrm{E}\pi_2 = \frac{2}{3}\left((20 - q_1^H - q_2)q_2 - 2q_2\right) + \frac{1}{3}\left((8 - q_1^L - q_2)q_2 - 2q_2\right)$$

$$= \frac{2}{3}(20q_2 - q_1^H q_2 - q_2^2) + \frac{1}{3}(8q_2 - q_1^L q_2 - q_2^2) - 2q_2$$

2b) Taking FOCs with respect to q_2,

$$\frac{40}{3} - \frac{2}{3}q_1^H - \frac{4}{3}q_2 + \frac{8}{3} - \frac{1}{3}q_1^L - \frac{2}{3}q_2 - 2 = 0$$

And solving for q_2, we obtain firm 2's best response function.

$$BRF_2 \rightarrow q_2(q_1^H, q_1^L) \rightarrow q_2 = 7 - \frac{2}{6}q_1^H - \frac{1}{6}q_1^L$$

3) Plugging q_1^H and q_1^L into firm 2's best response function, we find

$$q_2 = 7 - \frac{1}{3}\left[9 - \frac{1}{2}q_2\right] - \frac{1}{6}\left[3 - \frac{1}{2}q_2\right]$$

and solving for q_2, we obtain firm 2's equilibrium output level,

$$q_2 = 4.66$$

4) We can now find firm 1's equilibrium output level. First, plugging q_2=4.66 into $q_1^H = 9 - \frac{1}{2}q_2$, we obtain:

$$q_1^H = 9 - \frac{1}{2}(4.66) = 6.7$$

Similarly, plugging q_2=4.66 into $q_1^L = 3 - \frac{1}{2}q_2$,we obtain.

$$q_1^L = 3 - \frac{1}{2}(4.66) = 0.66$$

Summarizing, the BNE of this incomplete information Cournot game is:

$$(q_1^H, q_1^L, q_2) = (6.7, 0.66, 4.66)$$