EconS 424 - Strategy and Game Theory
Final Exam

Instructions:

- Write your answers to each exercise in a different page.
- Show all your work, and be as clear as possible in your answer. You can work in groups, but each student must submit his/her own exam.
- The due date of this take-home exam is Friday, April 27th, at 2:00p.m. in my mailbox. (You can also submit your exam in my office before this date.)
- I strongly recommend you to work a few exercises every day, rather than trying to solve all exercises in one day. Late submission will be subject to significant grade reduction.

Exercises from Watson (see scanned pages at the end of this exam):

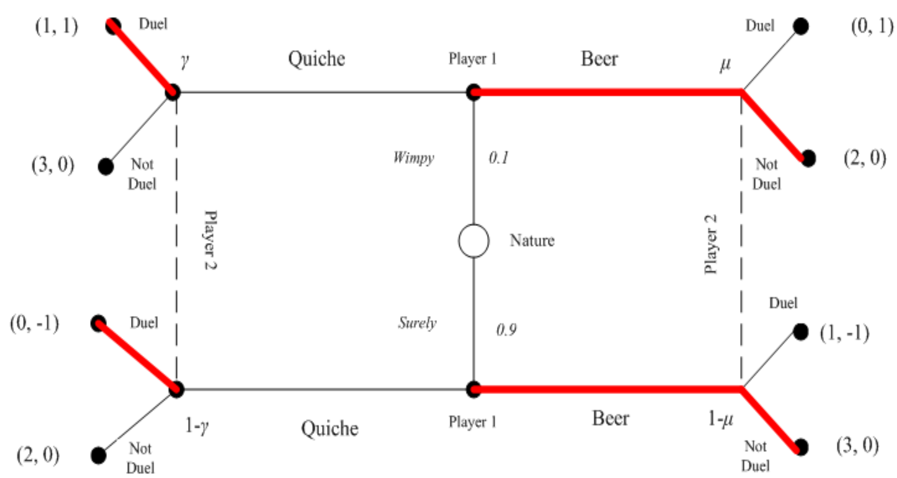
1. Exercise 2 from Chapter 23; and
2. Exercise 5 from Chapter 26.

Exercises from Harrington (both editions of the book work)**:**

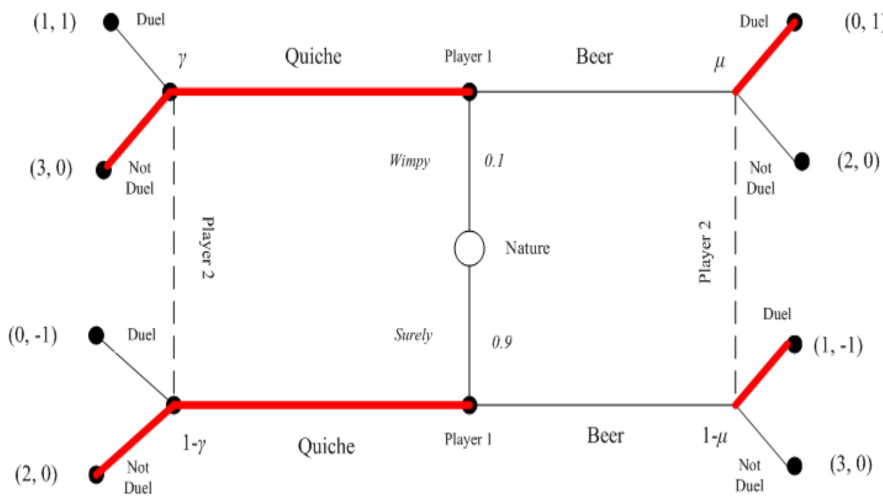
3. Exercise 9 from Chapter 11; and
4. Exercise 8 from Chapter 12.

5. [**Applying the Intuitive Criterion in the Far West (Beer-Quiche game)**] Consider the following sequential game with incomplete information. It represents a Saloon in the Far West. Player 1 just moved into town, and nobody else but him knows whether he is "Wimpy" or "Surely" (i.e., Weak or Strong). At that moment in the morning the Saloon is a quite place, and he is deciding whether to have Quiche for breakfast (something that he really enjoys if he is a Wimpy type) or a Beer (something he prefers to Quiche when he is of the Surely type). Then, player 2 (the typical character looking for trouble in this kind of films) enters into the Saloon and observes the newcomer having breakfast, but does not know whether he is Surely or Wimpy.

   (a) Check if the pooling equilibrium in which both types of player 1 have Beer for breakfast survives the Cho and Kreps' (1987) Intuitive Criterion.



   (b) Check if pooling equilibrium in which both types of player 1 have Quiche for breakfast survives the Cho and Kreps' (1987) Intuitive Criterion.



at a constant cost of 10 per unit. (That is, the cost function for each firm is $c(q) = 10q$.) Determining the Nash equilibrium of this game was the subject of a previous exercise.

(a) Suppose that this game is infinitely repeated. (The firms play the game each period, for an infinite number of periods.) Define δ as the discount factor for the firms. Imagine that the firms wish to sustain a collusive arrangement in which they all select the monopoly price $p^M = 60$ each period. What strategies might support this behavior in equilibrium? (Do not solve for conditions under which equilibrium occurs. Just explain what the strategies are. Remember, this requires specifying how the firms punish each other. Use the Nash equilibrium price as punishment.)

(b) Derive a condition on n and δ that guarantees that collusion can be sustained.

(c) What does your answer to part (b) imply about the optimal size of cartels?

Ch. 23

2. Examine the infinitely repeated tariff-setting game, where the stage game is the two-country tariff game in Chapter 10 (see also Exercise 3 in that chapter).

SCANNED IN THE NEXT TWO PAGES

(a) Compute the Nash equilibrium of the stage game.

(b) Find conditions on the discount factor such that zero tariffs ($x_1 = x_2 = 0$) can be sustained each period by a subgame perfect equilibrium. Use the grim-trigger strategy profile.

(c) Find conditions on the discount factor such that a tariff level of $x_1 = x_2 = k$ can be sustained by a subgame perfect equilibrium, where k is some fixed number between 0 and 100.

3. Repeat the analysis of goodwill presented in this chapter for the following stage game:

1 \ 2	X	Y	Z
A	5, 5	0, 3	4, 8
B	0, 0	4, 4	0, 0

4. Consider an infinite-period repeated prisoners' dilemma game in which a long-run player 1 faces a sequence of short-run opponents. (You dealt with games like this in Exercise 8 of Chapter 22.) Formally, there is an infinite number of players—denoted $2^1, 2^2, 2^3, \ldots$—who play as player 2 in the stage game. In period t, player 1 plays the following prisoners' dilemma with player 2^t.

cost of production, because a firm can grab the entire market by doing so. But in the Cournot model, firms have to raise output more than just a little to grab significant market share. In addition, large quantity increases cause large price decreases, which have a negative effect on profit. There is thus a sense in which price-setting environments are more competitive than quantity-setting environments in markets with homogeneous products.

TARIFF SETTING BY TWO COUNTRIES

National governments can influence international trade (trade between consumers and firms of different countries) by imposing barriers that restrict trade. The most common of these barriers are taxes on the importation of foreign commodities, commonly referred to as *tariffs*. A large country (or union of countries) usually benefits by setting a small import tariff, assuming that other countries do not raise their tariffs, too. Consider, for example, the European Union (EU) as an importer of bananas. Because the EU is a large economy in regard to its share of world trade, an increase in the EU's banana tariff causes the world's quantity demand for bananas to decrease and the international price of bananas to fall. Simultaneously, the tariff drives up the price of bananas in the EU. Thus, the tariff creates a wedge between the international price of bananas and the price of bananas in the EU. When this wedge is large enough, the tariff revenue may be larger than the loss in consumer welfare incurred by Europeans due to the higher prices for bananas in the EU. Similar reasoning holds for the United States as an importer of European cheese.

Thus, the United States and the EU have unilateral incentives to impose tariffs (the United States sets a tariff on the importation of cheese, and the EU sets a tariff on bananas). Unfortunately, the United States and the EU are both worse off when tariffs are uniformly high, relative to uniformly low tariffs. Thus, the tariff-setting game is a form of the prisoners' dilemma. The two economies would benefit by cooperating to keep tariffs low, instead of narrowly pursuing their individual interests. In other words, they would benefit by finding a way to enforce free trade.

A game-theoretic model can be used to illustrate the strategic aspects of tariffs. Suppose there are two countries that are labeled 1 and 2. Let x_i be the tariff level of country i (in percent), for $i = 1, 2$. If country i picks x_i and the other country (j) selects x_j, then country i gets a payoff of $2000 + 60x_i + x_i x_j - x_i^2 - 90x_j$ (measured in billions of dollars). Assume that x_1 and x_2 must be between 0 and 100 and that the countries set tariff levels simultaneously and independently.[5] Exercise 3 at the end of this chapter asks you to compute the Nash equilibrium of this game.

A MODEL OF CRIME AND POLICE

Game theory and the Nash equilibrium concept can be used to study the interaction between criminals and law-enforcement agencies. Gary Becker, a Nobel Prize-winning economist, led the way on this kind of research and showed that economic analysis is extremely useful in this policy arena. According to Becker's theory, "The optimal amount of enforcement is shown to depend on, among other things, the cost of catching and convicting offenders, the nature of punishments—for example, whether they are fines or prison terms—and the response of offenders to changes in enforcement."[6] Becker also argued that, with the optimal enforcement system, crime does occur.

Here is a game that illustrates how the government balances the social cost of crime with law-enforcement costs and how criminals balance the value of illegal activity with the probability of arrest. The game has two players: a criminal (C) and the government (G). The government selects a level of law enforcement, which is a number $x \geq 0$. The criminal selects a level of crime, $y \geq 0$. These choices are made simultaneously and independently. The government's payoff is given by $u_G = -xc^4 - y^2/x$ with the interpretation that $-y^2/x$ is the negative effect of crime on society (moderated by law enforcement) and c^4 is the cost of law enforcement, per unit of enforcement. The number c is a positive constant. The criminal's payoff is given by $u_C = y^{1/2}/(1 + xy)$, with the interpretation that $y^{1/2}$ is the value of criminal activity when the criminal is not caught, whereas $1/(1 + xy)$ is the probability that the criminal evades capture. Exercise 4 of this chapter asks you to compute the Nash equilibrium of this game.

THE MEDIAN VOTER THEOREM

Consider an expanded version of the location-choice model from Chapter 8, where two political candidates (players 1 and 2) decide where to locate on the political spectrum. Suppose the policy space is given by the interval [0, 1], with the location 0 denoting extreme liberal and 1 denoting extreme conservative. Citizens (the voters) are distributed across the political spectrum, not

[5] You can read more about tariff games in J. McMillan, *Game Theory in International Economics* (New York: Harwood Academic Publishers, 1986).

[6] G. Becker, "Crime and Punishment: An Economic Approach," *Journal of Political Economy* 76(2): 169–217, at p. 170.

(c) Explain why the predictions of parts (a) and (b) are the same in regard to equilibrium but different in regard to rationalizability. (Hint: The answer has to do with the scope of the players' beliefs.)

4. Demonstrate that, for the Cournot game discussed in this chapter, the only rationalizable strategy is the Bayesian Nash equilibrium.

Ch. 26

5. Consider a differentiated duopoly market in which firms compete by selecting prices and produce to fill orders. Let p_1 be the price chosen by firm 1 and let p_2 be the price of firm 2. Let q_1 and q_2 denote the quantities demanded (and produced) by the two firms. Suppose that the demand for firm 1 is given by $q_1 = 22 - 2p_1 + p_2$, and the demand for firm 2 is given by $q_2 = 22 - 2p_2 + p_1$. Firm 1 produces at a constant marginal cost of 10 and no fixed cost. Firm 2 produces at a constant marginal cost of c and no fixed cost. The payoffs are the firms' individual profits.

(a) The firms' strategies are their prices. Represent the normal form by writing the firms' payoff functions.

(b) Calculate the firms' best-response functions.

(c) Suppose that $c = 10$ so the firms are identical (the game is symmetric). Calculate the Nash equilibrium prices.

(d) Now suppose that firm 1 does not know firm 2's marginal cost c. With probability $1/2$ nature picks $c = 14$ and with probability $1/2$ nature picks $c = 6$. Firm 2 knows its own cost (that is, it observes nature's move), but firm 1 only knows that firm 2's marginal cost is either 6 or 14 (with equal probabilities). Calculate the best-response functions of player 1 and the two types ($c = 6$ and $c = 14$) of player 2 and calculate the Bayesian Nash equilibrium quantities.

6. Find the Bayesian Nash equilibrium of the game pictured here. Note that Exercise 3 of Chapter 24 asked you to convert this into the normal form.