ECONS 424 – STRATEGY AND GAME THEORY
MIDTERM EXAM #2

<u>DUE DATE:</u> MONDAY, APRIL 9TH 2018, IN CLASS

Instructions:

- This exam has 6 exercises, plus a bonus exercise at the end.
- Write your answers to each exercise in a different page.
- Show all your work and be as clear as possible in your answer. You can work in groups, but each student must submit his/her exam.
- The due date of this take-home exam is Monday, April 9th, in class. I strongly recommend you work a few exercises every day, rather than trying to solve all exercises in one day.
- Since this is a take-home exam, late submission will be subject to significant grade reduction.

Exercise #1. Hawk-Dove game. Consider the following payoff matrix representing the Hawk-Dove game. Intuitively, Players 1 and 2 compete for a resource, each of them choosing to display an aggressive posture (hawk) or a passive attitude (dove). Assume that payoff $V > 0$ denotes the value that both players assign to the resource, and $C > 0$ is the cost of fighting, which only occurs if they are both aggressive by playing hawk in the top left-hand cell of the matrix.

		Player 2	
		Hawk	*Dove*
Player 1	*Hawk*	$\frac{V-C}{2}, \frac{V-C}{2}$	$V, 0$
	Dove	$0, V$	$\frac{V}{2}, \frac{V}{2}$

a) Show that if $C \leq V$, the game is strategically equivalent to a Prisoner's Dilemma game.

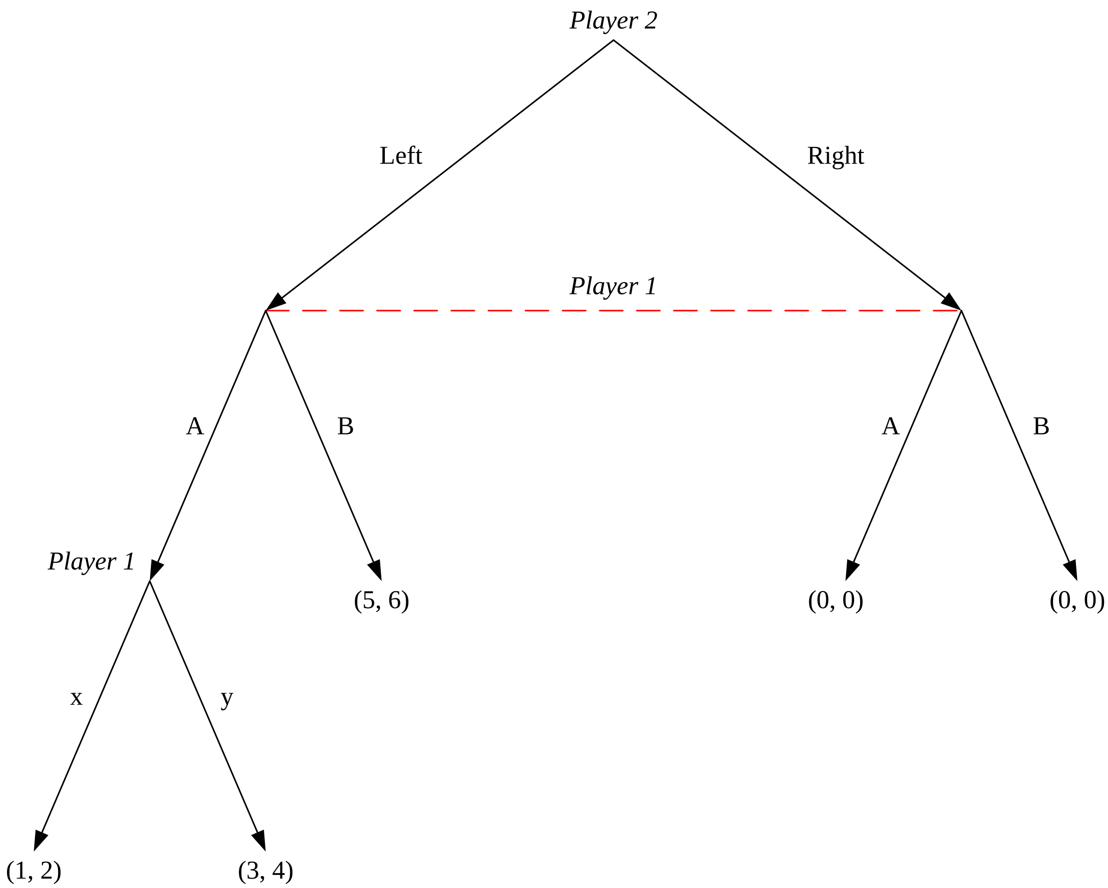
b) The Hawk-Dove game commonly assumes that the value of the resource is less than the cost of a fight, i.e., $C > V > 0$. Find the set of pure strategy Nash equilibria.

Exercise #2. Cournot competition and cost disadvantages. Consider two firms competing in quantities (a la Cournot), facing inverse demand function $p(Q) = a - Q$, where Q denotes aggregate output, that is, $Q = q_1 + q_2$. Firm 1's marginal cost of production is $c_1 > 0$, while firm 2's marginal cost is $c_2 > 0$, where $c_2 \geq c_1$ indicating that firm 2 suffers a cost disadvantage relative to firm 1. Firms face no fixed costs.

For compactness, let us represent firm 2's marginal costs relative to c_1, so that $c_2 = \alpha c_1$, where parameter $\alpha \geq 1$ indicates the cost disadvantage that firm 2 suffers relative to firm 1. Intuitively, when α is close to 1 the cost disadvantage diminishes, while when α is significantly above 1 firm 2's cost disadvantage is severe. This allows us to represent both firms' marginal costs in terms of c_1, without the need to use c_2.

a. Write down every firm's profit maximization problem and find its best response function.
b. Interpret how each firm's best response function is affected by a marginal increase in alpha. [*Hint*: Recall that an increase in parameter alpha indicates that firms' marginal costs are more similar.]
c. Find the Nash equilibrium of this Cournot game.
d. Identify the regions of (a, α)-pairs for which equilibrium output levels of firm 1 and firm 2 are positive.
e. How are equilibrium output levels affected by a marginal increase in a? And by a marginal increase in α?

Exercise #3 – Sequential move game. Consider the following game tree, where player 2 chooses firstly whether to go Left (L) or Right (R). Then, without observing what player 2 chose, player 1 responds by playing A or B. Finally, player 1 is again called to move after playing A in the case in which player 2 initially chose Left. In this event player 1 can choose between action *x* and *y*.



a) How many strategies does player 1 have in this extensive form game?
b) Represent the game in its normal form payoff matrix.
c) Find the pure strategy Nash equilibria of the normal form game you represented in *b)*.
d) How many proper subgames can you identify? Draw them and explain why these can be considered proper subgames.
e) Find the subgame perfect Nash equilibrium (SPNE) of this extensive form game.

Exercise #4. Stackelberg competition with efficiency changes. Consider a Stackelberg model of output competition where firm 1 is the industry leader and firm 2 is the follower. Firms face a linear inverse demand curve $p(Q) = 10 - Q$, where $Q = q_1 + q_2$ denotes the aggregate output sold by both firms

a) Assuming that every firm i's cost function is $TC_i(q_i) = 4q_i$, find the equilibrium in this Stackelberg game.

b) Assume now that firm 2's cost function becomes $TC_2(q_2) = 2q_2$ after investing in the cost-saving technology. Find the equilibrium in this Stackelberg game. How are your results from part (a) affected?

c) Repeat parts (a) and (b), but assuming that firm 2 is the industry leader while firm 1 is the follower.

Exercise #5. Temporary punishments from deviation. Consider two firms competing in quantities (a la Cournot), facing linear inverse demand $p(Q) = 100 - Q$, where $Q = q_1 + q_2$ denotes aggregate output. For simplicity, assume that firms face a common marginal cost of production $c = 10$.

a) *Unrepeated game.* Find the equilibrium output each firm produces when competing a la Cournot (that is, when they simultaneously and independently choose their output levels) in the unrepeated version of the game (that is, when firms interact only once). In addition, find the profits that each firm earns in equilibrium.

b) *Repeated game* **-** *Collusion.* Assume now that the CEOs from both companies meet to discuss a collusive agreement that would increase their profits. Set up the maximization problem that firms solve when maximizing their *joint* profits (that is, the sum of profits for both firms). Find the output level that each firm should select to maximize joint profits. In addition, find the profits that each firm obtains in this collusive agreement.

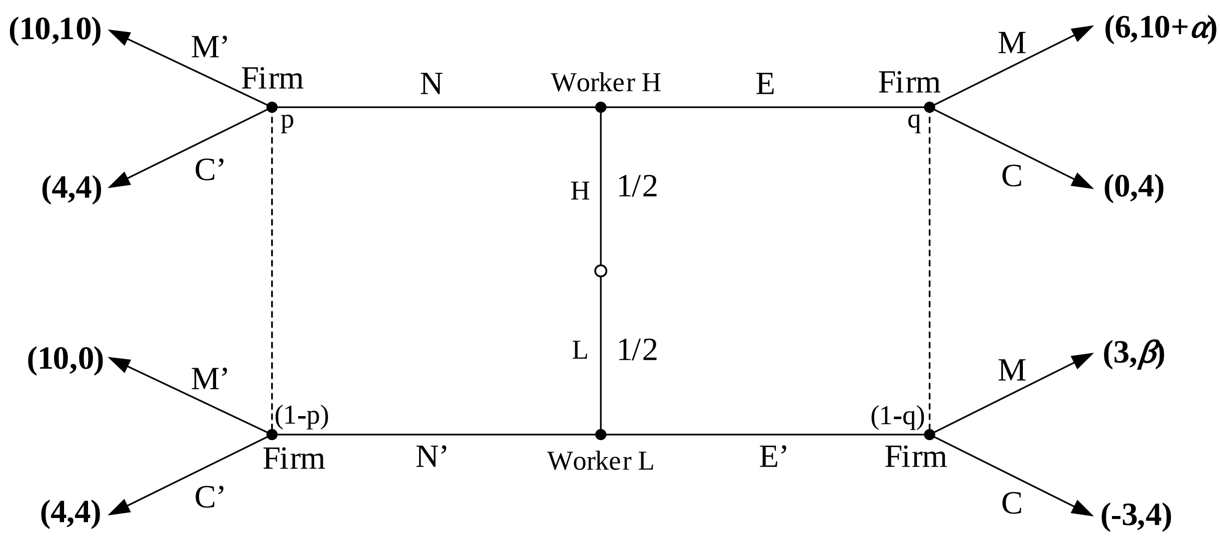
c) *Repeated game – Permanent punishment.* Consider a grim-trigger strategy in which every firm starts colluding in period 1, and it keeps doing so as long as both firms colluded in the past. Otherwise, every firm deviates to the Cournot equilibrium thereafter (that is, every firm produces the Nash equilibrium of the unrepeated game found in part a forever). In words, this says that the punishment of deviating from the collusive agreement is *permanent*, since firms never return to the collusive outcome. For which discount factors this grim-trigger strategy can be sustained as the SPNE of the infinitely-repeated game?

d) *Repeated game – Temporary punishment.* Consider now a “modified” grim-trigger strategy. Like in the grim-trigger strategy of part (c), every firm starts colluding in period 1, and it keeps doing so as long as both firms colluded in the past. However, if a deviation is detected by either firm, every firm deviates to the Cournot equilibrium during only 1 period, and then every firm returns to cooperation (producing the collusive output). Intuitively, this implies that the punishment of deviating from the collusive agreement is now *temporary* (rather than permanent) since it lasts only one period. For which discount factors this “modified” grim-trigger strategy can be sustained as the SPNE of the infinitely-repeated game?

e) Consider again the temporary punishment in part (d) but assume now that it lasts for two periods. How are your results from part (d) affected? Interpret.

f) Consider again the temporary punishment in part (d) but assume now that it lasts for three periods. How are your results from part (d) affected? Interpret.

Exercise #6. Job-market signaling with productivity-enhancing education Consider a variation of the job-market signaling game we discussed in class (see point #18 in the EconS 424 website). Still assume two types of worker (high or low productivity), two messages he can send (education or no education), and two responses by the firm (hiring the worker as a manager or as a cashier). However, let us now allow for education to have a productivity-enhancing effect. In particular, when the worker acquires education (graphically represented in the right-hand side of the game tree below), the firm's payoff from hiring him as a manager is $10 + \alpha$ when the worker is of high productivity, and β when the worker is of low productivity. In words, parameter α (β) represents the productivity-enhancing effect of education for the high (low, respectively) productivity worker respectively, where we assume that $\alpha \geq \beta \geq 0$. For simplicity, we assume that education does not increase the firm's payoff when the worker is hired as a cashier, providing a payoff of 4 regardless of the worker's productivity. All other payoffs for both the firm and the worker are unaffected relative to the figure we saw in class.



Job-market signaling game with productivity-enhancing education

a) Consider the separating strategy profile EN'. Under which conditions of parameters α and β can this strategy profile be supported as a PBE?
b) Consider now the pooling strategy profile NN'. Under which conditions of parameters α and β can this strategy profile be supported as a PBE?

How are your results in parts (a) and (b) affected by an increase in parameter α? And an increase in parameter β? Interpret.

BONUS EXERCISE.[1] **Public good game with incomplete information**. Consider a public good game between two players, A and B. The utility function of every player *i* is

$$u_i(g_i) = (w_i - g_i)^{1/2}\left[m\left(g_i + g_j\right)\right]^{1/2}$$

where the first term, $w_i - g_i$, represents the utility that the player receives from money (i.e., the amount of his wealth not contributed to the public good which he can dedicate to private uses). The second term indicates the utility he obtains from aggregate contributions to the public good, $g_i + g_j$, where $m \geq 0$ denotes the return from aggregate contributions. Since public goods are non-rival in consumption, player *i*'s utility from this good originates from both his own donations, g_i, and in those of player j, g_j.

Consider that the wealth of player A, $w_A > 0$, is common knowledge among the players; but that of player B, w_B, is privately observed by player B but not observed by player A. However, player A knows that player B's wealth level w_B can be high (w_B^H) or low (w_B^L) with equal probabilities, where $w_B^H > w_B^L > 0$. Let us next find the Bayesian Nash Equilibrium (BNE) of this public good game where player A is uninformed about the exact realization of parameter w_B.

a) Starting with the privately informed player B, set up his utility maximization problem, and separately find his best response function, first, when $w_B = w_B^H$ and, second, when $w_B = w_B^L$.

b) Find the best response function of the uninformed player A.

c) Using your results in parts (a) and (b), find the equilibrium contributions to the public good by each player. For simplicity, you can assume that player A's wealth is $w_A = \$14$, and those of player B are $w_B^H = \$20$ when his wealth is high, and $w_B^L = \$10$ when his wealth is low. [*Hint*: You will find one equilibrium contribution for player A, but two equilibrium contributions for player B as his contribution is dependent on his wealth level w_B]

[1] This exercise will not hurt your grade. Solving it correctly can bring your grade in the exam to 120 out of 100 points.