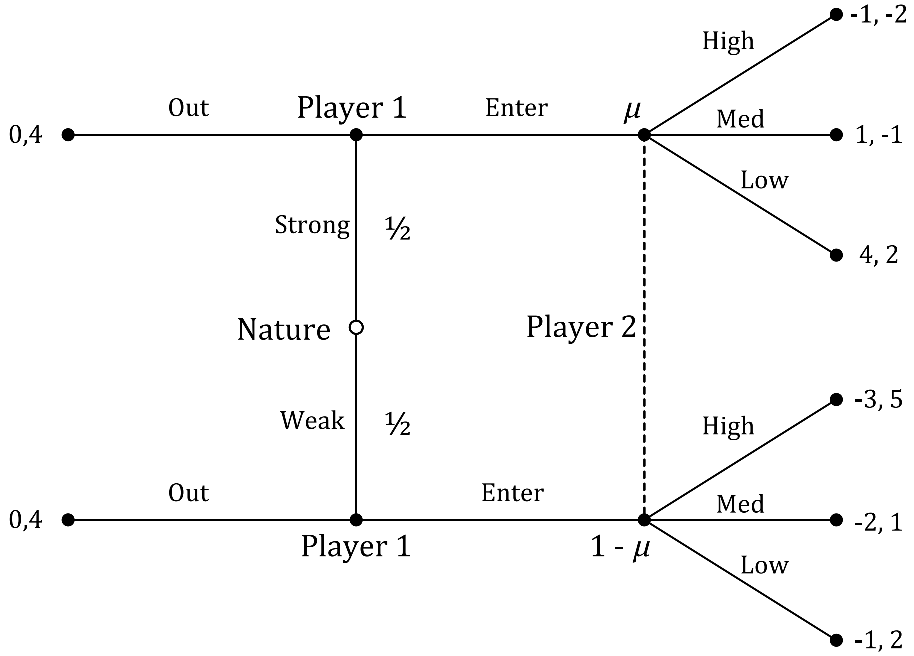
# EconS 503 - Advanced Microeconomics II
# Handout on The Intuitive Criterion

## 1. Exercise on Multiple PBEs

Consider the game depicted below where an informed entrant (Player 1) chooses whether to enter a market, and an uninformed incumbent (Player 2) responds with a High, Medium, or Low price.



a) Find all pure strategy Bayesian Nash Equilibria.

### Answer:

We can draw the normal form of the game as follows

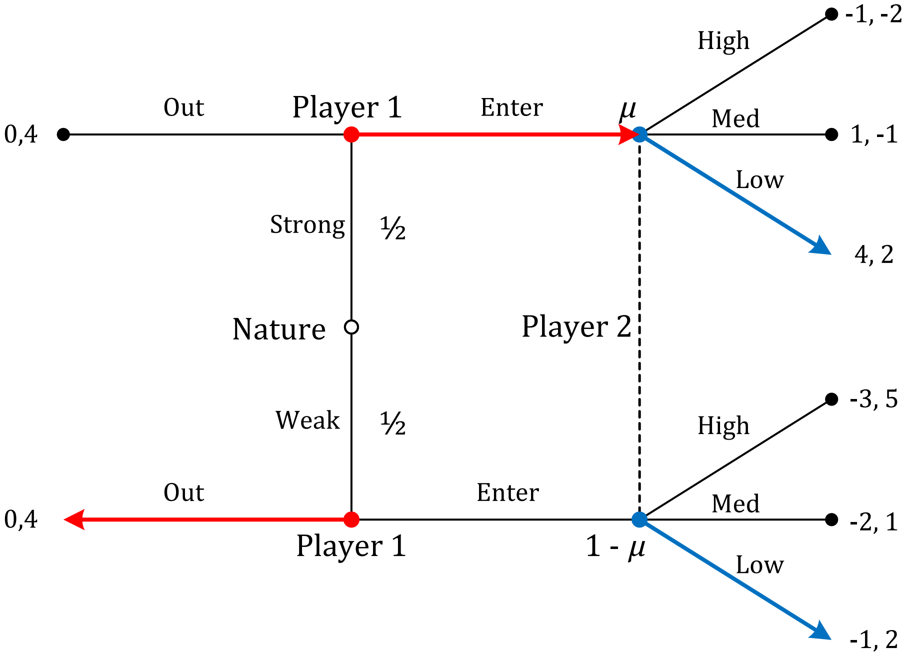
| | | Player 2 | | |
|---|---|---|---|---|
| | | $H_E$ | $M_E$ | $L_E$ |
| Player 1 | $O_W O_S$ | $\underline{0},\underline{4}$ | $0,\underline{4}$ | $0,\underline{4}$ |
| | $O_W E_S$ | $-\frac{1}{2}, 1$ | $\underline{\frac{1}{2}}, \frac{3}{2}$ | $\underline{2},\underline{3}$ |
| | $E_W O_S$ | $-\frac{3}{2},\underline{\frac{9}{2}}$ | $-1, \frac{5}{2}$ | $-\frac{1}{2}, 3$ |
| | $E_W E_S$ | $-2, \frac{3}{2}$ | $-\frac{1}{2}, 0$ | $\frac{3}{2},\underline{2}$ |

which gives rise to two pure strategy Bayesian Nash Equilibira, $(O_W O_S, H_E)$ and $(O_W E_S, L_E)$.

b) Are the strategies found in part (a) perfect Bayesian Equilibria?

## Answer:

Starting with the Separating Strategy profile $(O_W E_S, L_E)$, it is clear that if only the strong type enters the market, $\mu = 1$. In this case, the incumbent will respond with low prices as shown in the figure below, since his payoff of 2 is greater than what he would receive if he chose high prices $(-2)$, or medium prices $(-1)$.



Now, we need to check if player 1 has any incentive to deviate. If he is the strong type, he will receive a payoff of 4 for entering the market, and a payoff of 0 if he deviates to Out. If he is the weak type, he receives a payoff of 0 from staying out of the market and a payoff of $-1$ for deviating to enter. Thus, his payoffs are always larger by utilizing his equilibrium strategies, and the separating strategy profile $(O_W E_S, L_E)$ is a perfect Bayesian Equilibrium.

Next, we consider the Pooling Strategy Profile $(O_W O_S, H_E)$. In this case, Enter is off the equilibrium path, and thus $\mu \in (0, 1)$. We can express Player 2's expected payoffs as a function of $\mu$ below

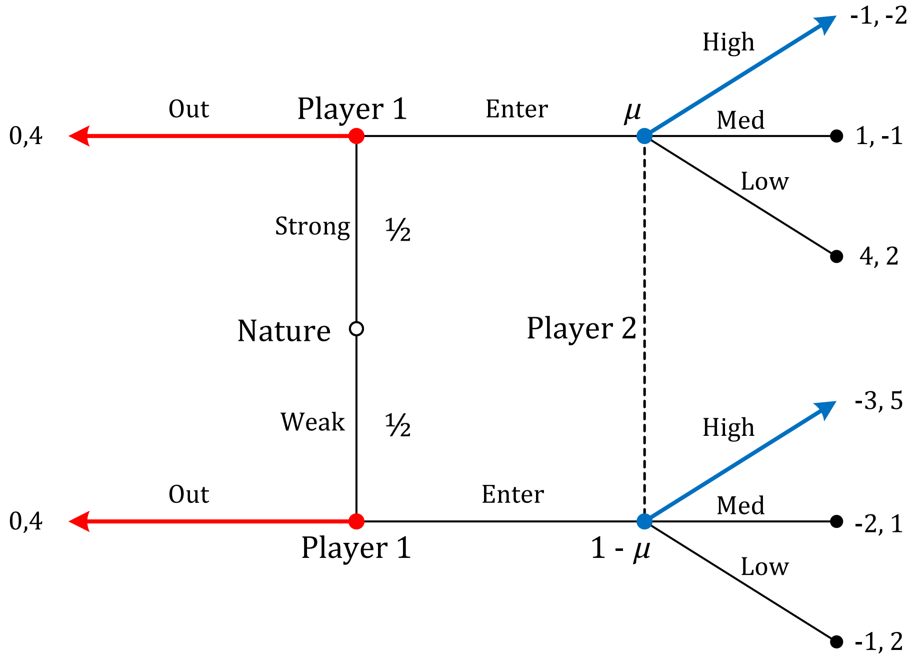
$$\begin{aligned} EU_2(High|Enter) &= -2\mu + 5(1-\mu) = 5 - 7\mu \\ EU_2(Med|Enter) &= -1\mu + 1(1-\mu) = 1 - 2\mu \\ EU_2(Low|Enter) &= 2\mu + 2(1-\mu) = 2 \end{aligned}$$

From these three expected payoff, we can see that choosing a low price will always yield a higher payoff than choosing a medium price, and choosing a high price will yield a higher expected payoff if

$$5 - 7\mu \geq 2 \implies \mu \leq \frac{3}{7}$$

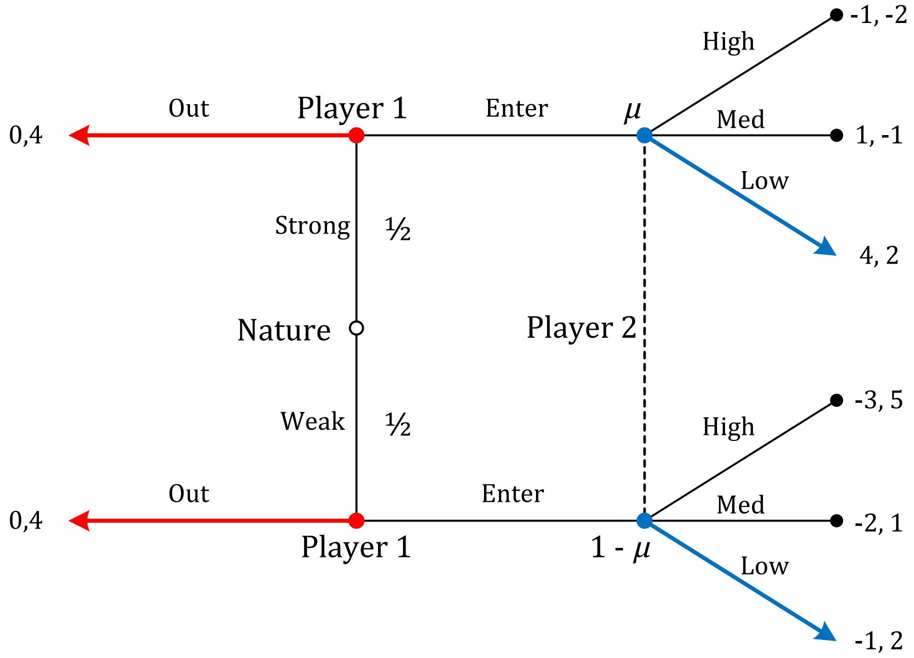
We can now break this analysis into two cases:

**Case 1:** $\mu \leq \frac{3}{7}$. In this case, the incumbent responds to Enter with High prices, as depicted below.



Checking to see if the entrant has any incentives to deviate, if he is the strong type, he receives a payoff of 0 for staying out and a payoff of $-1$ for deviating to Enter. Likewise, if he is the weak type, he receives a payoff of 0 for staying out and a payoff of $-3$ if he deviates to enter. Thus, he does not have any incentive to deviate from his prescribed strategy profile if $\mu \leq \frac{3}{7}$.

**Case 2:** $\mu \geq \frac{3}{7}$. In this case, the incumbent responds to Enter with Low prices, as depiced below.



Checking for deviations for player 1, if he is the strong type, staying out yields him a payoff of 0 while deviating to Enter yields him a payoff of 4. Thus, player 1 does have a profitable deviation if he is the high type, and this cannot be a perfect Bayesian Equilibrium if $\mu \geq \frac{3}{7}$.

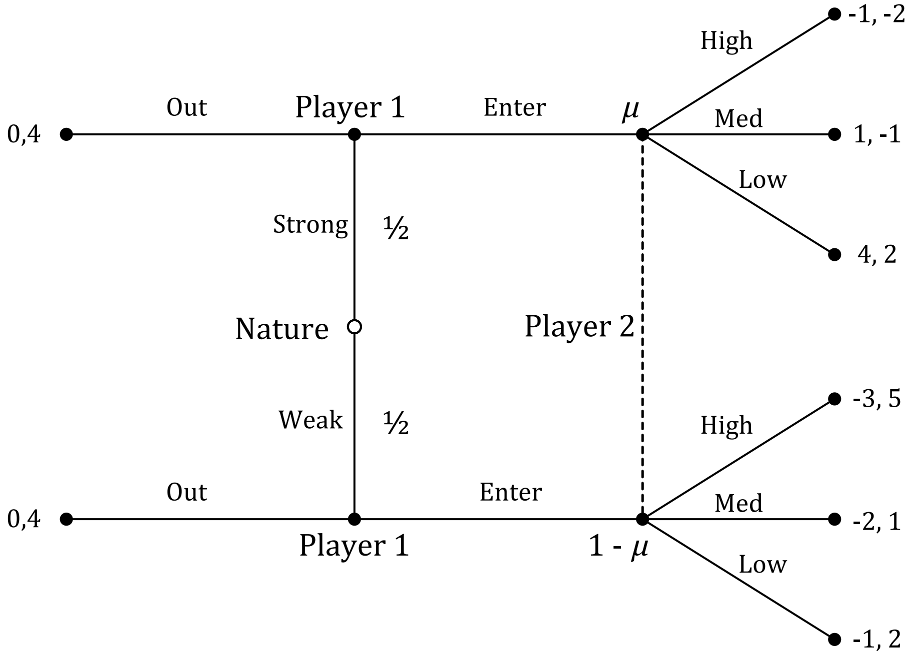
In summary, the Pooling Strategy Profile $(O_W O_S, H_E)$ can only be supported as a perfect Bayesian Equilibrium if $\mu \leq \frac{3}{7}$.

c) Do the strategies found in part (b) survive the Intuitive Critereon?

## Answer:

The Separating Strategy Profile $(O_W E_S, L_E)$ trivially survives the Intuitive Critereon because of no off-the-equilibrium beliefs.

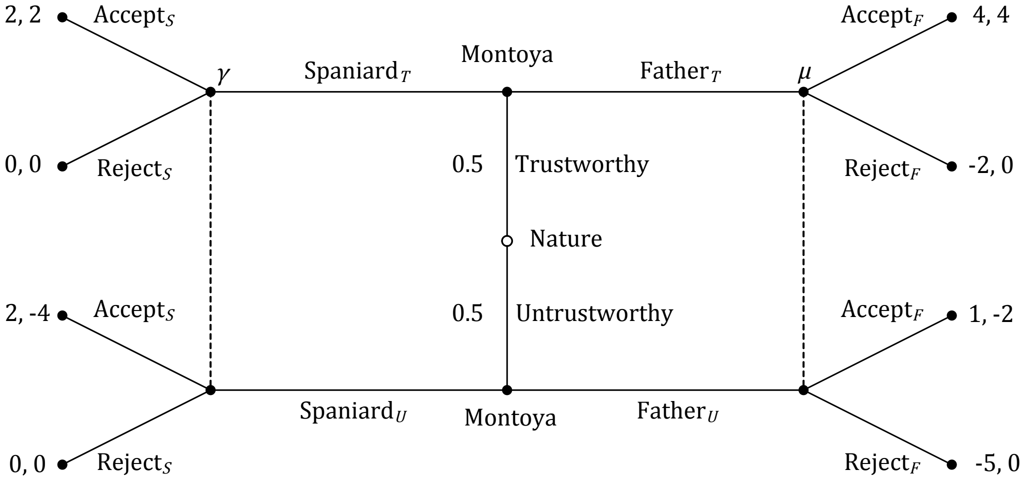
Regarding the Pooling Strategy Profile $(O_W O_S, H_E)$, first we look to see if either type would be irrational to deviate to Entering. As shown below, the weak type would only be able to receive a strictly lower payoff by deviating to Enter



The incumbent will intuitively know this, and thus, if he receives an off-the-equilibrium signal of Enter, he will know that it came from the strong type. Therefore, $\mu = 1$ in this case, which is outside of the bound of $\mu \leq \frac{3}{7}$ and the Pooling Strategy Profile $(O_W O_S, H_E)$ does not survive the Intuitive Critereon.

## 2. Revisiting the Cliff - Based on *The Princess Bride*

Recall the "Signaling with a Spaniard" game from last week



We know that the Pooling Strategy Profile (Father$_T$Father$_U$, Reject$_S$Accept$_F$) can be supported as a perfect Bayesian Equilibrium if $\gamma \leq \frac{2}{3}$ and the Pooling Strategy Profile (Spaniard$_T$Spaniard$_U$, Reject$_S$Reject$_F$) can be supported as a perfect Bayesian Equilibrium if $\mu \leq \frac{1}{3}$. Do these strategies survive the Intuitive Critereon?

## Answer:

Starting with (Father$_T$Father$_U$,Reject$_S$Accept$_F$), first, we need to see if either type of Montoya would be irrational for choosing to signal Spaniard. Indeed, the Trustworthy Montoya would receive a strictly lower payoff from signaling Spaniard (Either 2 or 0) than he would from signaling Father (4). Thus, Dread Pirate Roberts will know that only an Untrustworthy Montoya will signal Spaniard, and he will fix $\gamma = 0$. Fortunately, this is within the bounds of the perfect Bayesian Equilibrium and thus, the Pooling Strategy Profile (Father$_T$Father$_U$,Reject$_S$Accept$_F$) survives the Intuitive Critereon.

With the other Pooling Strategy Profile (Spaniard$_T$Spaniard$_U$,Reject$_S$Reject$_F$), it would be rational for either type of Montoya to signal Father, as each has the potential for a profitable deviation. Thus, we cannot apply the Intuitive Criterion to this perfect Bayesian Equilibrium, and it survives.

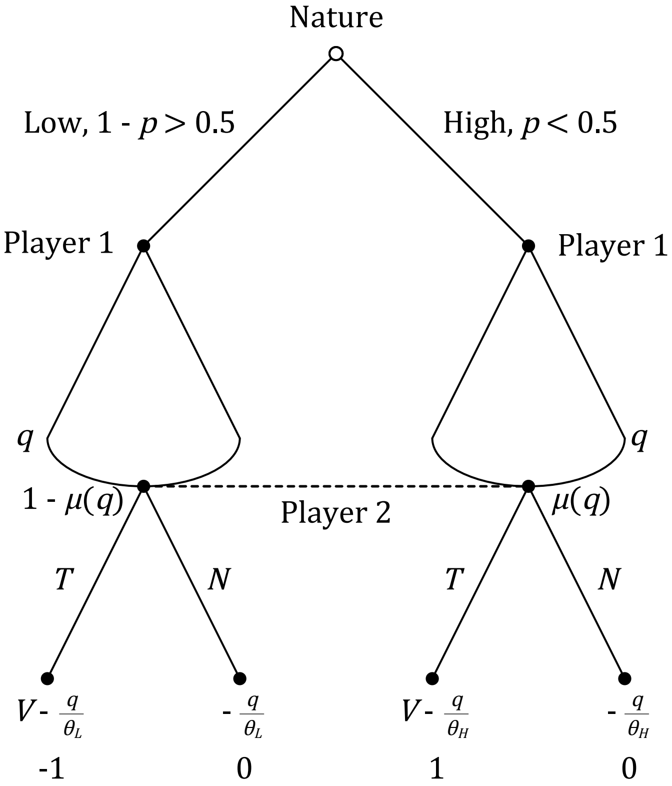
# 3. Publish or Perish - Based on Tadelis 16.6

Inagine that ant newly minted Ph.D. who starts a tenure-track assistant professor job (player 1) is one of two types: high-ability ($\theta_H$) or low-ability ($\theta_L$), where $\theta_H > \theta_L > 0$. The assistant professor knows his type, but the department that hires him (player 2) knows only that he has high ability with probability $p < \frac{1}{2}$. The assistant professor first chooses how hard to work, which is effectively how many papers to publish ($q$) in period 1 (the pre-tenure period). After observing how many papers the assistant professor published, the department decides whether to grant him tenure ($T$) or not to do so ($N$). If the department chooses to grant tenure then the assistant professor's payoff is $v_1(q, T|\theta) = V - \frac{q}{\theta}$, where $V$ is the value of being tenured (common knowledge). The department's payoff is 1 if it tenures a high-ability type and $-1$ if it tenures a low-ability type. If the department denies tenure, it gets a payoff of 0 and the assistant professor's payoff is $v_1(q, N|\theta) = -\frac{q}{\theta}$. Denote by $\mu(q)$ the department's belief that the professor is a high-ability type given that he published $q$ papers.

a) Depict the extensive form of this game.

## Answer:

We can depict the extensive form of this game as follows.



b) If there is a pooling perfect Beyesian equilibrium, will the assistant professor be tenured? Does he write any papers? What then is the unique outcome of the pooling perfect Bayesian Equilibrium?

## Answer:

In a Pooling Strategy Profile, the assistant professor cannot condition the number of papers he writes based on his type, and thus the beliefs of the department will be the same as their priors, i.e., $\mu(q) = p < \frac{1}{2}$. The expected payoff for responding with tenure is

$$EU_2(T|q) = 1p + (-1)(1-p) = 2p - 1$$

and since $p < \frac{1}{2}$, $EU_2(T|q) < 0 = EU_2(N|q)$. Thus, regardless of how many papers the assistant professor writes, he will be denied tenure. His optimal strategy is to then choose $q = 0$ and receive a payoff of 0.

c) Does this Pooling Strategy Profile survive the Intuitive Critereon? (i.e., is there a way for the High type of assistant professor to signal to the department that he is definitely a high type?)

## Answer:

It would be irrational for a Low type assistant professor to set $q$ such that $V - \frac{q}{\theta_L} < 0$, or $q > V\theta_L$. If the high type were to produce that many papers, the department would know with certainty that he is of the high type ($\mu(q) = 1$) and award him tenure. We'll assume that the high type publishes $q = V\theta_L + \varepsilon$ papers, which for simplicity, we'll denote as $q = V\theta_L$. Now that he is receiving tenure, his payoff is

$$V - \frac{q}{\theta_H} = V - \frac{V\theta_L}{\theta_H} = V\left(1 - \frac{\theta_L}{\theta_H}\right)$$

and since $\theta_H > \theta_L > 0$, we know that $V\left(1 - \frac{\theta_L}{\theta_H}\right) > 0$, the payoff the assistant professor would receive from publishing zero papers and being denied tenure. Thus, if the department were to observe some $q > 0$, they can assume that it comes from the high type, and set $\mu(q) = 1 > p$, and therefore this Pooling Strategy Profile does not survive the Intuitive Critereon.