EconS 503 - Microeconomic Theory II
Homework #7 - Due date: Monday, March 26th, in class.

1. **Signaling when the expert receives imprecise signals.** Consider the following signaling model between an expert (E), such a special interest group, and a decision maker (DM), such as a politician. For simplicity, assume that the state of the world is discrete, either $\theta = 1$ or $\theta = 0$ with prior probability $p \in (0, 1)$ and $1 - p$, respectively. The expert privately observes an informative but noisy signal s, which also takes two discrete values $s \in \{0, 1\}$. The precision of the signal is given by the conditional probability
$$\text{prob}(s = k|\theta = k) = q,$$
where $k = \{0, 1\}$, and $q > \frac{1}{2}$. In words, the probability that the signal s coincides with the true state of the world θ is q (precise signal), while the probability of an imprecise signal where $s \neq \theta$ is $1 - q$. The time structure of the game is as follows:

   1) Nature chooses θ according to the prior p.
   2) Expert observes signal s and reports a message $m \in \{0, 1\}$
   3) Decision maker observes m and responds with $x \in \{0, 1\}$
   4) θ is observed and payoffs are realized

   The payoff function for the decision maker is
$$u(x, \theta) = \left(\theta - \frac{1}{2}\right) x$$
while that of the expert is
$$v(m, \theta) = \begin{cases} 1, & \theta = m \\ 0, & \theta \neq m \end{cases}$$
which, in words, indicates that the expert's payoff is 1 when the message she sends coincides with the true realization of the state of the world, but becomes zero otherwise. Importantly, her payoff is unaffected by the signal, which she only uses to infer the actual realization of parameter θ. Intuitively, $v(m, \theta)$ is often understood as a "reputation function" since it provides the expert with a payoff of 1 only when his message was an accurate representation of the true state of the world (which in this model he does not precisely observe).

   (a) Is there a Perfect Bayesian equilibrium (PBE) in which the expert reports his signal truthfully?

2. **Policy announcements as signals.** Consider Downs' (1957) model of voting with a continuum of voters with policy ideals in the interval $[0, 1]$, distributed according to cummulative distribution function $F(x)$ with positive and continuous density in $[0, 1]$. The median voter $x = m$ satisfies $F(m) = \frac{1}{2}$, and is either low (L) or high (H), where $L < H$, with equal probabilities. The time structure of the game is the following:

1) Political candidate 1 privately observes the position of the median voter (that is, $m = L$ or $m = H$), and announces a policy position p_1.

2) Candidate 2 observes p_1, and updates its beliefs about the position of the median voter. Candidate 2 then responds announcing his own policy p_2.

3) After observing policies p_1 and p_2, voters vote for the candidate who is closest to their ideal policy. In case of a tie, you can assume that candidates evenly share votes.

Candidates only care about winning the election and assign a payoff of 1 to winning, $\frac{1}{2}$ to a tie, and 0 to losing.

(a) Find at least one separating Perfect Bayesian Equilibria (PBEs).

(b) Find at least one pooling Perfect Bayesian Equilibrium (PBE).