

EconS 503 - Microeconomic Theory II
Homework #6 - Due date: Monday, March 19th, in class.

1. **Exercises from textbooks:**

- (a) Watson, Chapter 29: Exercises 7, 8 and 10 (see exercises at the end of this assignment).
- (b) MWG, Chapter 9: Exercise 9.C.7.
- (c) Bolton and Dewatripont, Exercise 6 (see page 650 since all exercises are at the end of the book).

2. **[Entry deterrence with a sequence of potential entrants]** The following entry model is inspired on the original paper of Kreps and Wilson (*JET*, 1982). Consider an incumbent monopolist building a reputation as a tough competitor who does not allow entry without a fight. The entrant first decides whether to enter the market, and, if he does, the monopolist chooses whether to fight or acquiesce. If the entrant stays out, the monopolist obtains a profit of $a > 1$, and the entrant gets 0. If the entrant enters, the monopolist gets 0 from fighting and -1 from acquiescing if he is a "tough" monopolist, and -1 from fighting and 0 from acquiescing if he is a "normal" monopolist. The entrant obtains a profit of b if the monopolist acquiesces and $b - 1$ if he fights, where $0 < b < 1$. Suppose the entrant believes the monopolist to be tough (normal) with probability p ($1 - p$, respectively), while the monopolist observes his own type.

- (a) Depict a game tree representing this incomplete information game.
- (b) Solve for the PBE of this game.
- (c) Suppose the monopolist faces two entrants in sequence, and the second entrant observes the outcome of the first game (there is no discounting). Depict the game tree, and solve for the PBE. [*Hint:* you can use backward induction to reduce the game tree as much as possible before checking for the existence of separating or pooling PBEs. For simplicity, focus on the case in which prior beliefs satisfy $p \leq b$.]

3. **[PBEs in bargaining]** A buyer and a seller are bargaining. The seller owns an object for which the buyer has a value v ; the seller's value is zero. The buyer knows v but the seller does not. The seller's beliefs about v , which are common knowledge, are that $v = 30$ with probability $\frac{1}{2}$ and $v = 10$ with probability $\frac{1}{2}$. There are two periods of bargaining; there is no discounting (i.e., $\delta = 1$).

- In the first period, the seller makes an offer p_1 that represents a price that the buyer will need to pay to buy the object. The buyer can accept or reject the offer. If the buyer accepts, the offer is implemented and the game ends. If the buyer rejects, the game continues to the second period.

- In the second period, the seller (again) makes an offer p_2 , which is the price the buyer will need to pay to buy the object. The buyer can accept or reject the offer. If the buyer accepts, the offer is implemented and the game ends. If the buyer rejects, then the seller keeps the object and the game ends.

If the buyer buys the object in the first period, then the payoffs are p_1 for the seller and $v - p_1$ for the buyer. Similarly, if the buyer buys the object in the second period, then the payoffs are p_2 for the seller and $v - p_2$ for the buyer. If the buyer does not buy the object, then the payoffs are zero for each player.

- Provide an extensive-form representation of this game.
 - Find a Perfect Bayesian equilibrium in which the seller believes that any buyer that rejects a first-period offer is the type with valuation $v = 10$ with probability 1. (Justify your answer, and remember to fully specify the Perfect Bayesian equilibrium.)
4. **Cheap talk with three types.** Consider the cheap talk model with three types discussed in class (Investing recommendations game). Let us focus on the partially separating strategy profile where the Analyst (sender) recommends Buy both when the stock outperforms the market and when its neutral, but recommends Hold when the stock underperforms the market. In class, we made a simplifying assumption on off-the-equilibrium beliefs (after the Investor receives a Sell recommendation), denoted by γ_1 , γ_2 , and $1 - \gamma_1 - \gamma_2$.
- Without restricting off-the-equilibrium beliefs, find under which conditions the above partially separating strategy profile can be sustained as a PBE of this game.
 - Consider now the pooling strategy profile where the Analyst recommends Buy regardless of the stock's type. Under which conditions can this strategy profile be supported as a PBE?
5. **Cheap talk with a continuum of types.** Consider the Crawford-Sobel cheap talk game with a continuum of types. In class we discussed the maximal number of partitions n that can be sustained as a PBE of the game.
- Find the ex-ante expected utility that the sender obtains in equilibrium. (By "ex-ante" we mean before observing the realization of parameter θ .)
 - Find the ex-ante expected utility that the receiver obtains in equilibrium.
 - How are the above ex-ante expected utilities affected by an increase in the preference divergence parameter δ ?