EconS 503 - Microeconomic Theory II
Homework #5 - Due date: March 2nd, 2018

1. [**Cournot competition when all firms are uninformed**]. Consider Example 8.10 in the *Advanced Microeconomic Theory* textbook (MIT Press) about a Cournot duopoly operating under asymmetric information. Assume now that every firm i is privately informed about its marginal cost (that is, $MC_i = 1/4$ or $MC_i = 0$, both equally likely), but does not observes its rival's marginal costs.

   (a) Find the best response function for every firm i, $q_i^k(q_j^H, q_j^L)$, where $k = \{H, L\}$ denotes firm i's marginal cost (high or low).

   (b) Use your results from part (a) to find the Bayesian Nash Equilibrium (BNE) of the game.

2. [**All pay auction**] Consider an all-pay auction in which the winning bidder is that submitting the highest bid, but all bidders must pay their bid. Assume that valuations are uniformly distributed on $[0, 1]$.

   (a) In a setting with $N = 2$ bidders, show that the symmetric equilibrium bidding function $b_i(v_i) = \frac{1}{2}v_i^2$ can be sustained as a BNE.

   (b) In a setting with $N \geq 2$ bidders, show that the symmetric equilibrium bidding function $b_i(v_i)$ that can be sustained as a BNE is of the form $b_i(v_i) = kv_i^N$, where $k > 0$.

   (c) In a setting with $N \geq 2$ bidders, let us now allow for every bidder i's valuation v_i to be drawn from a general cdf $F(v_i)$ (not necessarily uniform), satisfying i.i.d. Find the symmetric equilibrium bidding function $b_i(v_i)$ that can be sustained as a BNE.

3. [**Reservation price**] Consider a FPA where the seller sets a reservation price, i.e., no bids are allowed below the reservation price r. Assume that every bidder i's valuation v_i is drawn from a general cdf $F(v_i)$ (not necessarily uniform), satisfying i.i.d.

   (a) Explain why all those bidders with valuations above r are strictly better off bidding than not bidding. That is, the equilibrium bidding strategy is a mapping $b_i(v_i)$ with domain $[r, 1]$.

   (b) Show that the equilibrium bidding function is

   $$b_i(v_i) = \frac{F(r)^{N-1}r + \int_r^{v_i}(N-1)xF(x)^{N-2}dx}{F(v_i)^{N-1}}$$

   (c) Integrate by parts to show that the equilibrium bidding function can expressed as follows

   $$b_i(v_i) = v_i - \int_r^{v_i}\left(\frac{F(x)}{F(v_i)}\right)^{N-1}dx$$

(d) If a cdf F_A stochastically dominates another cdf F_B, does it follow that equilibrium bids will be higher with F_A than with F_B?

(e) Show that the equilibrium bid increases in the number of bidders, N.

4. [**An application of the Revenue Equivalence Theorem**] In 1991 US Vice-President Quayle proposed that the loser in lawsuit be required to transfer an amount equal to her legal expense to the winner. Quayle claimed this would reduce the amount spent on legal services. (Under current US rules each party pays its own legal costs.) Let us model this argument by assuming that each party $i = 1, 2$ has a privately observed value v_i for winning a lawsuit, which is independently drawn from cdf $F(v)$ (where we assume that $F(\underline{v}) = 0$ at the lowest value $\underline{v}$). The two parties independently and simultaneously decide how much to spend on legal services, and the party that spends the most money wins.

(a) Obtain an expression for the amount each player spends under current US rules, by using revenue equivalence with an ascending auction for the prize of winning the lawsuit. What additional assumption(s), if any, did you have to make to use the revenue equivalence theorem?

(b) Without doing any more calculations, use our model to evaluate Quayle's claim. What additional assumption(s), if any, have you made?

(c) In European legal systems the loser usually pays a fraction of the winner's actual expense. Without doing any more calculations, do you think this rule will increase or reduce expected legal expenses?

(d) Use the revenue equivalence theorem to obtain a differential equation for the amount $l(v)$ each party spends under Quayle's rules. Show that

$$l(v) = \frac{v^2}{3} \frac{3 - v}{(2 - v)^2}$$

satisfies your equation when valuations are drawn from a uniform distribution, i.e., $F(v) = v$.

(e) Very briefly, describe in words how satisfactory is the model.