

EconS 424 - Strategy and Game Theory
Homework #5 - Due date: Monday, March 19th, in class.

1. **Harrington, Chapter 13:**

- Exercise 8 (first edition). It corresponds to Exercise 7 in the second edition of the book.

2. [**Collusion when 2 firms compete in quantities**] Consider two firms competing as Cournot oligopolists in a market with demand

$$p(q_1, q_2) = a - b(q_1 + q_2)$$

where $a, b > 0$. Both firms have total costs, $TC(q_i) = cq_i$ where $c > 0$ is the marginal cost of production, and $a > c$.

- (a) Considering that firms only interact once (playing an unrepeated Cournot game), find the equilibrium output for every firm, the market price, and the equilibrium profits for every firm.
 - (b) Now assume that they could form a cartel. Which is the output that every firm should produce in order to maximize the profits of the cartel? Find the market price, and profits of every firm. Are their profits higher when they form a cartel than when they compete as Cournot oligopolists?
 - (c) Study under which conditions can the cartel agreement be supported in the infinitely repeated game. For simplicity, let us use the following grim-trigger strategy: first, firms start cooperating (choosing the cartel output), and they continue to do so as long as all firms choose this level of output. If some firm deviates, however, all firms revert to the Cournot output level you found in part (a). Assume that both firms assign the same weight to future payoffs (i.e., they both have the same discount factor δ).
3. [**Collusion when n firms compete in quantities**] Consider n firms producing homogenous goods and choosing quantities in each period for an infinite number of periods. Demand in the industry is given by $p(Q) = 1 - Q$, Q being the sum of individual outputs. All firms in the industry are identical: they have the same constant marginal costs $c < 1$, and the same discount factor $\delta \in (0, 1)$. Consider the following trigger strategy:

- Each firm sets the output q^m that maximizes joint profits at the beginning of the game, and continues to do so unless one or more firms deviate.
- After a deviation, each firm sets the quantity q^c , which is the Nash equilibrium of the one-shot Cournot game.

- (a) Find the condition on the discount factor that allows for collusion to be sustained in this industry.
- (b) Indicate how the number of firms in the industry, n , affects the possibility of reaching the tacit collusive outcome.
- (c) Do you know any other punishment strategy under which firms can sustain the collusive outcome under weaker conditions?