EconS 503 - Advanced Microeconomics II
Handout on Cheap Talk

1. Cheap talk with Stockbrokers

(From Tadelis, Ch. 18, Exercise 18.2) A stockbroker can give his client one of three recommendations regarding a certain stock: buy (B), hold (H), or sell (S). The stock can be one of three kinds: a winner (W), mediocre (M), or a loser (L). The stockbroker knows the type of stock but the client knows only that each type is equally likely. The game proceeds as follows: first. the stockbroker makes a recommendation $a_1 \in \{B, H, S\}$ to the client, after which the client chooses an action $a_2 \in \{B, H, S\}$ and payoffs are determined. The payoffs to the stockbroker (player 1) and client (player 2) depend on the type of stock and the action taken by the client (the pairs are (v_1, v_2) where v_i is player i's payoff) as follows (Note that this is not actually a normal form representation of the game, just a payoff table):

Type of stock θ	Player 2's action a_2: B	H	S
W	$(2, 2)$	$(-1, -1)$	$(-2, -2)$
M	$(0, 1)$	$(1, 0)$	$(0, -1)$
L	$(-2, 0)$	$(-1, 1)$	$(2, 0)$

a) Find a babbling perfect Bayesian equilibrium of this game.

Answer:

First, note that the extensive form of the game can be depicted as follows:

See Figure 1 at the end of the handout

In a babbling equilibrium, the message that the stockbroker sends reveals no information to the client, and is thus useless. We can show that this will be the case due to the stockbroker having incentive to lie. When the stock is a winner ($\theta = W$), the preferred outcomes are B for the stockbroker and B for the client; however, when the stock is mediocre ($\theta = M$), the preferred outcome are H for the stockbroker and B for the client. Likewise, when the stock is a loser ($\theta = L$), the preferred outcomes are S for the stockbroker and H for the client. Thus, since the interests of the stockbroker and client are not in alignment, the stockbroker has incentive to lie, and the message will not be useful to the client.
Thus, the client is best off by choosing the action that yields the highest expected utility, i.e.,

$$
\begin{aligned}
EU_2(B|\theta) &= \frac{1}{3}(2) + \frac{1}{3}(1) + \frac{1}{3}(0) = 1 \\
EU_2(H|\theta) &= \frac{1}{3}(-1) + \frac{1}{3}(0) + \frac{1}{3}(1) = 0 \\
EU_2(S|\theta) &= \frac{1}{3}(-2) + \frac{1}{3}(-1) + \frac{1}{3}(0) = -1
\end{aligned}
$$

which yields B as the babbling perfect Bayesian equilibrium of this game.

b) Is there a fully truthful perfect Bayesian equilibrium in which the stockbroker makes the recommendation that, if followed, maximizes the client's payoff?

Answer:

No. As we showed in the previous part, the stockbroker only has incentive to be truthful when the stock is a winner.

c) What is the most informative perfect Bayesian equilibrium of this game?

Answer:

Since the stockbroker has incentive to be truthful when the stock is a winner, and he it is not in his interest to send the signal B otherwise, we can specify the following strategy: The stockbroker, upon observing that the stock is a winner sends the message $a_1 = B$. If the stock is not a winner, the stockbroker mixes with 0.5 probability between signals $a_1 = H$ and $a_1 = S$ as shown in the figure below.

See Figure 2 at the end of the handout

In this case, the clent's best response is to choose B upon receiving a signal of B. When receiving a signal of H or S, the client knows that the stock is not a winner, and maximizes it's expected value

$$\begin{aligned} EU_2(B|\theta) &= \frac{1}{2}(1) + \frac{1}{2}(0) = \frac{1}{2} \\ EU_2(H|\theta) &= \frac{1}{2}(0) + \frac{1}{2}(1) = \frac{1}{2} \\ EU_2(S|\theta) &= \frac{1}{2}(-1) + \frac{1}{2}(0) = -\frac{1}{2} \end{aligned}$$

and thus, the client is indifferent between B and H.

2. Labor Market Signaling

(From Macho-Stadler and Perez-Castrillo, Ch. 5, Exercise 1). Assume that in the market there exists two types of worker, differentiated by their productivity. Type-k^G workers have productivity $k = 2$, while the productivity of the type-k^B workers is $k = 1$. The cost of achieving a given level of education is greater for type-k^B workers that for type k^G workers. In particular, the cost of e units of education for a type-k individual is $c(e;k) = \frac{e}{k}$. The utility function of a type-k individual is $U(w, e; k) = w - c(e; k)$.

a) Does a worker's education level influence his productivity? What would be the optimal education level if firms had the same information as workers as to the value of k?

Answer:

No. Education is merely a signal to the firm of the worker's productivity, not the productivity itself. If firms could perfectly observe productivity, there would be no reason to obtain education (unless education increased productivity) and thus, $e = 0$.

For parts (b) through (c), now assume that a worker's productivity is not observable by firms, but that their education level is. Furthermore, assume that firms believe that education greater than or equal to a certain level e^O is a signal of high productivity, while education less than this level signals low productivity. Hence firms offer wages according to $w(e) = 2$ if $e \geq e^O$ and $w(e) = 1$ if $e < e^O$.

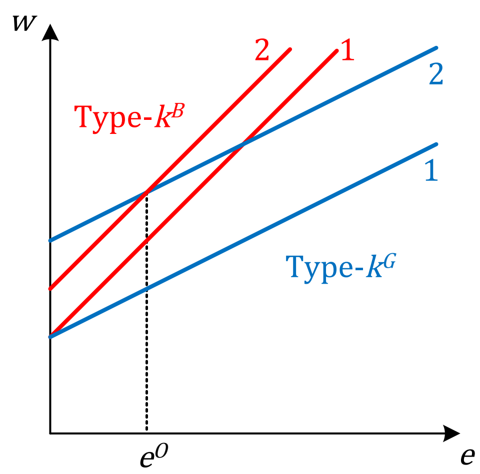
b) Given these wages, calculate the level of education that each type of agent will choose.

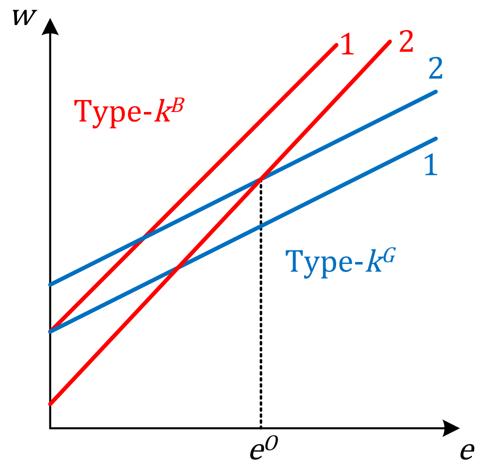
Answer:

Each type of worker will only obtain education if it is profitable, or if the gains to education are greater than what could be received by obtaining no education. This can be expressed as

$$\begin{aligned} U(2, e^O; k) &\geq U(1, 0; k) \\ 2 - \frac{e^O}{k} &\geq 1 - \frac{0}{k} \end{aligned}$$

Solving this expression yields $k \geq e^O$. Thus, if $e^O \leq 2$, the type k^G agents will have incentive to obtain education, and if $e^O \leq 1$, both types of agent have incentive to obtain education. The figures below illustrate this effect. In the first figure, e^O is low enough such that it induces both types of agent to obtain the education to receive the higher paying job (denoted as 2 on the indifference curves), whereas the second figure shows a situation where e^O is such that only the type k^G agent will obtain education. If e^O were to shift further to the right, the type-k^G indifference curve for the higher wage would eventually cross below the lower wage's indifference curve.





c) Find the necessary condition on e^O so that education is an effective signal of productivity.

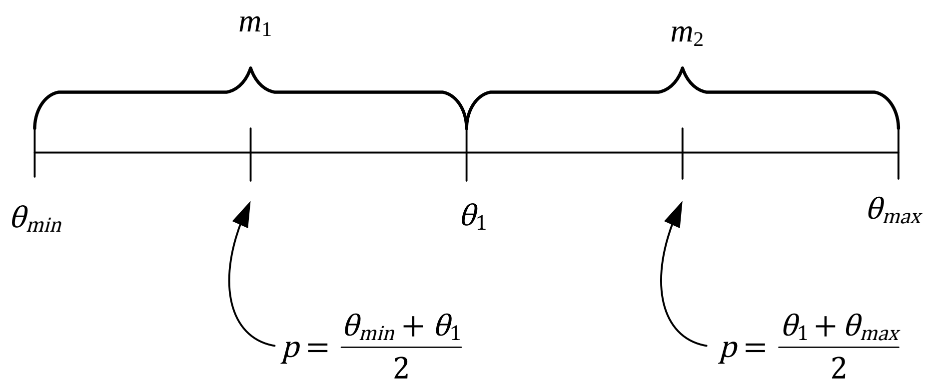
Answer:

As shown in part (b), if the cutoff value of e^O is either too low or too high, both types of agent will choose the same level of education, making the signal ineffective. Thus, the necessary condition is that $1 \leq e^O \leq 2$.
Remark: Don't worry about the weak inequalities used in this problem. We can assume that when faced with indifference, the agents choose our desired outcomes.

3. Cheap Talk with a continuum of types

Let us extend the environmental regulation exercise you did in class about Cheap Talk messages, to a setting in which the sender (lobbyist) can have a continuum of types (rather than only two), uniformly distributed in the interval $[\theta_{min}, \theta_{max}]$. Answer the following questions, assuming the same utility functions $-(p-\theta)^2$ for the politician and $-(p-\theta-b)^2$ for the lobbyist.

a) Consider the partially informative strategy profile in Figure 1. In particular, all lobbyists with a type $\theta < \theta_1$ send the same message m_1, while those above this cutoff send a different message, m_2. What are the receiver's beliefs in this partially informative equilibrium?



Answer:

If message m_1 is received, the receiver's beliefs are totally concentrated on the fact that the true state of the world must be within some of the values of interval 1, where $\theta \in [\theta_{min}, \theta_1)$, and never from interval 2, where $\theta \in [\theta_1, \theta_{max}]$:

$$\mu(\theta \in [\theta_{min}, \theta_1)|m_1) = 1 \quad \text{and} \quad \mu(\theta \in [\theta_1, \theta_{max}]|m_1) = 0$$

Similarly if message m_2 is received, the receiver is convinced that the true state of the world must be within some of the values of interval 2, and never from interval 1:

$$\mu(\theta \in [\theta_1, \theta_{max}]|m_2) = 1 \quad \text{and} \quad \mu(\theta \in [\theta_{min}, \theta_1)|m_2) = 0$$

b) What is the receiver's optimal action (or policy), given the beliefs you specified in the previous question?

Answer:

If a message on the first interval is received, then beliefs are updated (concentrated on the fact that the true state must be in this first interval). Hence, the optimal action of the receiver is $p = \frac{\theta_{min}+\theta_1}{2}$. This action minimizes the distance between p and the expected value of θ in the first interval, which is $\frac{\theta_{min}+\theta_1}{2}$.

If a message on the second interval is received, then beliefs are updated (concentrated on the fact that the true state must be in this second interval), and hence, the optimal action of the receiver is $p = \frac{\theta_1+\theta_{max}}{2}$. This action minimizes the distance between p and the expected value of θ in the second interval, which is $\frac{\theta_1+\theta_{max}}{2}$.

c) What is the value of θ_1 that makes a sender with type θ_1 to be indifferent between sending message m_1 (inducing an action of $p = \frac{\theta_{min}+\theta_1}{2}$) and sending a message of m_2 (inducing an action of $p = \frac{\theta_1+\theta_{max}}{2}$)? [Hint: your expression of θ_1 should depend on θ_{max}, θ_{min}, and the parameter reflecting the divergence between the receiver and the sender's preferences, b.]

Answer:

If θ_1 is the true state, then the sender sends any message in the first interval. We must check that the sender does not want to deviate to send m_2. The sender observing a true state of the world in the first interval who has the highest incentives to deviate towards a message on the second interval is precisely θ_1. In fact, he is indifferent between sending a message on the first and on the second interval:

$$-\left(\frac{\theta_{min}+\theta_1}{2} - (\theta_1 + b)\right)^2 = -\left(\frac{\theta_1+\theta_{max}}{2} - (\theta_1 + b)\right)^2$$

Rearranging the expression yields,

$$(\theta_1 + b) - \frac{\theta_{min}+\theta_1}{2} = \frac{\theta_1+\theta_{max}}{2} - (\theta_1 + b)$$

Further simplifying,

$$4(\theta_1 + b) = \theta_{max} + \theta_{min} + 2\theta_1$$

Solving for θ_1, we obtain,

$$\theta_1 = \frac{1}{2}(\theta_{max} + \theta_{min}) - 2b$$

d) *Numerical example.* Let us now consider a numerical example. Assume that $\theta_{min} = 0$ and $\theta_{max} = 1$, and rewrite the expression of θ_1 you found in part (c). We know that θ_1 must be above 0 and below 1, since otherwise it would be beyond the lower bound θ_{min} or upper bound θ_{max}, i.e., $0 < \theta_1 < 1$. Plug in this inequality the value for θ_1 that you found in the previous questions, and solve for parameter b. What is the range of values for parameter b that support this partially informative PBE with two steps (two partitions)?

Answer:

When $\theta_{min} = 0$ and $\theta_{max} = 1$, the above expression becomes,

$$\theta_1 = \frac{1}{2}(\theta_{max} + \theta_{min}) - 2b = \frac{1}{2} - 2b$$

In addition, if $\theta_1 = \frac{1}{2} - 2b$ must be larger than 0 or smaller than 1, we obtain that

$$-\frac{1}{2} < -2b < \frac{1}{2}, \text{ and hence } -\frac{1}{4} < b < \frac{1}{4}$$

Intuitively, if the preferences between the receiver (politician) and the sender (lobbyist) are not very divergent ($|b| < \frac{1}{4}$), we can obtain a 2-step equilibrium which is partially informative about the true state of the world. That is, some (although not full) information is transmitted from the sender to the receiver. Otherwise (if $|b| > 1/4$) we cannot support such a partially informative equilibrium in two steps, and we would only have a pooling equilibrium in which all type of lobbyists in $[\theta_{min}, \theta_{max}]$ send the same message to the politician.

Figure 1 – Cheap Talk with Stockbrokers

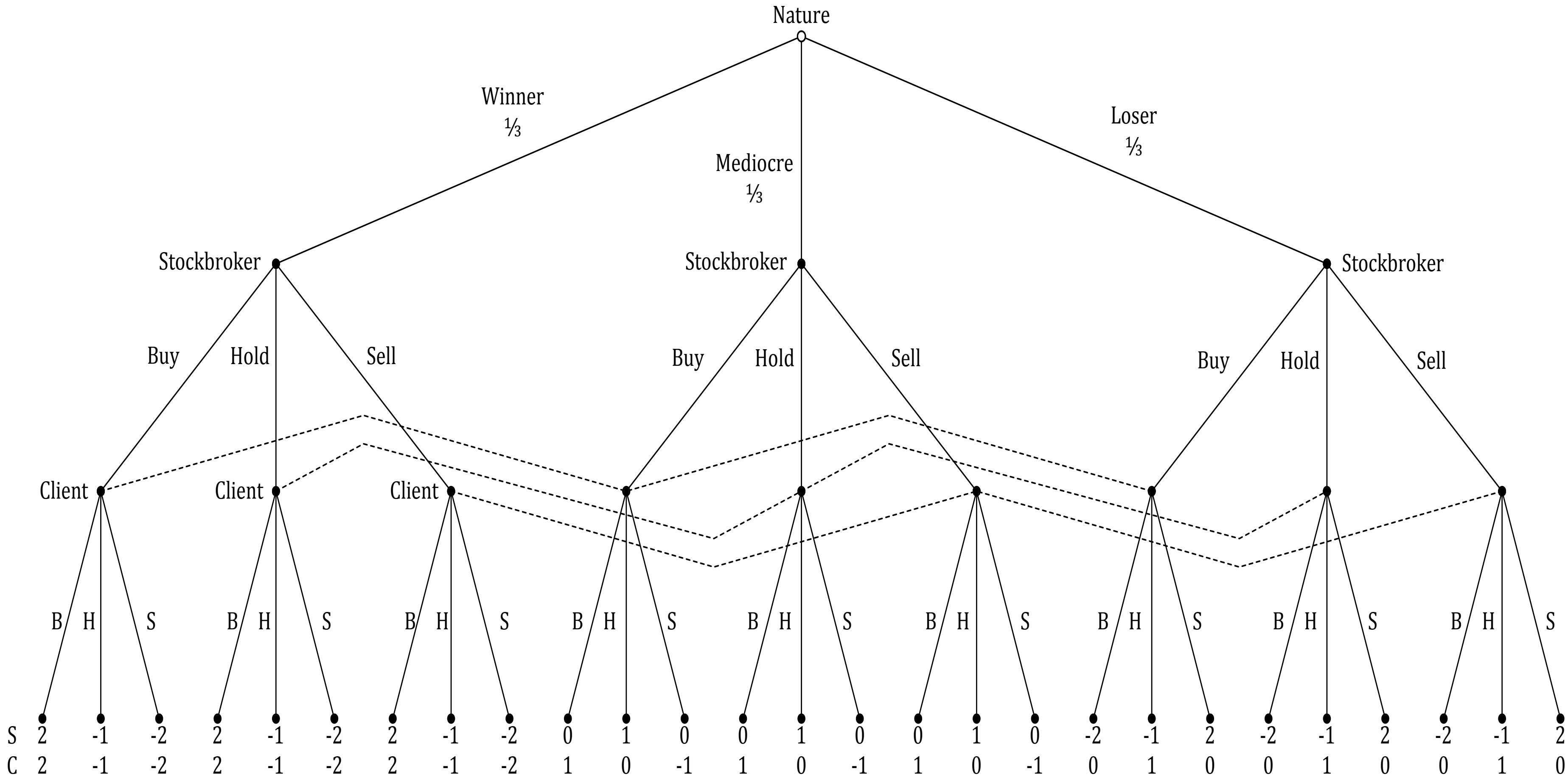


Figure 2 – Cheap Talk with Stockbrokers (Informative PBE)