EconS 503 - Advanced Microeconomics II
Handout on Adverse Selection

1. MWG 13.B.3

Consider a positive selection version of the model discussed in Section 13.B in which $r(\cdot)$ is a continuous, strictly decreasing function of θ. Let the density of workers of type θ be $f(\theta)$, with $f(\theta) > 0$ for all $\theta \in [\underline{\theta}, \bar{\theta}]$.
(a) Show that the more capable workers are the ones choosing to work at any given wage.

Answer:

Suppose firms offer a wage of w. All workers of type θ, with $r(\theta) \leq w$, will accept the wage and work. Suppose there exists a θ^* with $r(\theta^*) = w$. Then all workers of type $\theta \geq \theta^*$ will work, since $r(\theta) \leq r(\theta^*) = w$ and $r(\cdot)$ is decreasing. Thus, the more capable workers are the ones who will work at any given wage.

(b) Show that if $r(\theta) > \theta$ for all θ, then the resulting competitive equilibrium is Pareto efficient.

Answer:

Firms can offer the wage $w = \bar{\theta}$, and since $r(\bar{\theta}) > \bar{\theta}$ no workers of type $\bar{\theta}$ will work. From part (a), no worker of any type will work. Therefore, the competitive equilibrium is Pareto efficient, i.e. nobody will work.

(c) Suppose that there exists a $\hat{\theta}$ such that $r(\theta) < \theta$ for $\theta > \hat{\theta}$ and $r(\theta) > \theta$ for $\theta < \hat{\theta}$. Show that any competitive equilibrium with strictly positive employment necessarily involves too much employment relative to the Pareto optimal allocation of workers.

Answer:

- If $w = \hat{\theta}$, only workers of type $\theta \geq \hat{\theta}$ will accept the wage w and work. But

$$E\left[\theta \middle| \theta \geq \hat{\theta}\right] > \hat{\theta} = w,$$

  which implies that firms demand more workers than there are in supply, and the market will not clear.

- If $w < \hat{\theta}$, only workers of type $\theta \geq \theta^* > \hat{\theta}$, with $r(\theta^*) = w$, will accept the wage w and work (since $r(\cdot)$ is a decreasing function). But

$$E[\theta | \theta \geq \theta^*] > \theta^* = w,$$

  which implies that firms demand more workers than there are in supply, and the market will not clear.

Thus, to obtain market clearing, firms have to offer a wage $w > \hat{\theta}$, which implies that some workers of type $\theta < \hat{\theta}$ will accept the job, and there is over employment in the competitive equilibrium (in a equilibrium with perfect information only workers of type $\theta \geq \hat{\theta}$ will work).

2. Screening

Consider a situation where a principal has the following objective function

$$u^p(e, w) = w - 16e.$$

She may hire an agent that can be of type l or h. There is an equal probability that each agent is of each type, and the utility function of each agent is

$$u^i(e, w) = w - \theta_i e^2$$

where $i = \{l, h\}$. Let $\theta_l = 1$ and $\theta_h = 2$. The reservation utility of the agents is zero.
(a) Find the contract or contracts that will be offered by the principal when there is symmetric information.

Answer:

Since the principal can observe the type of each agent, we can set this up as two different maximization problems. For the high type,

$$\max_{w_h, e_h} \quad w_h - 16e_h$$

and this is subject to the participation constraint of the agent,

$$w_h - 2e_h^2 \geq 0$$

For a profit maximizing principal, he will offer the lowest w_h that will cause the high-type agent to enter the market, implying that the participation constraint is binding. Hence,

$$w_h = 2e_h^2$$

and we can substitute this into our objective function, obtaining

$$\max_{e_h} \quad 2e_h^2 - 16e_h$$

with first-order condition

$$4e_h - 16 = 0 \implies e_h = 4$$

and inserting the value for e_h into the participation constraint gives $w_h = 32$. Hence, for the high type, he will be offered the contract $(4, 32)$.
For the low type, we follow similar calculations to obtain the contract $(8, 64)$.

(b) Find the contract or contracts that will be offered by the principal under asymmetric information (i.e., the principal cannot observe the types of each agent).

Answer:

In this case, the principal will maximize his expected utility

$$\max_{w_l, w_h, e_l, e_h} \quad \frac{1}{2}(w_l - 16e_l) + \frac{1}{2}(w_h - 16e_h)$$

subject to the participation constraints for each agent

$$\begin{aligned} w_l - e_l^2 &\geq 0 \qquad (PC_l) \\ w_h - 2e_h^2 &\geq 0 \qquad (PC_h) \end{aligned}$$

and the incentive compatibility constraints for each agent

$$\begin{aligned} w_l - e_l^2 &\geq w_h - e_h^2 \qquad (IC_l) \\ w_h - 2e_h^2 &\geq w_l - 2e_l^2 \qquad (IC_h) \end{aligned}$$

We know that not all of these constraints will bind in equilibrium. We can use the information from symmetric information to find which incentive compatibility constraint will bind. Starting with the low type, plugging in the symmetric information values yields

$$64 - 8^2 \ngeq 32 - 4^2$$

which does not hold. Hence, the low type would prefer to pretend to be the high type. This implies that IC_l will bind in equilibrium. For completeness, we also check for the high type, plugging in the values from the symmetric information equilibrium

$$32 - (2)4^2 \geq 64 - (2)8^2$$

which holds. Hence, the high type would not prefer to pretend to be the low type, and IC_h will not bind in equilibrium.

Since we know that the low type would prefer the high type's contracts, we also know that if the high type enters the market, the low type will also enter the market (as he would be happy with the high type's contract). This implies that PC_l does not bind and PC_h does bind. Summarizing, our two binding constraints are

$$\begin{aligned} w_h - 2e_h^2 &= 0 \qquad (PC_h) \\ w_l - e_l^2 &= w_h - e_h^2 \qquad (IC_l) \end{aligned}$$

Solving PC_h for w_h and substituting into IC_l yields

$$\begin{aligned} w_h &= 2e_h^2 \\ w_l &= 2e_h^2 - e_h^2 + e_l^2 = e_h^2 + e_l^2 \end{aligned}$$

and substituting these values into our objective function gives

$$\max_{e_l, e_h} \quad \frac{1}{2}(e_h^2 + e_l^2 - 16e_l) + \frac{1}{2}(2e_h^2 - 16e_h)$$

with first-order conditions

$$\begin{aligned} e_h + 2e_h - 8 &= 0 \\ e_l - 8 &= 0 \end{aligned}$$

yielding $e_l = 8$ and $e_h = 2\frac{2}{3}$. Substituting these values back into the binding constraints gives two contracts, $(e_l, w_l) = (8, 71\frac{1}{9})$ and $(e_h, w_h) = \left(2\frac{2}{3}, 14\frac{2}{9}\right)$.

(c) Compare between the contract or contracts when the agents' types are observable and when they are unobservable.

Answer:

When the agents' types are observable, there is no distortion of workers' effort. However, when the agents' types are unobservable, the low type still exerts 8 units of effort as in the case of symmetric information, but his wage increases from \$64 to \$$71\frac{1}{9}$, which difference represents the information rent that induces him not to take the high type contract (from the binding IC_l). Whereas, the high type exerts less effort (i.e., reducing from 8 to $2\frac{2}{3}$ units) when his type is unobservable; and as a result, receives a lower level of wage (i.e., reducing from \$64 to \$$14\frac{2}{9}$).

3. Example: Monopoly Coffee Shop

Consider a college coffee shop which has a marginal cost of 5 per ounce of coffee. The shop serves two types of customer, a coffe hound (likely an Economics student; a high type with $\theta_H = 20$), and a regular Joe (a low type with $\theta_L = 15$). The consumer's utility function is $u_C(q,p) = \theta_i v(q) - p$, where $v(q)$ is the utility that the customer receives from consuming a certain quality (q) coffee, and p is the price paid. The coffee shop's utility function is $u_S(q,p) = p - cq$. For simplicity, assume that $v(q) = 2\sqrt{q}$.

(a). Under complete information, what prices and quantities would the coffee shop offer to each type of consumer?

Answer:

The coffee shop will set up its maximization problem for each type. Starting with the high type,

$$\max_{p_H, q_H} \quad p_H - cq_H$$

subject to the high type's participation constraint

$$20v(q_H) - p_H \geq 0 \qquad (PC_H)$$

In equilibrium, the participation constraint will bind, as if it were not binding, the coffee shop could increase its profits by raising its price. Hence, we can set

$$p_H = 20v(q_H)$$

and substituting this into the objective function yields

$$\max_{q_H} \quad 20v(q_H) - cq_H$$

with first-order condition

$$20v'(q_H) - c = 0$$

Substituting the values of $v'(q_H) = \frac{1}{\sqrt{q_H}}$ and $c = 5$, and solving for q_H gives the solution

$$\frac{20}{\sqrt{q_H}} = 5 \implies q_H = 16$$

and plugging this back into PC_H gives our price,

$$p_H = 20(2\sqrt{q_H}) = 160$$

giving the contract $(p_H, q_H) = (160, 16)$.
We can follow similar calculations to obtain the contract for the low type of $(p_L, q_L) = (90, 9)$.

For parts (b)-(d), assume now that the coffee shop cannot observe the type of each consumer. The coffee shop can now offer contracts in the form of $(p(q), q)$, with function $p(q)$ being as general as you can imagine.

(b) Consider the case where the coffee shop uses linear pricing, i.e., $p(q) = pq$ and customers pay p for every unit they buy. What are the optimal contracts offered?

Answer:

First, we want to derive the demand for coffee for each type of consumer, (this is essentially the second stage of a two-stage game). Every customer with type θ_i pays p per unit of q purchased. Setting up their maximization problem,

$$\max_{q_i} \quad \theta_i v(q_i) - pq_i \quad \text{for all } i = \{H, L\}$$

with first-order condition

$$\theta_i v'(q_i) - p = 0$$

Solving for q (recall that $v'(q) = \frac{1}{\sqrt{q}}$), we obtain

$$q_i = \left(\frac{\theta_i}{p}\right)^2 \equiv D_i(p)$$

Hence, a θ_i-customer's utility is

$$\theta_i * 2\sqrt{\left(\frac{\theta_i}{p}\right)^2} - p\left(\frac{\theta_i}{p}\right)^2 = \frac{\theta_i^2}{p}$$

Now, we can move to the first-stage of the game, where the firm will set its price, setting up the expected profit maximization problem,

$$\max_{p} \quad \frac{1}{2}\left(pD_H(p) - cD_H(p)\right) + \frac{1}{2}(pD_L(p) - cD_L(p))$$

simplifying,

$$\max_{p} \quad \frac{1}{2}(p-c)\left[D_H(p) + D_L(p)\right] = \frac{1}{2}(p-c)\left[\left(\frac{\theta_H}{p}\right)^2 + \left(\frac{\theta_L}{p}\right)^2\right]$$

and taking first-order conditions with respect to p yields

$$\frac{1}{2}\left[\left(\frac{\theta_H}{p}\right)^2 + \left(\frac{\theta_L}{p}\right)^2\right] + \frac{1}{2}(p-c)\left[-2\frac{\theta_H^2}{p^3} - 2\frac{\theta_L^2}{p^3}\right] = 0$$

Rearranging,

$$2\frac{(p-c)}{p}\left[\left(\frac{\theta_H}{p}\right)^2 + \left(\frac{\theta_L}{p}\right)^2\right] = \left[\left(\frac{\theta_H}{p}\right)^2 + \left(\frac{\theta_L}{p}\right)^2\right]$$

and simplifying,

$$2(p-c) = p \Rightarrow p = 2c$$

Thus, the firm will charge twice its marginal cost to each consumer, yielding a solution $(q_H, q_L, p) = (4, 2.25, 10)$

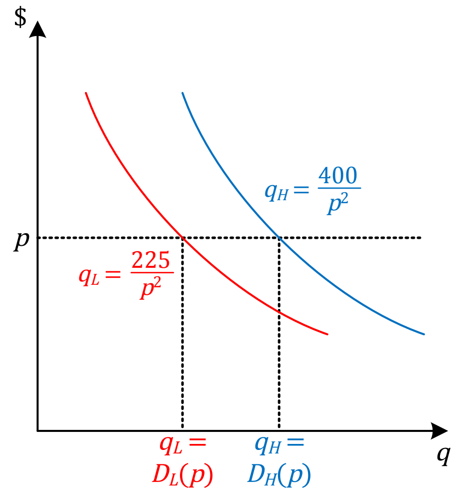
(c) Now consider non-linear pricing, where the coffee shop can use a single two-part tariff for all types of customers, (that is, the coffee shop sets some fixed price F, then some price per unit p and each type of customer decides to take it or leave it). What is the optimal price and quantity sold?

Answer:

From the UMP of each type of consumer, we obtained the first-order condition of

$$\begin{aligned} p &= \theta_i v'(q_i) \Rightarrow \\ q_i &= \left(\frac{\theta_i}{p}\right)^2 \equiv D_i(p) \end{aligned}$$

Plotting them on the same figure, we find:



where functions $\theta_i v'(q)$ are decreasing in q by the concavity of $v(\cdot)$, i.e., $v''(\cdot) < 0$ for all q. Hence, $D_H(p) > D_L(p)$, thus implying that net surpluses, $S_i(p)$, satisfy

$$S_H(p) = \theta_H v\left(D_H(p)\right) - pD_H(p) > \theta_L v(D_l(p)) - pD_L(p) = S_L(p)$$

That is, $S_H(p) > S_L(p)$.
If the firm seeks the participation of both types of customers, we need the fee to satisfy

$$F \leq S_L(p) < S_H(p)$$

More explicitly:

- In the second stage, every customer with type θ_i purchases if and only if $F \leq S_i(p)$.
- In the first stage, the firm anticipates the customers' decision rule of $F \leq S_i(p)$, and chooses the single two part tariff that maximizes profits.

Mathematically,

$$\begin{aligned}\max_{F,p} \quad & \frac{1}{2}\left[F + (p-c)D_L(p)\right] + \frac{1}{2}\left[F + (p-c)D_H(p)\right] \\ & = F + (p-c)\underbrace{\left[\frac{1}{2}D_L(p) + \frac{1}{2}D_H(p)\right]}_{D(p)\text{, i.e., expected demand}} \\ & \text{subject to} \quad F \leq S_i(p) \text{ for all } i = \{H, L\}\end{aligned}$$

However, the seller can increase F, until $F = S_L(p)$. Raising it any further would lead the low-type customers to reject the purchase.
Plugging that into the above program helps us obtain an unconstrained PMP (with only one choice variable p), as follows

$$\max_{p} \quad S_L(p) + (p-c)D(p)$$

Taking first-order conditions with respect to p yields,

$$S_L'(p) + D(p) + (p - c)D'(p) = 0$$

Solving for p and rearranging, we obtain a price of the single two-part tariff, p^{STPT}, of

$$p^{STPT} = \underbrace{c - \overbrace{\frac{D(p)}{D'(p)}}^{+}}_{\substack{p^{LP},\\ \text{price under linear pricing}}} + \overbrace{\frac{S_L'(p)}{D'(p)}}^{+}$$

where the final term is positive due to $S_L'(p) < 0$ and $D'(p) < 0$.

Remark: $S_i'(p)$ can be found by applying the Envelope Theorem on

$$S_i(p) = \theta_i \cdot v(D_i(p)) - p \cdot D_i(p)$$

In particular, second-order effects are absent, so that $D_i(p)$ is unaffected by a price change. As a consequence,

$$S_i'(p) = 0 - D_i(p) = -D_i(p) < 0$$

Hence, prices in each setting are ranked as follows:

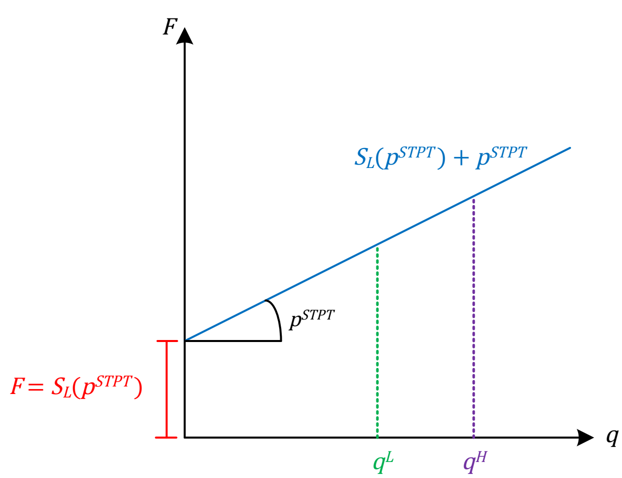
$$p^{STPT} > p^{LP} > c \text{ (price under perfect competition)}$$

The firm then sets a single two-part tariff

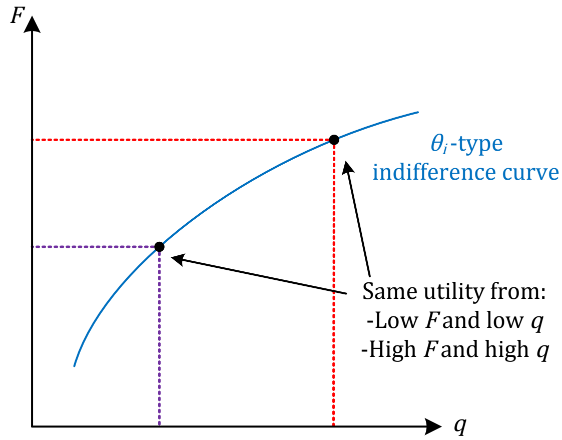
$$(F, p) = (S_L(p^{STPT}), p^{STPT})$$

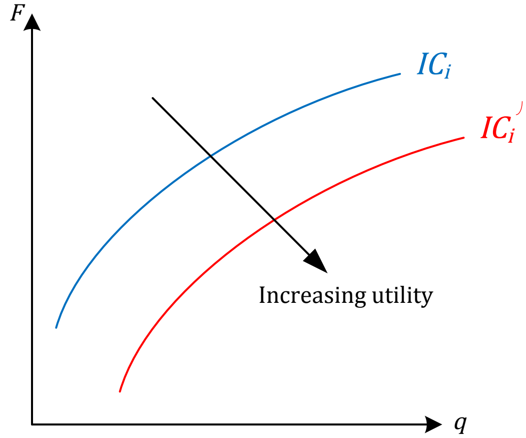
Practice: Solve out the values of the single two-part tariff using the values given at the start of the problem.

In addition, $q_H = D_H(p^{STPT}) > D_L(p^{STPT}) = q_L$
We can depict this two-part tariff in the (F, q)-quadrant, as follows.

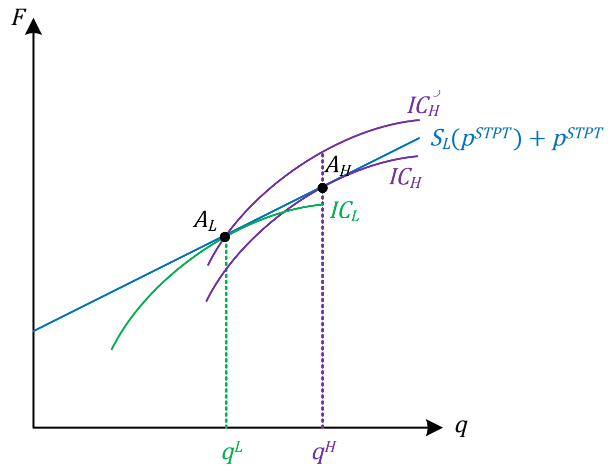


Graphical representation of the indifference curves using the same (F, q)-quadrant:





We can now superimpose IC on top of the two-part tariff obtaining:



Some points to note:

- Customer θ_H is better off at A_H than at A_L.
- Customer θ_L is better off at A_L than at A_H.

Motivation to move to other contracts (two two-part tariffs):

- The seller could do better if he sets a contract that yields point A_H to θ_H-buyer (since this buyer is indifferent about accepting the contract meant for him or that of the θ_L-customer.)

(d) Extend the model as developed in part (c) to allow for several two-part tariffs (where the coffee shop tailors the fixed cost, F_i of consuming q_i units for each type of consumer) . What is the optimal price and quantity sold?

Answer:

We can set up the monopolist's profit maximization problem, as follows:

$$\max_{F_L,q_L,F_H,q_H} \quad \frac{1}{2}\left[F_H - cq_H\right] + \frac{1}{2}\left[F_L - cq_L\right]$$

subject to the participation constraints,

$$\begin{aligned} \theta_L v(q_L) - F_L &\geq 0 & (PC_L) \\ \theta_H v(q_H) - F_H &\geq 0 & (PC_H) \end{aligned}$$

and the incentive compatibility constraints,

$$\begin{aligned} \theta_L v(q_L) - F_L &\geq \theta_L v(q_H) - F_H & (IC_L) \\ \theta_H v(q_H) - F_H &\geq \theta_H v(q_L) - F_L & (IC_H) \end{aligned}$$

Recall from class that only one of each type of constraints will bind from the consumer. The method presented in the lecture notes can be used (**Practice**: Work this out!) to show that IC_H and PC_L are the two binding constraints. Rearranging the constraints gives

$$\begin{aligned} F_L &= \theta_L v(q_L) = 30\sqrt{q_L} \\ F_H &= \theta_H v(q_H) - \theta_H v(q_L) + F_L = 40\sqrt{q_H} - 10\sqrt{q_L} \end{aligned}$$

Substituting these values into the maximization problem yields

$$\max_{q_L,q_H} \quad \frac{1}{2}\left[40\sqrt{q_H} - 10\sqrt{q_L} - cq_H\right] + \frac{1}{2}\left[30\sqrt{q_L} - cq_L\right]$$

with first-order conditions

$$\begin{aligned} \frac{1}{2}\left[-\frac{5}{\sqrt{q_L}} + \frac{15}{\sqrt{q_L}} - c\right] &= 0 \\ \frac{1}{2}\left[\frac{20}{\sqrt{q_H}} - c\right] &= 0 \end{aligned}$$

Solving these expressions, then substituting back into our binding constraints, yields contracts of $(F_H, q_H) = (140, 16)$ and $(F_L, q_L) = (60, 4)$