EconS 424 - Signalling Games II

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Harrington, Ch. 11 Exercise 7

- Consider the signalling game on the next slide.
    - Nature chooses one of three types for the sender.
    - After learning her type, the sender chooses one of three actions.
    - The receiver observes the sender's action, but not her type, and then chooses one of two actions.
- Determine if the separating strategy profile (y, y, z) can be supported as a PBE.

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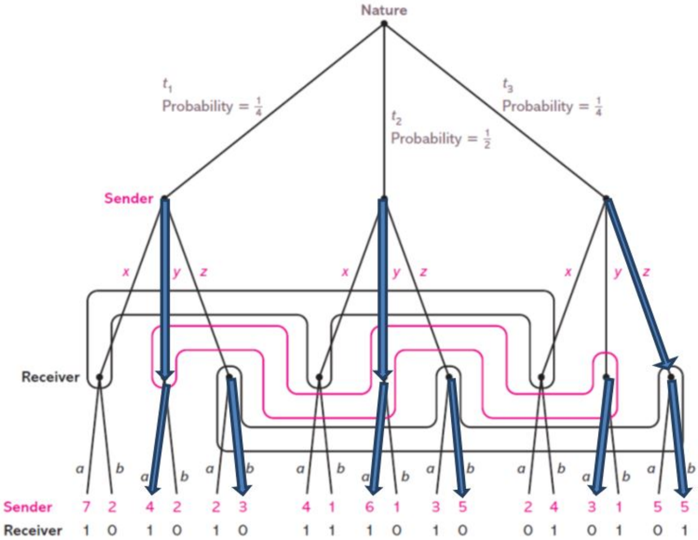

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- *Beliefs*:
  - After observing message y, the probability that such action originates from sender type t_1, t_2 and t_3, can be computed using Bayes' rule, as follows

$$prob(t_1|y) = \frac{\frac{1}{4} * 1}{\frac{1}{4} * 1 + \frac{1}{2} * 1 + \frac{1}{4} * 0} = \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3}$$

$$prob(t_2|y) = \frac{\frac{1}{2} * 1}{\frac{1}{4} * 1 + \frac{1}{2} * 1 + \frac{1}{4} * 0} = \frac{\frac{1}{2}}{\frac{3}{4}} = \frac{2}{3}$$

$$prob(t_3|y) = \frac{\frac{1}{4} * 0}{\frac{1}{4} * 1 + \frac{1}{2} * 1 + \frac{1}{4} * 0} = 0$$

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- After observing message z, the probability that such action originates from sender type t_1, t_2 and t_3, can be computed using Bayes' rule, as follows

$$prob(t_1|z) = \frac{\frac{1}{4} * 0}{\frac{1}{4} * 0 + \frac{1}{2} * 0 + \frac{1}{4} * 1} = 0$$

$$prob(t_2|z) = \frac{\frac{1}{2} * 0}{\frac{1}{4} * 0 + \frac{1}{2} * 0 + \frac{1}{4} * 1} = 0$$

$$prob(t_3|z) = \frac{\frac{1}{4} * 1}{\frac{1}{4} * 0 + \frac{1}{2} * 0 + \frac{1}{4} * 1} = \frac{\frac{1}{4}}{\frac{1}{4}} = 1$$

- Regarding message x, we know that this can only occur off-the-equilibrium path, since no type of sender selects this message in the strategy profile we are testing.
- Hence, the receiver's off-the-equilibrium beliefs are

$$prob(t_1|x) = \gamma_1 \in [0, 1]$$

  - (Recall that, as described in class, the use of Bayes' rule does not provide a precise value for γ_1, and we must leave the receiver's beliefs unrestricted in the interval $\gamma_1 \in [0, 1]$).

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- Similarly, the conditional probability that such message of x originates from a type t_2 sender is

$$prob(t_2|x) = \gamma_2 \in [0, 1]$$

And therefore,

$$prob(t_3|x) = 1 - \gamma_1 - \gamma_2$$

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- *Receiver*:
  - After observing y, he responds with either a or b depending on which action yields him the highest expected utility.
  - In particular,

$$EU_2(a|y) = \frac{1}{3} * 1 + \frac{2}{3} * 1 = 1$$

$$EU_2(b|y) = \frac{1}{3} * 0 + \frac{2}{3} * 0 = 0$$

Hence, the receiver selects a after observing y.

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- After observing z, the receiver similarly compares his utility from a and b, as follows.
  - (Note that in this case, the receiver does not need to compute expected utilities, since he is convinced to be dealing with a t_3-type of sender, i.e., in the node at the right-hand side of the game tree)

$$EU_2(a|z) = 0$$

$$EU_2(b|z) = 1$$

Hence, the receiver selects b after observing z.

- After observing x (off-the-equilibrium path), the receiver compares his expected utility from selects a and b, as follows

$$EU_2(a|x) = \gamma_1 * 1 + \gamma_2 * 1 + (1 - \gamma_1 - \gamma_2) * 0 = \gamma_1 + \gamma_2$$

$$EU_2(b|x) = \gamma_1 * 0 + \gamma_2 * 1 + (1 - \gamma_1 - \gamma_2) * 1 = 1 - \gamma_1$$

  Hence, after observing x, the receiver chooses a iff $\gamma_1 + \gamma_2 > 1 - \gamma_1$, or $\gamma_2 > 1 - 2\gamma_1$.

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- *Sender*:
  - If his type is t_1,

$$EU_1(y|t_1) = 4$$

$$EU_1(z|t_1) = 3$$

$$EU_1(x|t_1) = \begin{cases} 7 \text{ if } \gamma_2 > 1 - 2\gamma_1 \\ 2 \text{ if } \gamma_2 < 1 - 2\gamma_1 \end{cases}$$

Note that we need the second condition on $EU_1(x|t_1)$ (otherwise P1 deviates).

- If his type is t_2,

$$EU_1(y|t_2) = 6$$

$$EU_1(z|t_2) = 5$$

$$EU_1(x|t_2) = \begin{cases} 4 \text{ if } \gamma_2 > 1 - 2\gamma_1 \\ 1 \text{ if } \gamma_2 < 1 - 2\gamma_1 \end{cases}$$

There is no incentive to deviate for P1 under all parameter conditions for $EU_1(x|t_2)$.

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- And finally, if his type is t_3,

$$EU_1(y|t_3) = 3$$

$$EU_1(z|t_3) = 5$$

$$EU_1(x|t_3) = \begin{cases} 2 \text{ if } \gamma_2 > 1 - 2\gamma_1 \\ 4 \text{ if } \gamma_2 < 1 - 2\gamma_1 \end{cases}$$

  There is no incentive to deviate for P1 under all parameter conditions for $EU_1(x|t_3)$.

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- One example of $\gamma_2 < 1 - 2\gamma_1$ being satisfied is that after observing the off-the-equilibrium message of x, the receiver believes:

$$\gamma_1 = \frac{1}{4} \text{ and } \gamma_2 = \frac{1}{2}$$

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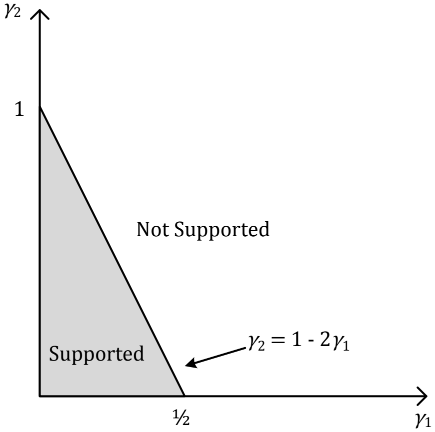
- The following figure represents all combinations of γ_1 and γ_2 for which the above strategy profile can be sustained as a PBE of this game.



Watson, Ch. 29 Exercise 5

- Consider an investment game played between two people.
- Player 1 owns the asset that can be put to productive use only if both players make an investment.
  - For example, the asset might be a motorcycle that is in need of repair.
  - Player 1 might be an expert in electrical systems, so his investment would be to perform the electrical repairs on the bike.
  - Player 2 might be a mechanical specialist, whose investment would be to repair the engine mechanics.
- At the beginning of the game, player 1 decides whether to invest in the asset (perform the electrical repair).
- Player 1's choice is observed by player 2.

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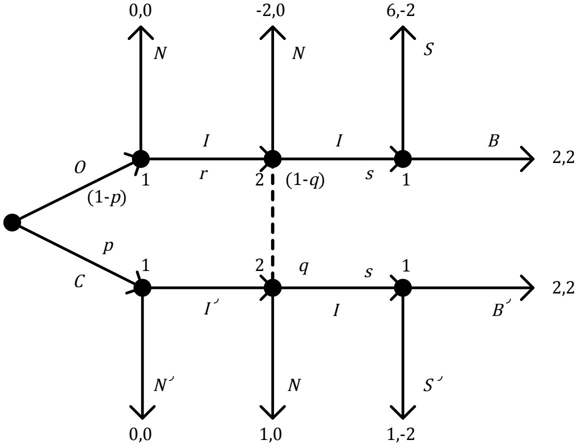
- **Player 1:**
- If player 1 decides not to invest (N), then the game ends with zero payoffs.
- If player 1 invests (I), then player 2 must decide whether to invest (repair the engine).
  - If player 2 fails to invest (N), then the asset is of no productive use; in this case, the game ends and player 1 gets a negative payoff owing to his wasted investment.

Watson, Ch. 29 Exercise 5

- **Player 2:**
- If player 2 invests (I), then the asset is made productive, creating a net value of 4. That is, investment by both players puts the motorcycle in operating condition so that it can be enjoyed at the local park for off-road vehicles.
- But because player 1 owns the asset, he determines how it will be used.
  - He can decide to be benevolent (B) by sharing the asset with player 2 (that is, allowing player 2 to ride the bike) or he can be selfish (S) and hoard the asset.

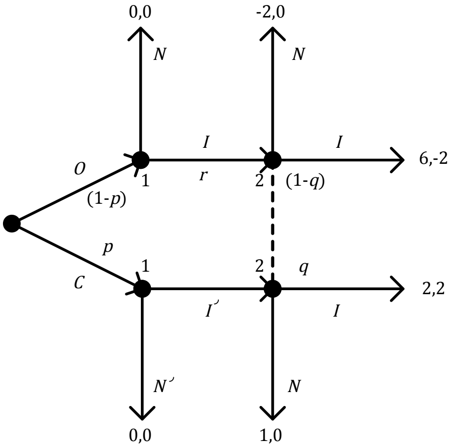
Watson, Ch. 29 Exercise 5



- Let us check if the following semi-separating strategy profile can be sustained as a PBE of this game:

$$\textbf{P1 chooses:} \begin{cases} p_c = 1 \text{ if he selects } I' \text{ with probability } 1 \\ r = \frac{p}{1-p} \text{ he selects } I \text{ with probability } r = \frac{p}{1-p} \end{cases}$$

- Reducing the game (solving the proper subgames at the right-hand side of the tree),



Watson, Ch. 29 Exercise 5

- Player 2 must be indifferent between N and I, as follows

$$EU_2(N) = EU_2(I)$$

$$0 * (1 - q) + 0 * q = -2 * (1 - q) + 2q$$

$$0 = -2 + 2q + 2q$$

$$2 = 4q \rightarrow q = \frac{1}{2} \rightarrow \text{ Belief for } P_2$$

Watson, Ch. 29 Exercise 5

- Now, we must use player 2's beliefs that we found in the previous step, $q = \frac{1}{2}$, in order to find what mixed strategy player 1 uses. For that, we use Bayes' rule as follows:

$$q = \frac{1}{2} = \frac{p * p_c}{p * p_c + (1-p) * r}$$

  - where p_c denotes the probability with which player 1 chooses I' (when his type is C in the lower part of the tree),
  - whereas r represents the probability with which player 1 chooses I (when his type is O in the upper part of the tree).

- Since in this semi-separating strategy profile we have that $p_c = 1$, the above ratio becomes

$$\frac{1}{2} = \frac{p}{p + (1-p) * r}$$

- Solving for probability r, we obtain

$$r = \frac{p}{1-p}$$

  recalling that probability r represents the probability with which player 1 chooses I (when his type is O).
- Hence, at this stage of our solution we know everything regarding player 1:
  - he chooses I with probability $r = \frac{p}{1-p}$ when his type is O, and
  - he selects I' using pure strategies (with 100% probability) when his type is C, i.e., $p_c = 1$.

Watson, Ch. 29 Exercise 5

- Finally, note that if player 1 mixes with probability $r = \frac{p}{1-p}$ when his type is O, it must be that player 2 makes him indifferent between I and N.
- That is,

$$EU_1(I|O) = EU_1(N|O)$$

$$s\,(6) + (1 - s) * (\text{-}2) = 0$$

where s denotes the probability with which player 2 chooses I. Solving for probability s, we obtain

$$s = \frac{1}{4}$$

Watson, Ch. 29 Exercise 5

- We can now summarize, with all your previous results, the Semi-Separating PBE of this game:
  - Player 1:
    - chooses I with probability $r = \frac{p}{1-p}$ when his type is O, and
    - selects I' using pure strategies when his type is C, i.e., $p_c = 1$.
  - Player 2 responds I with probability $s = 1/4$, and his beliefs are $q =1/2$.