EconS 424 - Signalling Games I

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Watson, Ch. 29 Exercise 3

- There is a firm and a worker. In this game, nature chooses the "type" of the firm (player 1). With probability p, the firm is of high quality (H) and, with probability $1-p$, the firm is of low quality (L).
- The firm chooses either to offer a job to the worker (O) or not to offer a job (N).
  - If no job is offered, the game ends and both parties receive 0.
  - If the firm offers a job, the worker either accepts (A) or rejects (R) the offer.

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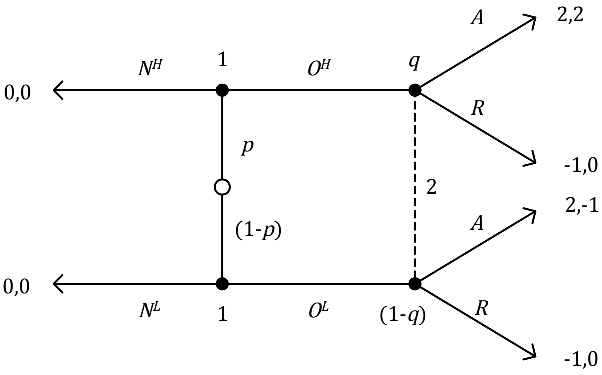
- **Firm:**
- The worker's effort on the job brings the firm a profit of 2.
- If the worker rejects an offer of employment, then the firm gets a payoff of -1 (associated with being jilted).

- **Worker:**
- Rejecting an offer yields a payoff of 0 to the worker.
- Accepting an offer yields the worker a payoff of 2 if the firm is of high quality, and -1 if the firm is of low quality.
- The worker does not observe the quality of the firm directly.

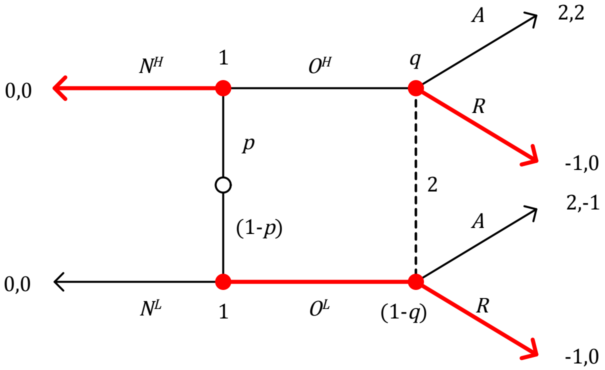
Watson, Ch. 29 Exercise 3

- Let us find all PBEs of this game.

Watson, Ch. 29 Exercise 3



- **First strategy profile candidate to be PBE:** $N^H O^L$: Let us start with the separating equilibrium where the informed player 1 (the firm) makes a job offer to the worker (O) only when the firm is of low quality, O^L, but not to make a job offer otherwise, i.e., N^H.
  - The figure below shades the branches corresponding to this strategy.



Watson, Ch. 29 Exercise 3

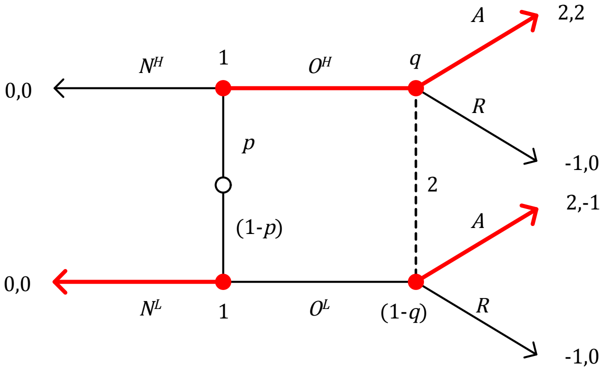
- Given this strategy from the firm, the worker's beliefs after observing a job offer must be

$$q = \frac{p * 0}{p * 0 + (1 - p) * 1} = 0$$

- Intuitively, this implies that after observing O, the worker concentrates all his beliefs on being in the lower node of the information set, i.e., low-quality firm.

- Given these beliefs, it is optimal for the worker to reject the job offer, since $0 > -1$.
  - For future reference, we shade the branch in which the worker rejects the firm's job offer, an action that is independent on the firm's type (something that the worker cannot observe).
- If the worker rejects, then the high-quality firm prefers to not make a job offer, N^H, since $0 > -1$, as prescribed by this strategy profile.
  - However, the low-quality firm prefers to not make job offers, N^L, which contradicts the initially proposed strategy profile $N^H O^L$.
- Then, the separating strategy profile $N^H O^L$ cannot be sustained as a PBE of this game.

- **Second strategy profile candidate to be PBE:** $O^H N^L$ Let us continue with the separating equilibrium where the informed player (firm) makes a job offer to the worker (O) only when the firm is of high quality, O^H, but not to make a job offer otherwise, i.e., N^L.
  - The figure below shades the branches corresponding to this strategy.



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- Given this strategy from the firm, the worker's beliefs after observing a job offer must be

$$q = \frac{p * 1}{p * 1 + (1 - p) * 0} = 1$$

- Intuitively, this implies that after observing O, the worker concentrates all his beliefs on being in the upper node of the information set, i.e., high-quality firm.

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- Given these beliefs, it is optimal for the worker to accept the job offer, since $2 > 0$.
  - For future reference, we shade the branch in which the worker accepts the firm's job offer, an action that is independent on the firm's type (something that the worker cannot observe).
- If the worker accepts, then the high-quality firm prefers to make a job offer, O^H, since $2 > 0$, as prescribed by this strategy profile.
  - The low-quality firm also prefers to make a job offer, O^L, which contradicts the initially proposed strategy profile $O^H N^L$.
- Then, the separating strategy profile $O^H N^L$ cannot be sustained as a PBE of this game either.

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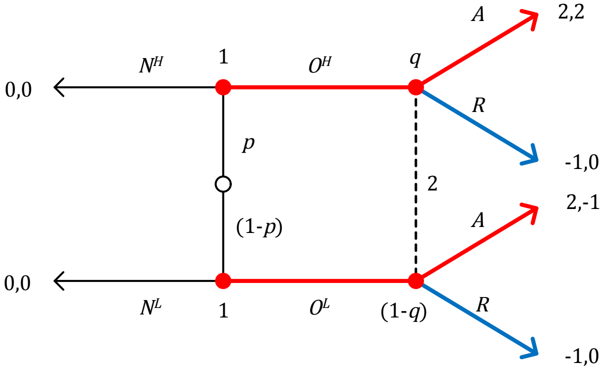
- **Third strategy profile candidate to be PBE:** $O^H O^L$. Let us continue with the pooling equilibrium where both types of firm (high and low-quality) make a job offer to the worker.
  - The figure below shades the branches corresponding to this strategy for player 1.



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- Given this strategy from the firm, the worker's beliefs after observing a job offer must be

$$q = \frac{p * 1}{p * 1 + (1 - p) * 1} = p$$

- Intuitively, this implies that after observing O, the worker cannot infer additional information about the firm's quality, since all firms make job offers. (Note that this result radically differs from that in the two separating strategy profiles described above.)

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- Given these beliefs, it is optimal for the worker to accept the job offer if $EU_2(A) > EU_2(R)$, where

$$EU_2(A) = 2p + (1-p)(-1) = 3p - 1$$

$$EU_2(R) = 0 + 0$$

$$\rightarrow 3p - 1 = 0$$

$$p = \frac{1}{3} \text{ for } A = R$$

$$p > \frac{1}{3} \text{ for } A > R$$

$$p < \frac{1}{3} \text{ for } A < R$$

- Hence, if $p > \frac{1}{3}$, the worker accepts the job offer and we can shade branch A in the figure.
  - Otherwise, the worker rejects the job offer and we must shade branch R.

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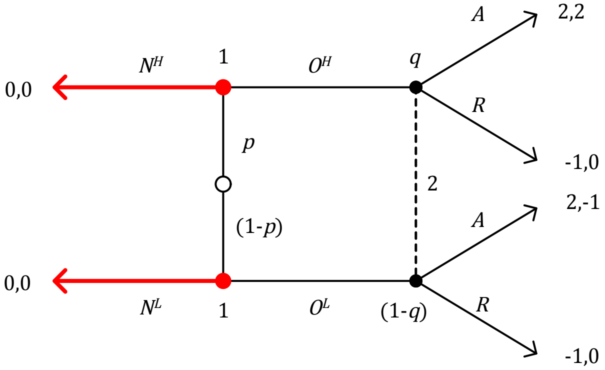
- Let us divide our analysis according to the value of p:
  - If $p > \frac{1}{3}$, the worker accepts the job offer. The high-quality firm prefers to make a job offer (as prescribed) since $2 > 0$, while the low-quality firm also wants to make a job offer (as prescribed) since $2 > 0$.
    - Hence, the pooling strategy profile $O^H O^L$ can be sustained as PBE of this game when $p > \frac{1}{3}$.
  - If $p < \frac{1}{3}$, the worker rejects the job offer. The high-quality firm does not want to make a job offer since making it yields a payoff of -1 (since the offer is rejected) whereas not making such an offer yields a payoff of 0.
    - Hence, the pooling strategy profile $O^H O^L$ cannot be sustained as PBE of this game when $p < \frac{1}{3}$.

Watson, Ch. 29 Exercise 3

- **Fourth strategy profile candidate to be PBE:** $N^H N^L$. Let us continue with the pooling equilibrium where no type of firm (high or low-quality) make a job offer to the worker.
  - The figure below shades the branches corresponding to this strategy for player 1.



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- Given this strategy from the firm, the worker's beliefs after observing a job offer must be

$$q = \frac{p * 0}{p * 0 + (1 - p) * 0} = \frac{0}{0}$$

Hence, q can be any number in the $[0, 1]$ interval.

- Given these beliefs, it is optimal for the worker to accept the job offer if $EU_2(A) > EU_2(R)$, where

$$EU_2(A) = 2q + (-1)(1-q)$$

$$EU_2(R) = 0$$

$$\rightarrow 2q + (-1)(1-q) = 0$$

$$q = \frac{1}{3} \text{ for } A = R$$

$$q < \frac{1}{3} \text{ for } R > A$$

$$q > \frac{1}{3} \text{ for } R < A$$

- Hence, if $q > \frac{1}{3}$, the worker accepts the job offer and we can shade branch A in the figure.
  - Otherwise, the worker rejects the job offer and we must shade branch R.

Watson, Ch. 29 Exercise 3

- Let us divide our analysis according to the value of q:
  - If $q > \frac{1}{3}$, the worker accepts the job offer. The high-quality firm prefers to make a job offer since $2 > 0$, and the low-quality firm also wants to make a job offer since $2 > 0$.
    - Hence, the pooling strategy profile $N^H N^L$ cannot be sustained as PBE of this game when $q > \frac{1}{3}$.
  - If $q < \frac{1}{3}$, the worker rejects the job offer. The high-quality firm does not want to make a job offer: making it yield a payoff of -1 (since the offer is rejected) whereas not making such an offer yields a payoff of 0. Similarly for the low-quality firm.
    - Hence, no type of firm makes job offers and the pooling strategy profile $N^H N^L$ can be sustained as PBE of this game when $q < \frac{1}{3}$.

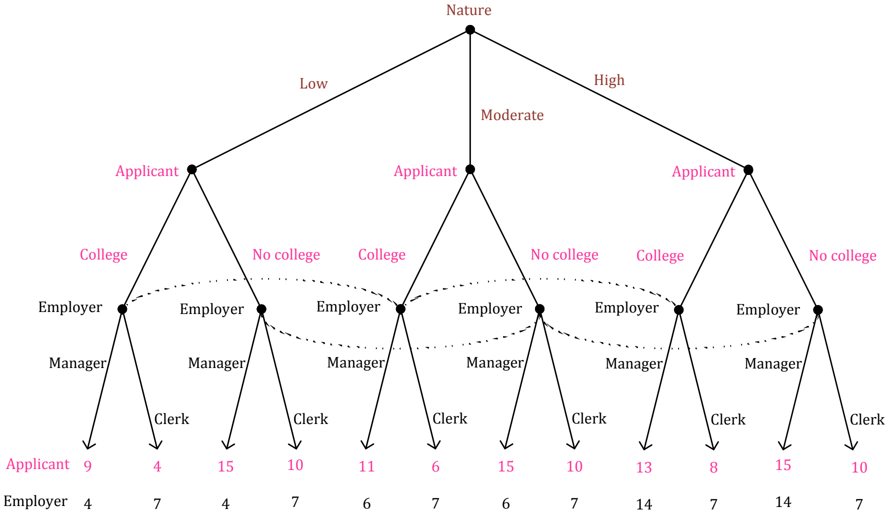
Harrington, Ch. 11 Exercise 5

- Consider a two-player game between a prospective employee, whom we'll refer to as the applicant, and an employer.
- The applicant's type is her intellect, which may be low, medium or high, with probability $\frac{1}{3}$, $\frac{1}{2}$, and $\frac{1}{6}$, respectively.
- After the applicant learns her type, she decides whether or not to go to college. The personal cost in gaining a college degree is higher when the applicant is less intelligent, because a less smart student has to work harder if she is to graduate.
- Assume that the cost of gaining a college degree is 2, 4, and 6 for an applicant who is of high, moderate, and low intelligence, respectively.

Harrington, Ch. 11 Exercise 5

- The employer decides whether to offer the applicant a job as a manager or as a clerk.
- The applicant's payoff to being hired as a manager is 15, while the payoff to being a clerk is 10. These payoffs are independent of the applicant's type.
- The employer's payoff from hiring someone as a clerk is 7 (and is the same regardless of intelligence and whether or not the person has a college degree).
- If the applicant is hired as a manager, then the employer's payoff increases with the applican't intellect, from 4, to 6, to 14, depending on whether the applicant has low, moderate, or high intellect, respectively.
  - Note that the employer's payoff does not depend on whether or not the applicant has a college degree.
- The extensive form of this game is shown on the next slide.

Harrington, Ch. 11 Exercise 5



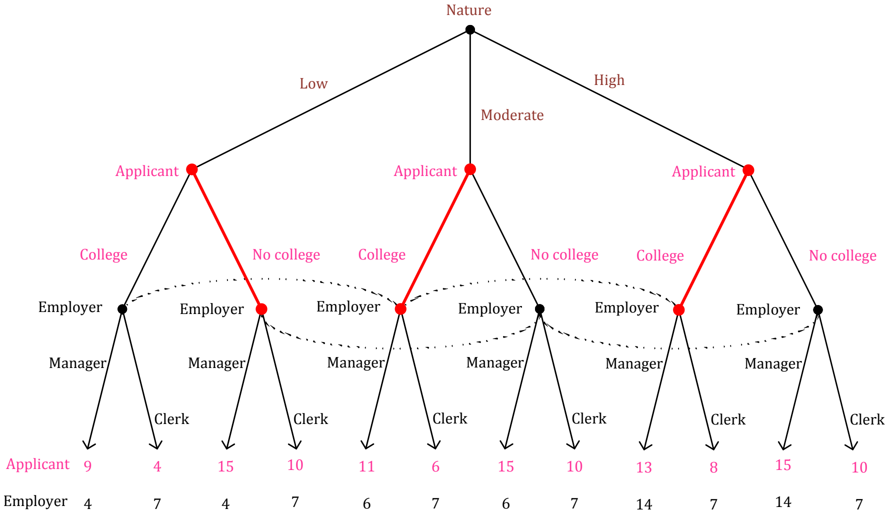
Harrington, Ch. 11 Exercise 5

- Find a PBE in which students of low intellect do not go to college and those of moderate and high intellect do.

Harrington, Ch. 11 Exercise 5

- The figure on the next slide shades the branches corresponding to the strategy profile we need to check, where the applicant goes to college both when his innate ability is moderate and high, but does not go to college when his ability is low.

Harrington, Ch. 11 Exercise 5



- *Step 1* - Employer's beliefs:
  - If the student did not go to college, then he is low intellect with probability 1
  - If the student did go to college, then he is low intellect with probability zero.

$$\mu_2 \text{ moderate intellect with probability: } \frac{\frac{1}{2}*1}{\frac{1}{2}*1+\frac{1}{6}*1}=\frac{3}{4}$$

$$1-\mu_2 \text{ high intellect with probability: } \frac{\frac{1}{6}*1}{\frac{1}{2}*1+\frac{1}{6}*1}=\frac{1}{4}$$

- *Step 2* - Firm's best response:
  - If the student did not go to college, then hire him as a clerk. (She believes the student is of low intellect, then payoff $7 > 4$).
  - If the student did go to college, then the employer must compute the expected utility from hiring the student as a manager and as a clerk, as shown below.

$$EU_F(M) = \overbrace{\frac{\frac{1}{2}*1}{\frac{1}{2}*1+\frac{1}{6}*1}}^{\mu_2} *6 + \overbrace{\frac{\frac{1}{6}*1}{\frac{1}{2}*1+\frac{1}{6}*1}}^{1-\mu_2} *14 = \frac{32}{4} = 8$$

$$EU_F(C) = \frac{\frac{1}{2}*1}{\frac{1}{2}*1+\frac{1}{6}*1} *7 + \frac{\frac{1}{6}*1}{\frac{1}{2}*1+\frac{1}{6}*1} *7 = 7$$

Harrington, Ch. 11 Exercise 5

- Clearly, $EU_F(M) > EU_F(C)$, implying that if the student did go to college, the employer hires him as a manager.
- The employer' s beliefs are consistent with respect to the student not going to college, since only when his type is low intellect does he not go to college.

- *Step 3* - Student's Strategy:
  - If of low intellect, do not go to college.
  - If of moderate or high intellect, go to college.

- The next table evaluates the applicant's incentives to deviate to a different strategy:

	No College $\rightarrow$ Clerk	College $\rightarrow$ Manager
Low Intellect	$10 >$	$(15-6) = 9$
Moderate Intellect	$10 <$	$(15-4) = 11$
High Intellect	$10 <$	$(15-2) = 13$

- It is clear that the applicant has no incentives to deviate away from his strategy.
- Thus, this strategy profile can be supported as a PBE.