EconS 424 - Strategy and Game Theory
Homework #3 - Due date: February 14th, in class.

1. **Exercises from Harrington:**

   (a) Chapter 6: Exercise 10 and 12.

   (b) Chapter 7: Exercise 12.

2. **Mixed strategy Nash equilibrium with N players.** Consider a game with N players. Simultaneously and independenly, the players choose between X and Y. That is, the strategy space for each player i is $S_i = \{X, Y\}$. The payoff of each player who selects X is

$$2m_x - m_x^2 + 3$$

   where m_x is the number of players who choose X. The payoff of each player who selects Y is

$$4 - m_y$$

   where m_y is the number of players who choose Y. Note that $m_x + m_y = n$.

   (a) For the case of only two players, $n = 2$, represent this game in the normal form, and find the pure-strategy Nash equilibria (if any).

   (b) Suppose now that $n = 3$. How many psNE does this game have?

   (c) Continue to assume $n = 3$. Determine whether this game has a symmetric msNE in which each player selects X with probability p.

3. **[Cournot competition with N firms]** There are three identical firms in the industry. The inverse demand function is $p(Q) = 1 - Q$, where $Q = q_1 + q_2 + q_3$ denotes aggregate output. To facilitate your calculations, assume that the marginal cost for all firms is zero, $c = 0$.

   (a) Find the best response function for each firm. Interpret.

   (b) Compute the Cournot equilibrium.

   (c) Show that if two of the three firms merge (transforming the industry into a duopoly), the profit of these two firms decreases. Explain.

   (d) What happens if all three firms merge?

   BONUS EXERCISE:

4. **[Cournot mergers with efficiency gains]** Consider an industry with three identical firms each selling a homogenous good and producing at a cost $c > 0$. Industry demand is given by $p(Q) = 1 - Q$, where $Q = q_1 + q_2 + q_3$ denotes aggregate output. Competition in the marketplace is in quantities (a la Cournot).

   (a) Find the equilibrium quantities, price and profits.

(b) Consider now a merger between two of the three firms, resulting in duopolistic structure of the market. The merger might give rise to efficiency gains, in the sense that the firm resulting from the merger produces at a cost $e \times c$, with $e \leq 1$ (whereas the outsider still has a cost c).

1. Find the post-merger equilibrium quantities, price and profits.
2. Under which conditions does the merger reduce prices?
3. Under which conditions is the merger beneficial to the merging firms?