Advanced Microeconomic Theory

Chapter 7: Monopoly
Outline

• Barriers to Entry
• Profit Maximization under Monopoly
• Welfare Loss of Monopoly
• Multiplant Monopolist
• Price Discrimination
• Advertising in Monopoly
• Regulation of Natural Monopolies
• Monopsony
Barriers to Entry
Barriers to Entry

• *Entry barriers*: elements that make the entry of potential competitors either impossible or very costly.

• Three main categories:

  1) **Legal**: the incumbent firm in an industry has the legal right to charge monopoly prices during the life of the patent

     -- *Example*: newly discovered drugs
Barriers to Entry

2) **Structural**: the incumbent firm has a cost or demand advantage relative to potential entrants.
   – superior technology
   – a loyal group of customers
     ▪ positive network externalities (Facebook, eBay)

3) **Strategic**: the incumbent monopolist has a reputation of fighting off newcomers, ultimately driving them off the market.
   – price wars
Profit Maximization under Monopoly
Profit Maximization

• Consider a demand function $x(p)$, which is continuous and strictly decreasing in $p$, i.e., $x'(p) < 0$.

• We assume that there is price $\bar{p} < \infty$ such that $x(p) = 0$ for all $p > \bar{p}$.

• Also, consider a general cost function $c(q)$, which is increasing and convex in $q$. 
Profit Maximization

- \( \bar{p} \) is a “choke price”
- No consumers buy positive amounts of the good for \( p > \bar{p} \).

\[ x(p) = 0 \text{ for all } p > \bar{p} \]

\[ x'(p) < 0 \]
Profit Maximization

• Monopolist’s decision problem is

\[
\max_p \ p x(p) - c(x(p))
\]

• Alternatively, using \( x(p) = q \), and taking the inverse demand function \( p(q) = x^{-1}(p) \), we can rewrite the monopolist’s problem as

\[
\max_{q \geq 0} \ p(q)q - c(q)
\]
Profit Maximization

• Differentiating with respect to \( q \),

\[
p(q^m) + p'(q^m)q^m - c'(q^m) \leq 0
\]

• Rearranging,

\[
\frac{d[p(q)q]}{dq} \leq \frac{c'(q^m)}{MC}
\]

with equality if \( q^m > 0 \).

• Recall that total revenue is \( TR(q) = p(q)q \)
Profit Maximization

• In addition, we assume that $p(0) \geq c'(0)$.
  – That is, the inverse demand curve originates above the marginal cost curve.
  – Hence, the consumer with the highest willingness to pay for the good is willing to pay more than the variable costs of producing the first unit.

• Then, we must be at an interior solution $q^m > 0$, implying

\[
p(q^m) + p'(q^m)q^m = c'(q^m)
\]

\[
\begin{array}{c}
MR \\
MC
\end{array}
\]
Profit Maximization

• Note that

\[ p(q^m) + p'(q^m)q^m = c'(q^m) \]

• Then, \( p(q^m) > c'(q^m) \), i.e.,

monopoly price > MC

• Moreover, we know that in competitive equilibrium \( p(q^*) = c'(q^*) \).

• Then, \( p^m > p^* \) and \( q^m < q^* \).
Profit Maximization
Profit Maximization

• Marginal revenue in monopoly

\[ MR = p(q^m) + p'(q^m)q^m \]

MR describes two effects:

– A **direct (positive) effect**: an additional unit can be sold at \( p(q^m) \), thus increasing revenue by \( p(q^m) \).

– An **indirect (negative) effect**: selling an additional unit can only be done by reducing the market price of all units (the new and all previous units), ultimately reducing revenue by \( p'(q^m)q^m \).

- **Inframarginal units** – initial units before the marginal increase in output.
Profit Maximization

• Is the above FOC also sufficient?
  – Let’s take the FOC $p(q^m) + p'(q^m)q^m - c'(q^m)$, and differentiate it wrt $q$,
    \[ p'(q) + p'(q) + p''(q)q - c''(q) \leq 0 \]
    \[
    \frac{dMR}{dq} \leq \frac{dMC}{dq}
    \]
  – That is, \( \frac{dMR}{dq} \leq \frac{dMC}{dq} \).
  – Since MR curve is decreasing and MC curve is weakly increasing, the second-order condition is satisfied for all $q$. 
Profit Maximization

\[ p^m \]

\[ q^m \]

\[ p \]

\[ q \]

MC(q)

MR(q)

x(p)
Profit Maximization

• What would happen if MC curve was decreasing in $q$ (e.g., concave technology given the presence of increasing returns to scale)?
  – Then, the slopes of MR and MC curves are both decreasing.
  – At the optimum, MR curve must be steeper MC curve.
Profit Maximization

\[ p^m \]

\[ q^m \]

\[ x(p) \]

\[ MC(q) \]

\[ MR(q) \]
Profit Maximization: Lerner Index

• Can we re-write the FOC in a more intuitive way? Yes.

  – Just take $MR = p(q) + p'(q)q = p + \frac{\partial p}{\partial q} q$ and multiply by $\frac{p}{p}$,

    $$MR = p \frac{p}{p} + \frac{\partial p}{\partial q} q = p + \frac{1}{\varepsilon_d} p$$

  – In equilibrium, $MR(q) = MC(q)$. Hence, we can replace MR with MC in the above expression.
Profit Maximization: Lerner Index

• Rearranging yields

\[
\frac{p - MC(q)}{p} = - \frac{1}{\varepsilon_d}
\]

• This is the Lerner index of market power
  – The price mark-up over marginal cost that a monopolist can charge is a function of the elasticity of demand.

• Note:
  – If \( \varepsilon_d \rightarrow \infty \), then \( \frac{p - MC(q)}{p} \rightarrow 0 \implies p = MC(q) \)
  – If \( \varepsilon_d \rightarrow 0 \), then \( \frac{p - MC(q)}{p} \rightarrow \infty \implies \) substantial mark-up
Profit Maximization: Lerner Index

• The Lerner index can also be written as

\[ p = \frac{MC(q)}{1 + \frac{1}{\varepsilon_d}} \]

which is referred to as the \textit{Inverse Elasticity Pricing Rule} (IEPR).

• \textit{Example} (Perloff, 2012):
  
  – Prilosec OTC: \( \varepsilon_d = -1.2 \). Then price should be
    \[ p = \frac{MC(q)}{1 + \frac{1}{-1.2}} = 5.88MC \]
  – Designed jeans: \( \varepsilon_d = -2 \). Then price should be
    \[ p = \frac{MC(q)}{1 + \frac{1}{-2}} = 2MC \]
Profit Maximization: Lerner Index

- **Example 1** (linear demand):
  - Market inverse demand function is
    \[ p(q) = a - bq \]
    where \( b > 0 \)
  - Monopolist’s cost function is \( c(q) = cq \)
  - We usually assume that \( a > c \geq 0 \)
    - To guarantee \( p(0) > c'(0) \)
    - That is, \( p(0) = a - b0 = a \) and \( c'(q) = c \), thus implying \( c'(0) = c \)
Profit Maximization: Lerner Index

• **Example 1** (continued):
  – Monopolist’s objective function

\[
\pi(q) = (a - bq)q - cq
\]

  – FOC: \( a - 2bq - c = 0 \)
  – SOC: \(-2b < 0\) (concave)

  ▪ Note that as long as \( b > 0 \), i.e., negatively sloped demand function, profits will be concave in output.
  ▪ Otherwise (i.e., Giffen good, with positively sloped demand function) profits will be convex in output.
Example 1 (continued):

- Solving for the optimal $q^m$ in the FOC, we find monopoly output

$$q^m = \frac{a - c}{2b}$$

- Inserting $q^m = \frac{a-c}{2b}$ in the demand function, we obtain monopoly price

$$p^m = a - b \left( \frac{a - c}{2b} \right) = \frac{a + c}{2}$$

- Hence, monopoly profits are

$$\pi^m = p^m q^m - c q^m = \frac{a - c}{4b}$$
Profit Maximization: Lerner Index

• Example 1 (continued):
Profit Maximization: Lerner Index

- **Example 1** (continued):
  - Non-constant marginal cost
  - The cost function is convex in output
  \[ c(q) = cq^2 \]
  - Marginal cost is
  \[ c'(q) = 2cq \]
Profit Maximization: Lerner Index

• **Example 2** (Constant elasticity demand):
  
  – The demand function is
    \[ q(q) = Ap^{-b} \]
  
  – We can show that \( \varepsilon(q) = -b \) for all \( q \), i.e.,
    \[ \varepsilon(q) = \frac{\partial q(p)}{\partial p} \frac{p}{q} = \left( -b \right)Ap^{-b-1} \frac{p}{Ap^{-b}} \]

    \[ \frac{p}{q} \]

    \[ = -b \frac{p^{-b}}{p} \frac{p}{p^{-b}} = -b \]
Example 2 (continued):

- We can now plug $\varepsilon(q) = -b$ into the Lerner index,

$$p^m = \frac{c}{1 + \frac{1}{\varepsilon(q)}} = \frac{c}{1 - \frac{1}{b}}$$

- That is, price is a constant mark-up over marginal cost.
Welfare Loss of Monopoly
Welfare Loss of Monopoly

- Welfare comparison for perfect competition and monopoly.

\[ p(q^m) \rightarrow p^m \rightarrow p^*_M \]

\[ \int_{q^m}^{q^*} [p(s) - c'(s)] ds \]

\[ MR = p(q) + p'(q)q \]
Welfare Loss of Monopoly

- **Consumer surplus**
  - Perfect competition: $A+B+C$
  - Monopoly: $A$

- **Producer surplus**:
  - Perfect competition: $D+E$
  - Monopoly: $D+B$

- **Deadweight loss of monopoly**: $C+E$

\[
DWL = \int_{q_m}^{q^*} [p(s) - c'(s)]ds
\]

- $DWL$ decreases as demand and/or supply become more elastic.
Welfare Loss of Monopoly

• Infinitely elastic demand
  \[ p'(q) = 0 \]
• The inverse demand curve becomes totally flat.
• Marginal revenue coincides with inverse demand:
  \[ MR(q) = p(q) + 0 \cdot q = p(q) \]
• Profit-maximizing \( q \)
  \[ MR(q) = MC(q) \implies p(q) = MC(q) \]
• Hence, \( q^m = q^* \) and \( DWL = 0. \)
Welfare Loss of Monopoly

- **Example** (Welfare losses and elasticity):
  - Consider a monopolist with constant marginal and average costs, \( c'(q) = c \), who faces a market demand with constant elasticity
    \[
    q(p) = p^e
    \]
    where \( e \) is the price elasticity of demand \((e < -1)\)
  - Perfect competition: \( p_c = c \)
  - Monopoly: using the IEPR
    \[
    p^m = \frac{c}{1 + \frac{1}{e}}
    \]
Welfare Loss of Monopoly

• Example (continued):
  – The consumer surplus associated with any price \( p_0 \) can be computed as

\[
CS = \int_{p_0}^{\infty} Q(P)dp = \int_{p_0}^{\infty} p^e dp = \frac{pe^{e+1}}{e+1} \bigg|_p^{\infty} - \frac{p_0^{e+1}}{e+1}
\]

  – Under perfect competition, \( p_c = c \),

\[
CS = -\frac{c^{e+1}}{e + 1}
\]

  – Under monopoly, \( p^m = \frac{c}{1+1/e} \),

\[
CS_m = -\frac{\left(\frac{c}{1 + 1/e}\right)^{e+1}}{e + 1}
\]
Welfare Loss of Monopoly

• **Example** (continued):

  – Taking the ratio of these two surpluses

  \[
  \frac{CS_m}{CS} = \left( \frac{1}{1 + 1/e} \right)^{e+1}
  \]

  – If \( e = -2 \), this ratio is \( \frac{1}{2} \)
    - CS under monopoly is half of that under perfectly competitive markets
Welfare Loss of Monopoly

• **Example** (continued):
  
  – The ratio \( \frac{CS_m}{CS} = \left( \frac{1}{1+1/e} \right)^{e+1} \) decreases as demand becomes more elastic.
**Welfare Loss of Monopoly**

- **Example** (continued):
  - Monopoly profits are given by
    \[
    \pi^m = p^m q^m - c q^m = \left( \frac{c}{1 + 1/e} - c \right) q^m
    \]
    where \( q^m(p) = p^e = \left( \frac{c}{1+1/e} \right)^e \).
  - Re-arranging,
    \[
    \pi^m = \left( \frac{-c/e}{1 + 1/e} \right) \left( \frac{c}{1 + 1/e} \right)^e
    \]
    \[
    = - \left( \frac{c}{1 + 1/e} \right)^{e+1} \cdot \frac{1}{e}
    \]
Welfare Loss of Monopoly

• **Example** (continued):
  – To find the transfer from CS into monopoly profits that consumers experience when moving from a perfectly competition to a monopoly, divide monopoly profits by the competitive CS

\[
\frac{\pi^m}{CS} = \left(\frac{e+1}{e}\right) \left(\frac{1}{1+1/e}\right)^{e+1} = \left(\frac{e}{1+e}\right)^e
\]

– If \( e = -2 \), this ratio is \( \frac{1}{4} \)
  ▪ One fourth of the consumer surplus under perfectly competitive markets is transferred to monopoly profits
Welfare Loss of Monopoly

• More social costs of monopoly:
  – Excessive R&D expenditure (patent race)
  – Persuasive (not informative) advertising
  – Lobbying costs (different from bribes)
  – Resources to avoid entry of potential firms in the industry
Comparative Statics
Comparative Statics

• We want to understand how $q^m$ varies as a function of monopolist’s marginal cost
Comparative Statics

• Formally, we know that at the optimum, $q^m(c)$, the monopolist maximizes its profits

$$\frac{\partial \pi(q^m(c), c)}{\partial q^m} = 0$$

• Differentiating wrt $c$, and using the chain rule,

$$\frac{\partial^2 \pi(q^m(c), c)}{\partial q^2} \frac{dq^m(c)}{dc} + \frac{\partial^2 \pi(q^m(c), c)}{\partial q \partial c} = 0$$

• Solving for $\frac{dq^m(c)}{dc}$, we have

$$\frac{dq^m(c)}{dc} = -\frac{\frac{\partial^2 \pi(q^m(c), c)}{\partial q \partial c}}{\frac{\partial^2 \pi(q^m(c), c)}{\partial q^2}}$$
Comparative Statics

• **Example:**
  
  – Assume linear demand curve \( p(q) = a - bq \)
  
  – Then, the cross-derivative is

\[
\frac{\partial^2 \pi(q^m(c), c)}{\partial q \partial c} = \frac{\partial}{\partial c} \left( \frac{\partial \left[ (a - bq)q - cq \right]}{\partial q} \right)
\]

\[
= \frac{\partial}{\partial c} [a - 2bq - c] = -1
\]

and

\[
\frac{dq^m(c)}{dc} = -\frac{\frac{\partial^2 \pi(q^m(c), c)}{\partial q \partial c}}{\frac{\partial^2 \pi(q^m(c), c)}{\partial q^2}} = -\frac{-1}{-2b} < 0
\]
Comparative Statics

• **Example** (continued):
  – That is, an increase in marginal cost, \( c \), decreases monopoly output, \( q^m \).
  – Similarly for any other demand.
  – Even if we don’t know the accurate demand function, but know the sign of

\[
\frac{\partial^2 \pi(q^m(c), c)}{\partial q \partial c}
\]
Comparative Statics

- **Example** (continued):
  - Marginal costs are increasing in $q$
  - For convex cost curve $c(q) = cq^2$, monopoly output is
    \[
    q^m(c) = \frac{a}{2(b + c)}
    \]
  - Here,
    \[
    \frac{dq^m(c)}{dc} = -\frac{a}{2(b + c)^2} < 0
    \]
Comparative Statics

• **Example** (continued):

  – Constant marginal cost
  – For the constant-elasticity demand curve \( q(p) = p^e \), we have \( p^m = \frac{c}{1+1/e} \) and

  \[
  q^m(c) = \left( \frac{ec}{1+e} \right)^e
  \]

  – Here,

  \[
  \frac{dq^m(c)}{dc} = \frac{e}{c} \left( \frac{ec}{1+e} \right)^e
  \]

  \[
  = \frac{e}{c} q^m < 0
  \]
Multiplant Monopolist
Multiplant Monopolist

- Monopolist produces output $q_1, q_2, \ldots, q_N$ across $N$ plants it operates, with total costs $TC_i(q_i)$ at each plant $i = \{1,2, \ldots, N\}$.

- Profits-maximization problem

\[
\max_{q_1, \ldots, q_N} [a - b \sum_{i=1}^N q_i] \sum_{i=1}^N q_i - \sum_{i=1}^N TC_i(q_i)
\]

- FOCs wrt production level at every plant $j$

\[
a - 2b \sum_{i=1}^N q_i - MC_j(q_j) = 0
\]

\[\iff MR(Q) = MC_j(q_j)
\]

for all $j$. 

Advanced Microeconomic Theory
Multiplant Monopolist

- Multiplant monopolist operating two plants with marginal costs $MC_1$ and $MC_2$. 

![Diagram showing a multiplant monopolist with marginal cost curves $MC_1$ and $MC_2$, and aggregate marginal cost $MC_{Total}$, with unique price $p^m$, and quantities $q_1$, $q_2$, and $Q_{Total}$.](image)
Multiplant Monopolist

• Total marginal cost is $MC_{total} = MC_1 + MC_2$ (i.e., horizontal sum)
• $Q_{total}$ is determined by $MR = MC_{total}$ (i.e., point A)
• Mapping $Q_{total}$ in the demand curve, we obtain price $p^m$ (both plants sell at the same price)
• At the MC level for which $MR = MC_{total}$ (i.e., point A), extend a line to the left crossing $MC_1$ and $MC_2$.
• This will give us output levels $q_1$ and $q_2$ that plants 1 and 2 produce, respectively.
Multiplant Monopolist

• **Example 1** (symmetric plants):
  
  – Consider a monopolist operating $N$ plants, where all plants have the *same* cost function $TC_i(q_i) = F + c q_i^2$. Hence, all plants produce the same output level $q_1 = q_2 = \cdots = q_N = q$ and $Q = Nq_j$. The linear demand function is given by $p = a - bQ$.

  – FOCs:
    
    $$a - 2b \sum_{j=1}^{N} q_j = 2cq_j \quad \text{or} \quad a - 2bNq_j = 2cq_j$$
    
    $$q_j = \frac{a}{2(bN + c)}$$
Multiplant Monopolist

• **Example 1** (continued):
  
  – Total output produced by the monopolist is
    
    $$Q = Nq_j = \frac{Na}{2(bN + c)}$$
    
    and market price is
    
    $$p = a - bQ = a - b \frac{Na}{2(bN + c)} = \frac{a(bN + 2c)}{2(bN + c)}$$
    
    – Hence, the profits of every plant $j$ are
      
      $$\pi_j = \frac{a^2}{4(bN+c)} - F,$$
      
    with total profits of
    
    $$\pi_{total} = \frac{Na^2}{4(bN + c)} - NF$$
Multiplant Monopolist

• **Example 1** (continued):

  – The optimal number of plants $N^*$ is determined by

  \[
  \frac{d\pi_{\text{total}}}{dN} = \frac{a^2}{4} \frac{c}{(bN + c)^2} - F = 0
  \]

  and solving for $N$

  \[
  N^* = \frac{1}{b} \left( \frac{a}{2} \sqrt{\frac{c}{F}} - c \right)
  \]

  – $N^*$ is decreasing in the fixed costs $F$, and also decreasing in $c$, as long as $a < 4\sqrt{cF}$.
Multiplant Monopolist

• **Example 1** (continued):
  
  – Note that when $N = 1$, $Q = q^m$ and $p = p^m$.
  
  – Note that an increase in $N$ decreases $q_j$ and $\pi_j$. 
Multiplant Monopolist

**Example 2** (asymmetric plants):

– Consider a monopolist operating two plants with marginal costs $MC_1(q_1) = 10 + 20q_1$ and $MC_2(q_2) = 60 + 5q_2$, respectively. A linear demand function is given by $p(Q) = 120 - 3Q$.

– Note that $MC_{total} \neq MC_1(q_1) + MC_2(q_2)$
  - This is a vertical (not a horizontal) sum.

– Instead, first invert the marginal cost functions

\[
MC_1(q_1) = 10 + 20q_1 \iff q_1 = \frac{MC_1}{20} - \frac{1}{2}
\]

\[
MC_2(q_2) = 60 + 5q_2 \iff q_2 = \frac{MC_2}{5} - 12
\]
Multiplant Monopolist

• **Example 2** (continued):
  – Second,

\[
Q_{\text{total}} = q_1 + q_2 = \frac{MC_{\text{total}}}{20} - \frac{1}{2} + \frac{MC_{\text{total}}}{5} - 12
\]

\[
= \frac{1}{4}MC_{\text{total}} - 12.5
\]

– Hence, \( MC_{\text{total}} = 50 + 4Q_{\text{total}} \)

– Setting \( MR(Q) = MC_{\text{total}} \), we obtain \( Q_{\text{total}} = 7 \) and 
  \( p = 120 - 3 \cdot 7 = 99 \).

– Since \( MR(Q_{\text{total}}) = 120 - 6 \cdot 7 = 78 \), then
  \[
  MR(Q_{\text{total}}) = MC_1(q_1) \Rightarrow 78 = 10 + 20q_1 \Rightarrow q_1 = 3.4
  \]
  \[
  MR(Q_{\text{total}}) = MC_2(q_2) \Rightarrow 78 = 60 + 5q_2 \Rightarrow q_2 = 3.6
  \]
Price Discrimination
Price Discrimination

• Can the monopolist capture an even larger surplus?
  
  – Charge \( p > p^m \) to those who buy the product at \( p^m \) and are willing to pay more
  
  – Charge \( c < p < p^m \) to those who do not buy the product at \( p^m \), but whose willingness to pay for the good is still higher than the marginal cost of production, \( c \).
  
  – With \( p^m \) for all units, the monopolist does not capture the surplus of neither of these segments.
Price Discrimination: First-degree

- **First-degree (perfect) price discrimination:**
  - The monopolist charges to every customer his/her maximum willingness to pay for the object.

  - *Personalized price:* The first buyer pays $p_1$ for the $q_1$ units, the second buyer pays $p_2$ for $q_2 - q_1$ units, etc.
Price Discrimination: First-degree

- The monopolist continues doing so until the last buyer is willing to pay the marginal cost of production.

- In the limit, the monopolist captures all the area below the demand curve and above the marginal cost (i.e., consumer surplus)
Price Discrimination: First-degree

- Suppose that the monopolist can offer a fixed fee, $r^*$, and an amount of the good, $q^*$, that maximizes profits.
- PMP:
  \[
  \max_{r,q} \quad r - cq \\
  \text{s.t. } u(q) \geq r
  \]
- Note that the monopolist raises the fee $r$ until $u(q) = r$. Hence we can reduce the set of choice variables
  \[
  \max_q \quad u(q) - cq
  \]
- FOC: $u'(q^*) - c = 0$ or $u'(q^*) = c$.
  - Intuition: monopolist increases output until the marginal utility that consumers obtain from additional units coincides with the marginal cost of production.
Price Discrimination: First-degree

• Given the level of production $q^*$, the optimal fee is $r^* = u(q^*)$

• *Intuition*: the monopolist charges a fee $r^*$ that coincides with the utility that the consumer obtains from $q^*$
Price Discrimination: First-degree

• **Example:**
  
  – A monopolist faces inverse demand curve \( p(q) = 20 - q \) and constant marginal costs \( c = \$2 \).
  
  – No price discrimination:
    \[
    MR = MC \implies 20 - 2q = 2 \implies q^m = 9
    \]
    \[
    p^m = \$11, \quad \pi^m = \$81
    \]
  
  – Price discrimination:
    \[
    p(Q) = MC \implies 20 - Q = 2 \implies Q = 18
    \]
    \[
    \pi = \$162
    \]
Price Discrimination: First-degree

- **Example** (continued):
Price Discrimination: First-degree

• Summary:
  – Total output coincides with that in perfect competition
  – Unlike in perfect competition, the consumer does not capture any surplus
  – The producer captures all the surplus
  – Due to information requirements, we do not see many examples of it in real applications
    ▪ Financial aid in undergraduate education (“tuition discrimination”)
Price Discrimination: First-degree

• **Example** (two-block pricing):
  
  – A monopolist faces a inverse demand curve $p(q) = a - bq$, with constant marginal costs $c < a$.
  
  – Under two-block pricing, the monopolist sells the first $q_1$ units at a price $p(q_1) = p_1$ and the remaining $q_2 - q_1$ units at a price $p(q_2) = p_2$. 

![Diagram of inverse demand curve and two-block pricing](diagram.png)
Price Discrimination: First-degree

- **Example** (continued):
  - Profits from the first \( q_1 \) units
    \[
    \pi_1 = p(q_1) \cdot q_1 - cq_1 = (a - bq_1 - c)q_1
    \]
  
  while from the remaining \( q_2 - q_1 \) units
  \[
  \pi_2 = p(q_2) \cdot (q_2 - q_1) - c \cdot (q_2 - q_1)
  = (a - bq_2 - c)(q_2 - q_1)
  \]
  
  - Hence total profits are
    \[
    \pi = \pi_1 + \pi_2
    = (a - bq_1 - c)q_1 + (a - bq_2 - c)(q_2 - q_1)
    \]
Price Discrimination: First-degree

- **Example** (continued):
  - FOCs:
    \[
    \frac{\partial \pi}{\partial q_1} = a - 2bq_1 - c - a + bq_2 + c = 0
    \]
    \[
    \frac{\partial \pi}{\partial q_2} = -b(q_2 - q_1) + a - bq_2 - c = 0
    \]
  - Solving for \( q_1 \) and \( q_2 \)
    \[
    q_1 = \frac{a - c}{3b} \quad q_2 = \frac{2(a - c)}{3b}
    \]
    which entails prices of
    \[
    p(q_1) = a - b \cdot \frac{a - c}{3b} = \frac{2a + c}{3} \quad p(q_2) = \frac{a + 2c}{3}
    \]
    where \( p(q_1) > p(q_2) \) since \( a > c \).
Price Discrimination: First-degree

- **Example** (continued):
  - The monopolist’s profits from each block are
    
    \[ \pi_1 = (p(q_1) - c) \cdot q_1 \]
    
    \[ = \left( \frac{2a + c}{3} - c \right) \cdot \frac{a - c}{3b} = \frac{2}{b} \left( \frac{a - c}{3} \right)^2 \]
    
    \[ \pi_2 = (p(q_2) - c)(q_2 - q_1) \]
    
    \[ = \left( \frac{a + 2c}{3} - c \right) \cdot \left( \frac{2(a - c)}{3b} - \frac{a - c}{3b} \right) = \frac{1}{b} \left( \frac{a - c}{3} \right)^2 \]
    
    - Thus, \( \pi = \pi_1 + \pi_2 = \frac{(a-c)^2}{3b} \), which is larger than those arising under uniform pricing, \( \pi^u = \frac{(a-c)^2}{4b} \).
Price Discrimination: Third-degree

- **Third degree price discrimination:**
  - The monopolist charges different prices to two or more groups of customers (each group must be easily recognized by the seller).
  - *Example:* youth vs. adult at the movies, airline tickets
  - Firm’s PMP:
    \[
    \max_{x_1, x_2} p_1(x_1)x_1 + p_2(x_2)x_2 - cx_1 - cx_2
    \]
  - FOCs:
    \[
    p_1(x_1) + p_1'(x_1)x_1 - c = 0 \quad \Rightarrow \quad MR_1 = MC
    \]
    \[
    p_2(x_2) + p_2'(x_2)x_2 - c = 0 \quad \Rightarrow \quad MR_2 = MC
    \]
  - FOCs coincides with those of a regular monopolist who serves two completely separated markets practicing uniform pricing.
Price Discrimination: Third-degree

- Example: \( p_1(x_1) = 38 - x_1 \) for adults and \( p_2(x_2) = 14 - \frac{1}{4}x_2 \) for seniors, with \( MC = $10 \) for both markets.

\[
MR_1(x_1) = MC \implies 38 - x_1 = 10 \implies x_1 = 14 \quad p_1 = $24
\]

\[
MR_2(x_2) = MC \implies 14 - \frac{1}{4}x_2 = 10 \implies x_2 = 8 \quad p_2 = $12
\]
Price Discrimination: Third-degree

- Using the Inverse Elasticity Pricing Rule (IERP), we can obtain the prices

\[ p_1(x_1) = \frac{c}{1 - 1/\varepsilon_1} \quad \text{and} \quad p_2(x_2) = \frac{c}{1 - 1/\varepsilon_2} \]

where \( c \) is the common marginal cost.

- Then, \( p_1(x_1) > p_2(x_2) \) if and only if

\[ \frac{c}{1 - 1/\varepsilon_1} > \frac{c}{1 - 1/\varepsilon_2} \implies 1 - \frac{1}{\varepsilon_2} < 1 - \frac{1}{\varepsilon_1} \]

\[ \frac{1}{\varepsilon_2} > \frac{1}{\varepsilon_1} \implies \varepsilon_2 < \varepsilon_1 \]

- Intuition: the monopolist charges lower price in the market with more elastic demand.
Price Discrimination: Third-degree

- **Example** (Pullman-Seattle route):
  - The price-elasticity of demand for business-class seats is -1.15, while that for economy seats is -1.52
  - From the IEPR,
    \[
    p_B = \frac{MC}{1 - 1/1.15} \implies 0.13p_B = MC
    \]
    \[
    p_E = \frac{MC}{1 - 1/1.52} \implies 0.34p_E = MC
    \]
  - Hence, \(0.13p_B = 0.34p_E\) or \(p_B = 2.63p_E\)
    - Airline maximizes its profits by charging business-class seats a price 2.63 times higher than that of economy-class seats
Price Discrimination: Second-degree

• **Second-degree price discrimination:**
  
  – The monopolist cannot observe the type of each consumer (e.g., his willingness to pay).
  
  – Hence the monopolist offers a menu of two-part tariffs, \((F_L, q_L)\) and \((F_H, q_H)\), with the property that the consumer with type \(i = \{L, H\}\) has the incentive to self-select the two-part tariff \((F_i, q_i)\) meant for him.
Price Discrimination: Second-degree

• Assume the utility function of type $i$ consumer

\[ U_i(q_i, F_i) = \theta_i u(q_i) - F_i \]

where

- $q_i$ is the quantity of a good consumed
- $F_i$ is the fixed fee paid to the monopolist for $q_i$
- $\theta_i$ measures the valuation consumer assigns to the good, where $\theta_H > \theta_L$, with corresponding probabilities $p$ and $1 - p$.

• The monopolist’s constant marginal cost $c$ satisfies $\theta_i > c$ for all $i = \{L, H\}$. 
Price Discrimination: Second-degree

• The monopolist must guarantee that
  1) both types of customers are willing to participate ("participation constraint")
     ▪ the two-part tariff meant for each type of customer provides him with a weakly positive utility level
  2) customers do not have incentives to choose the two-part tariff meant for the other type of customer ("incentive compatibility")
     ▪ type $i$ customer prefers $(F_i, q_i)$ over $(F_j, q_j)$ where $j \neq i$
Price Discrimination: Second-degree

• The participation constraints (PC) are
  \[ \theta_L u(q_L) - F_L \geq 0 \]
  \[ \theta_H u(q_H) - F_H \geq 0 \]
  \[ PC_L \]
  \[ PC_H \]

• The incentive compatibility conditions are
  \[ \theta_L u(q_L) - F_L \geq \theta_L u(q_H) - F_H \]
  \[ IC_L \]
  \[ \theta_H u(q_H) - F_H \geq \theta_H u(q_L) - F_L \]
  \[ IC_H \]
Price Discrimination: Second-degree

• Re-arranging the four inequalities, the monopolist’s profit maximization problem becomes:

$$\max_{F_L, q_L, F_H, q_H} \quad p[F_H - cq_H] + (1 - p)[F_L - cq_L]$$

$$\theta_L u(q_L) \geq F_L$$

$$\theta_H u(q_H) \geq F_H$$

$$\theta_L [u(q_L) - u(q_H)] + F_H \geq F_L$$

$$\theta_H [u(q_H) - u(q_L)] + F_L \geq F_H$$
Price Discrimination: Second-degree

- Both $PC_H$ and $IC_H$ are expressed in terms of the fee $F_H$
  - The monopolist increases $F_H$ until such fee coincides with the lowest of $\theta_H u(q_H)$ and $\theta_H [u(q_H) − u(q_L)] + F_L$ for all $i = \{L, H\}$
  - Otherwise, one (or both) constraints will be violated, leading the high-demand customer to not participate
Price Discrimination: Second-degree

\[ P_C_i \text{ is binding} \]

Maximal \( F_i \) that achieves participation and self-selection

\[ \theta_i u(q_i) \]

\[ \theta_i [u(q_i) - u(q_j)] + F_j \]

\[ F_i \]

\[ IC_i \text{ is binding} \]

Maximal \( F_i \) that achieves participation and self-selection

\[ \theta_i [u(q_i) - u(q_j)] + F_j \]

\[ \theta_i u(q_i) \]

\[ F_i \]
Price Discrimination: Second-degree

- **High-demand customer:**
  - Let us show that $IC_H$ is binding
  - An indirect way to show that
    \[ F_H = \theta_H [u(q_H) - u(q_L)] + F_L \]
    is to demonstrate that $F_H < \theta_H u(q_H)$
  - Proving this by contradiction, assume that
  \[ F_H = \theta_H u(q_H) \]
Price Discrimination: Second-degree

– Then, \( IC_H \) can be written as

\[
F_H - \theta_H u(q_L) + F_L \geq F_H
\]

\[
\Rightarrow F_L \geq \theta_H u(q_L)
\]

– Combining this result with the fact that \( \theta_H > \theta_L \),

\[
F_L \geq \theta_H u(q_L) > \theta_L u(q_L)
\]

which implies \( F_L > \theta_L u(q_L) \)

– However, this violates \( PC_L \)

  - We then reached a contradiction
  - Thus, \( F_H < \theta_H u(q_H) \)
  - \( IC_H \) is binding but \( PC_H \) is not.
Price Discrimination: Second-degree

- **Low-demand customer:**
  - Let us show that \( PC_L \) binding
  - Similarly as for the high-demand customer, an indirect way to show that
    \[
    F_L = \theta_L u(q_L)
    \]
    is to demonstrate that
    \[
    F_L < \theta_L [u(q_L) - u(q_H)] + F_H
    \]
  - Proving this by contradiction, assume that
    \[
    F_L = \theta_L [u(q_L) - u(q_H)] + F_H
    \]
– Then, $IC_H$ can be written as

$$\theta_H [u(q_H) - u(q_L)] + \theta_L [u(q_L) - u(q_H)] + F_H = F_H$$

$$\Rightarrow \theta_H [u(q_H) - u(q_L)] = \theta_L [u(q_L) - u(q_H)]$$

$$\Rightarrow \theta_H = \theta_L$$

which violates the initial assumption $\theta_H > \theta_L$

- We reached a contradiction
- Thus, $F_L < \theta_L [u(q_L) - u(q_H)] + F_H$
- $PC_L$ is binding but $IC_L$ is not
Price Discrimination: Second-degree

• In summary:
  – From $PC_L$ binding we have
    \[ \theta_L u(q_L) = F_L \]
  – From $IC_H$ binding we have
    \[ \theta_H [u(q_H) - u(q_L)] + F_L = F_H \]
  – In addition,
    • $PC_L$ binding implies that $IC_L$ holds, and
    • $IC_H$ binding entails that $PC_H$ is also satisfied,
    • That is, all four constraints hold.
Price Discrimination: Second-degree

- The monopolist’s expected PMP can then be written as unconstrained problem, as follows,

\[
\max_{q_L, q_H \geq 0} \quad p \left[ F_H - c q_H \right] + (1 - p) \left[ F_L - c q_L \right] \\
= p \left\{ \theta_H \left[ u(q_H) - u(q_L) \right] + \frac{F_L - c q_H}{F_H} \right\} \\
\phantom{= p \left\{ \theta_H \left[ u(q_H) - u(q_L) \right] + \frac{F_L - c q_H}{F_H} \right\}} + (1 - p) \left\{ \theta_L u(q_L) - c q_L \right\} \\
= p \left\{ \theta_H \left[ u(q_H) - u(q_L) \right] + \frac{\theta_L u(q_L) - c q_L}{F_L} \right\} \\
\phantom{= p \left\{ \theta_H \left[ u(q_H) - u(q_L) \right] + \frac{\theta_L u(q_L) - c q_L}{F_L} \right\}} + (1 - p) \left\{ \theta_L u(q_L) - c q_L \right\} \\
= p \left[ \theta_H u(q_H) - (\theta_H - \theta_L) u(q_L) - c q_H \right] \\
\phantom{= p \left[ \theta_H u(q_H) - (\theta_H - \theta_L) u(q_L) - c q_H \right]} + (1 - p) \left[ \theta_L u(q_L) - c q_L \right]
\]
Price Discrimination: Second-degree

• FOC with respect to $q_H$:
  
  \[ p[\theta_H u'(q_H) - c] = 0 \implies \theta_H u'(q_H) = c \]
  
  – which coincides with that under complete information.
  – That is, there is not output distortion for high-demand buyer
  – Informally, we say that there is “no distortion at the top”.

• FOC with respect to $q_L$:
  
  \[ p(-(\theta_H - \theta_L)u'(q_L)) + (1 - p)[\theta_L u'(q_L) - c] = 0 \]
  
  which can be re-written as
  
  \[ u'(q_L)[\theta_L - p\theta_H] = (1 - p)c \]
Price Discrimination: Second-degree

• Dividing both sides by \((1 − p)\), we obtain

\[ u'(q_L) \left[ \frac{\theta_L - \theta_H p}{1 - p} \right] = c \]

• The above expression can alternatively be written as

\[ u'(q_L) \left[ \theta_L - \frac{p}{1 - p} (\theta_H - \theta_L) \right] = c \]
Price Discrimination: Second-degree

- $u'(q_L) \cdot \theta_L$ depicts the socially optimal output $q_L^{SO}$, i.e., that arising under complete information
- The output offered to high-demand customers is socially efficient due to the absence of output distortion for high-type agents
- The output offered to low-demand customers entails a distortion, i.e., $q_L < q_L^{SO}$
- Per-unit price for high-type and low-type differs, i.e., $F_H \neq F_L$
  - Monopolist practices price discrimination among the two types of customers.

Advanced Microeconomic Theory
Price Discrimination: Second-degree

• Since constraint $PC_L$ binds while $PC_H$ does not, then only the high-demand customer retains a positive utility level, i.e., $\theta_H u(q_H) - F_H > 0$.

• The firm’s lack of information provides the high-demand customer with an “information rent.”
  – Intuitively, the information rent emerges from the seller’s attempt to reduce the incentives of the high-type customer to select the contract meant for the low type.
  – The seller also achieves self-selection by setting an attractive output for the low-type buyer, i.e., $q_L$ is lower than under complete information.
Price Discrimination: Second-degree

• **Example:**
  
  – Consider a monopolist selling a textbook to two types of graduate students, low- and high-demand, with utility function

  \[ U_i(q_i, F_i) = \frac{q_i^2}{2} - \theta_i q_i - F_i \]

  where \( i = \{L, H\} \) and \( \theta_H > \theta_L \).

  – Hence, the UMP of student type \( i \) is

  \[
  \max_{q_i} \frac{q_i^2}{2} - \theta_i q_i - F_i \quad \text{s.t.} \quad pq_i + F_i \leq w_i
  \]

  where \( w_i > 0 \) denotes the student’s wealth.
Price Discrimination: Second-degree

**Example** (continued):

– By Walras’ law, the constraint binds

\[ F_i = w_i - p q_i \]

– Then, the UMP can be expressed as

\[ \max_{q_i} \frac{q_i^2}{2} - \theta_i q_i - (w_i - p q_i) \]

– FOCs wrt \( q_i \) yields the direct demand function:

\[ q_i - \theta_i - p = 0 \quad \text{or} \quad q_i = \theta_i - p \]
Price Discrimination: Second-degree

• **Example** (continued):
  
  – Assume that the proportion of high-demand (low-demand) students is $\gamma$ ($1 - \gamma$, respectively).

  – The monopolist’s constant marginal cost is $c > 0$, which satisfies $\theta_i > c$ for all $i = \{L, H\}$.

  – Consider for simplicity that $\theta_L > \frac{\theta_H + c}{2}$.

  – This implies that each type of student would buy the textbook, both when the firm practices uniform pricing and when it sets two-part tariffs

    ▪ Exercise.
Advertising in Monopoly
Advertising in Monopoly

• **Advertising**: non-price strategy to capture surplus

• The monopolist must balance the additional demand that advertising entails and its associated costs ($A$ dollars)

• The monopolist solves

$$\max_A \ p \cdot q(p, A) - TC(q(p, A)) - A$$

where the demand function $q(p, A)$ depends on price and advertising.
Advertising in Monopoly

• Taking FOCs with respect to $A$,

$$p \cdot \frac{\partial q(p,A)}{\partial A} - \underbrace{\frac{\partial TC}{\partial q}}_{MC} \cdot \frac{\partial q(p,A)}{\partial A} - 1 = 0$$

Rearranging, we obtain

$$(p - MC) \frac{\partial q(p,A)}{\partial A} = 1$$

• Let us define the advertising elasticity of demand

$$\varepsilon_{q,A} = \frac{\% \text{ increase in } q}{\% \text{ increase in } A} = \frac{\partial q(p,A)}{\partial A} \cdot \frac{A}{q}$$

Or, rearranging,

$$\varepsilon_{q,A} \cdot \frac{q}{A} = \frac{\partial q(p,A)}{\partial A}$$
Advertising in Monopoly

• We can then rewrite the above FOC as

\[
(p - MC) \varepsilon_{q,A} \cdot \frac{q}{A} = 1
\]

\[
\frac{\partial q(p,A)}{\partial A}
\]

• Dividing both sides by \( \varepsilon_{q,A} \) and rearranging

\[
p - MC = \frac{1}{\varepsilon_{q,A}} \cdot \frac{A}{q}
\]

• Dividing both sides by \( p \)

\[
\frac{p - MC}{p} = \frac{1}{\varepsilon_{q,A}} \cdot \frac{A}{p \cdot q}
\]
Advertising in Monopoly

• From the Lerner index, we know that\[ \frac{p - MC}{p} = - \frac{1}{\varepsilon_{q,p}}. \]

Hence,

\[ - \frac{1}{\varepsilon_{q,p}} = \frac{1}{\varepsilon_{q,A}} \cdot \frac{A}{p \cdot q}. \]

• And rearranging

\[ - \frac{\varepsilon_{q,A}}{\varepsilon_{q,p}} = \frac{A}{p \cdot q}. \]

– The right-hand side represents the advertising-to-sales ratio.

– For two markets with the same \( \varepsilon_{q,p} \), the advertising-to-sales ratio must be larger in the market where demand is more sensible to advertising (higher \( \varepsilon_{q,A} \)).
Advertising in Monopoly

• **Example:**
  
  – If the price-elasticity in a given monopoly market is $\varepsilon_{q,p} = -1.5$ and the advertising-elasticity is $\varepsilon_{q,A} = 0.1$, the advertising-to-sales ratio should be

  \[
  \frac{A}{p \cdot q} = -\frac{0.1}{-1.5} = 0.067
  \]

  – Advertising should account for 6.7% of this monopolist’s revenue.
Regulation of Natural Monopolies
Regulation of Natural Monopolies

• **Natural monopolies**: Monopolies that exhibit *decreasing* cost structures, with the MC curve lying below the AC curve.

• Hence, having a single firm serving the entire market is cheaper than having multiple firms, as aggregate average costs for the entire industry would be lower.
Regulation of Natural Monopolies

• Unregulated natural monopolist maximizes profits at the point where MR=MC, producing $Q_1$ units and selling them at a price $p_1$.

• Regulated natural monopolist will charge $p_2$ (where demand crosses MC) and produce $Q_2$ units.

• The production level $Q_2$ implies a loss of $p_2 - c_2$ per unit.
Regulation of Natural Monopolies

• Dilemma with natural monopolies:
  – abandon the policy of setting prices equal to marginal cost, OR
  – continue applying marginal cost pricing but subsidize the monopolist for his losses

• Solution to the dilemma:
  – A multi-price system that allows for price discrimination
  – Charging some users a high price while maintaining a low price to other users
Regulation of Natural Monopolies

- Multi-price system:
  - a high price $p_1$
  - a low price $p_2$

- **Benefit**: $(p_1 - c_1)$ per unit in the interval from 0 to $q_1$

- **Loss**: $(c_2 - p_2)$ per unit in the interval $(q_2 - q_1)$

- The monopolist price discriminates iff
  $$ (p_1 - c_1)q_1 > (c_2 - p_2)(q_2 - q_1) $$
Regulation of Natural Monopolies

• An alternative regulation:
  – allow the monopolist to charge a price above marginal cost that is sufficient to earn a “fair” rate of return on capital investments

• Two difficulties:
  – what is a “fair” rate of return
  – overcapitalization
Regulation of Natural Monopolies

• Overcapitalization of natural monopolies:
  – Suppose a production function of the form \( q = f(k, l) \). An unregulated monopoly with profit function \( pf(k, l) - wl - rk \) has a rate of return on capital, \( r \). Suppose furthermore that the rate of return on capital investments, \( r \), is constrained by a regulatory agency to be equal to \( r_0 \).
Regulation of Natural Monopolies

- **PMP:**
  
  \[ L = pf(k, l) - wl - rk + \lambda[wl + r_0k - pf(k, l)] \]

  where \( 0 < \lambda < 1 \).

- **FOCs:**
  
  \[ \frac{\partial L}{\partial l} = pf_l - w + \lambda(w - pf_l) = 0 \]

  \[ \frac{\partial L}{\partial k} = pf_k - r + \lambda(r_0 - pf_k) = 0 \]

  \[ \frac{\partial L}{\partial \lambda} = wl + r_0k - pf(k, l) = 0 \]
Regulation of Natural Monopolies

- From the first FOC:
  \[ pf_l = w \]

- From the second FOC:
  \[ (1 - \lambda)pf_k = r - \lambda r_0 \]
  and re-arranging
  \[ pf_k = \frac{r - \lambda r_0}{1 - \lambda} = r - \frac{\lambda(r_0 - r)}{1 - \lambda} \]

- Since \( r_0 > r \) and \( 0 < \lambda < 1 \), then \( pf_k < r \).
- Hence, the firm would hire *more capital* than under unregulated condition, where \( pf_k = r \).
Regulation of Natural Monopolies

- $pf_k$ is the value of the marginal product of capital
  - It is decreasing in $k$ (due to diminishing marginal return, i.e., $f_{kk} < 0$)
- $r$ and $r - \frac{\lambda(r_0 - r)}{1 - \lambda}$ are the marginal cost of additional units of capital in the unregulated and regulated, respectively, monopoly
  - $r > r - \frac{\lambda(r_0 - r)}{1 - \lambda}$
- Example: electricity and water suppliers
Regulation of Natural Monopolies

• An alternative illustration of the overcapitalization (*Averch-Johnson effect*)

• Before regulation, the firm selects \((L^{BR}, K^{BR})\)

• After regulation, the firm selects \((L^{AR}, K^{AR})\), where \(K^{AR} > K^{BR}\) but \(L^{AR} < L^{BR}\)

• The overcapitalization result only captures the substitution effect of a cheaper input.
  – Output effect?
Monopsony
Monopsony

• **Monopsony**: A single buyer of goods and services exercises “buying power” by paying prices below those that would prevail in a perfectly competitive context.

• Monopsony (single buyer) is analogous to that of a monopoly (single seller).

• *Examples*: a coal mine, Walmart Superstore in a small town, etc.
Monopsony

• Consider that the monopsony faces competition in the product market, where prices are given at $p > 0$, but is a monopsony in the input market (e.g., labor services).

• Assume an increasing and concave production function, i.e., $f'(x) > 0$ and $f''(x) \leq 0$.
  – This yields a total revenue of $pf(x)$.

• Consider a cost function $w(x) \cdot x$, where $w(x)$ denotes the inverse supply function of labor $x$.
  – Assume that $w'(x) > 0$ for all $x$.
  – This indicates that, as the firm hires more workers, labor becomes scarce, thus increasing the wages of additional workers.
Monopsony

• The monopsony PMP is

\[
\max_x pf(x) - w(x)x
\]

• FOC wrt the amount of labor services \( x \) yields

\[
pf'(x^*) - w(x^*) - w'(x^*)x^* = 0
\]

\[
\Rightarrow pf'(x^*) = w(x^*) + w'(x^*)x^*
\]

\[\begin{align*}
A &= w(x^*) \\
B &= w'(x^*)x^*
\end{align*}\]

– \( A \): “marginal revenue product” of labor.
– \( B \): “marginal expenditure” (ME) on labor.

- The additional worker entails a monetary outlay of \( w(x^*) \).
- Hiring more workers make labor become more scarce, ultimately forcing the monopsony to raise the prevailing wage on all inframarginal workers, as captured by \( w'(x^*)x^* \).
Monopsony

- Monopsonist hiring and salary decisions.
  - The marginal revenue product of labor, $pf'(x)$, is decreasing in $x$ given that $f''(x) \leq 0$.
  - The labor supply, $w(x)$, is increasing in $x$ since $w'(x) > 0$.
  - The marginal expenditure (ME) on labor lies above the supply function $w(x)$ since $w'(x) > 0$.
  - The monopsony hires $x^*$ workers at a salary of $w(x^*)$. 
Monopsony

• A deadweight loss from monopsony is

\[ DWL = \int_{x^*}^{x^{PC}} [pf'(x) - w(x)]dx \]

• That is, the area below the marginal revenue product and above the supply curve, between \( x^* \) and \( x^{PC} \) workers.
Monopsony

• We can write the monopsony profit-maximizing condition, i.e., \( pf'(x^*) = w(x^*) + w'(x^*)x^* \), in terms of labor supply elasticity, using the following steps:

\[
pf'(x^*) = w(x^*) + \frac{\partial w(x^*)}{\partial x^*} x^*
\]

\[
= w(x^*) \left( 1 + \frac{\partial w(x^*)}{\partial x^*} \frac{x^*}{w(x^*)} \right)
\]

• And rearranging,

\[
pf'(x^*) = w(x^*) \left( 1 + \frac{1}{\frac{\partial x^* w(x^*)}{\partial w} \frac{x^*}{w(x^*)}} \right)
\]
Monopsony

• Since \( \frac{\partial x^*}{\partial w} \frac{w(x^*)}{x^*} \) represents the elasticity of labor supply \( \varepsilon \), then

\[
p f'(x^*) = w(x^*) \left( 1 + \frac{1}{\varepsilon} \right)
\]

• Intuitively, as \( \varepsilon \to \infty \), the behavior of the monopsonist approaches that of a pure competitor.
Monopsony

• The equilibrium condition above is also sufficient as long as

\[ pf''(x^*) - 2w'(x^*) - w''(x^*)x^* < 0 \]

• Since \( f''(x^*) < 0 \), \( w'(x^*) > 0 \) (by assumption), we only need that either:
  a) the supply function is convex, i.e., \( w''(x^*) > 0 \); or
  b) if it is concave, i.e., \( w''(x^*) < 0 \), its concavity is not very strong, that is

\[ pf''(x^*) - 2w'(x^*) < w''(x^*)x^* \]
Monopsony

• **Example:**

  – Consider a monopsonist with production function \( f(x) = ax \), where \( a > 0 \), and facing a given market price \( p > 0 \) per unit of output.
  – Labor supply is \( w(x) = bx \), where \( b > 0 \).
  – The marginal revenue product of hiring an additional worker is
    \[
    pf'(x) = pa
    \]
  – The marginal expenditure on labor is
    \[
    w(x) + w'(x)x = bx + bx = 2bx
    \]
Monopsony

- **Example** (continued):
  - Setting them equal to each other, \( pa = 2bx^* \), yields a profit-maximizing amount of labor:
    \[
    x^* = \frac{ap}{2b}
    \]
  - \( x^* \) increases in the price of output, \( p \), and in the marginal productivity of labor, \( a \); but decreases in the slope of labor supply, \( b \).
  - Sufficiency holds since
    \[
    pf''(x^*) - 2w'(x^*) = p0 - 2b < 0 = w''(x^*)x^*
    \]