Applying Repeated Games to Labor Economics: Efficiency wages

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Games with an Unbounded Horizon

- In the repeated Prisoner's Dilemma, the extent to which a player can "punish" or "reward" their rival in any round is fixed and predetermined.
- What happens in repeated games in which this extent is modifiable?
  - How does this affect the set of equilibria in the game?
- We will examine this question in the following example.

Games with an Unbounded Horizon

- According to the competitive economics model, in a perfect and frictionless market, there should be no unemployment:
    - If the supply of labor is greater than the demand for labor on the part of employers, workers will be prepared to work even at a lower wage.
    - The process of decrease in salaries will continue until the demand for employees equals the supply of labor.

Games with an Unbounded Horizon

- In practice, however, even in competitive markets, unemployment levels typically do not fall below 4.5 percent.
  - One possible reason for such unemployment is the process of job search on the part of the unemployed, and the search for workers by potential employers.
  - Another possible cause for the existence of a minimal level of unemployment is related to the ongoing and repeated interaction between employers and employees.
    - This leads to two interesting economic problems in the short run $\longrightarrow$

Games with an Unbounded Horizon

- **The Principal-Agent problem:**
  - This arises if the cooperation between the employer and employee is short-lived.
  - i.e., when an employee doesn't invest sufficient effort into the work process, he makes the firm worse off since the firm has to pay the employee for at least one unit (hour, day, etc.) of labor before firing him.
- However, that gives rise to the **Hold-up problem:**
  - The employer would not have hired the employee in the first place. But not hiring that employee would make the business unsustainable.
  - The employer would estimate that even after the employee had been trained for the job, he would not invest the effort required of him in that role.

Efficiency Wage

- However, when the relationship between the employer and the employee is expected to be long-lasting, an opportunity for overcoming these problems may present itself.
- **Efficiency Wage**.
  - This is an incentive to exert effort on the job, given a certain wage.
  - This wage is high enough for the employee not to want to get himself fired, as he knows he would forego this wage while he searches for a new job.

Efficiency Wage

- Let's assume that the employee's discount factor is $\delta < 1$, and that e is his "cost of effort"
  - The employee would be prepared to exert effort only in consideration of a wage greater than or equal to e.
- If the employee's monthly wage is w, then the difference $w - e$ is the net utility to the employee from his work in a given month when he exerts effort.
  - If he exerts zero effort, yet still gets paid, his payoff is w.
  - If he exerts zero effort, and doesn't get paid (he called in sick every day), his payoff is 0.

Efficiency Wage

- For the employee to want to exert effort while on the job (earning a payoff of V_1), it must be that

$$V_1 \geq V_0$$

  where V_1 as the discounted utility to the employee when he exerts effort at work (so he is never fired)

$$V_1 = (w - e) + \delta(w - e) + \delta^2(w - e) + ... = \frac{w - e}{1 - \delta}$$

- Let's now describe V_0

Efficiency Wage

- V_0 denotes the utility that the employee obtains from never exerting effort. He is immediately fired after one period, then he searches for a new job during $m > 0$ periods (being unemployed with a zero payoff). Hence,

$$\begin{aligned} V_0 &= w + \underbrace{\delta 0 + \ldots + \delta^m 0}_{\text{unemployed}} + \delta^{m+1} w + \\ &\quad + \underbrace{\delta^{m+2} 0 + \ldots + \delta^{m+m} 0}_{\text{unemployed}} + \delta^{m+m+1} w + \ldots \\ &= w + \delta^{m+1} w + \delta^{2(m+1)} w + \ldots \\ &= w \left[1 + \delta^{m+1} + \delta^{2(m+1)} + \ldots \right] \\ &= w \sum_{k=0}^{\infty} \delta^{(m+1)k} = \frac{w}{1 - \delta^{m+1}} \end{aligned}$$

Efficiency Wage

When $V_1 \geq V_0$,

$$\frac{w_1 - e}{1 - \delta} \geq \frac{w_1}{1 - \delta^{m+1}}$$

and solving for w_1 yields

$$w_1 \geq e \frac{1 - \delta^{m+1}}{\delta - \delta^{m+1}} > e$$

which is larger than e since $1 - \delta^{m+1} > \delta - \delta^{m+1}$ given that $1 > \delta$.

Efficiency Wage

- The less patient that an employee is (smaller δ), the higher his minimal wage w_1 must be since the temptation to earn w_1 instantly (as opposed to $w_1 - e$) becomes greater.
- The greater the anticipated number of months of unemployment, m, makes the threat of being fired much greater,
  - As a consequence, the minimal wage w_1 that will cause the employee to exert effort decreases.

Efficiency Wage: SPNE

- Let's assume that every employer has a positive profit even if he pays every employee the wage w_1 and each of the employees exerts a positive amount of effort.
  - But the employer will lose money if he keeps employees on the payroll that do not exert positive levels of effort.
- We will now show that there is a SPNE at which every employee is prepared to make an effort to work only at a wage of at least w_1.
  - While every employer who is seeking staff offers work to unemployed persons coming to her for a job at a monthly wage of w_1, and does not fire them as long as they exert a positive amount of effort.

Efficiency Wage: SPNE

- Let's see if there are any incentives for at least one of the two parties to deviate.
- Any employer may deviate from her strategy at the beginning of any month by changing w_1.
- Such deviation, however, is suboptimal:
  - Since the employee will exert himself even at a salary of w_1, the employer will make less of a profit if she offers the employee a wage that is higher than w_1.
  - If the employer hires the employee at a wage that is lower than w_1, the employee will not make any effort, and it is possible that the employer will make a loss for every month that the employee works there.

Efficiency Wage: SPNE

- For the employee, we can assume the following two things based on the one-deviation principle:
  - The employee will prefer to exert positive effort if his montly wage, w is at least w_1, and also
  - The employee will prefer to exert zero effort if his monthly wage, w, is less than w_1.
- Under the assumption that in the future, he will revert to his original strategy (to make an effort as long as $w \geq w_1$) and that any offer of work that he receives in the future will be at a monthly wage of w_1.
- Let's examine these two possible deviations.

Efficiency Wage: SPNE

- **First deviation:** if the employee exerts zero effort despite the fact that $w \geq w_1$, his discounted payoff will be

$$\underbrace{w}_{\text{First Month}} + (w_1 - e) \sum_{k=m+2}^{\infty} \delta^{k-1} = w + \delta^{m+1} \frac{w_1 - e}{1 - \delta}$$

  - Where in the first month, he will enjoy a wage of w without making any effort. This will get him fired and he will spend m months unemployed.
  - In the month $m + 2$, a subgame will commence in which an employer hires him to work at a wage of w_1 and the employee exerts a positive level of effort.

Efficiency Wage: SPNE

- *Not deviating:* However, if he adheres to his original strategy and regularly makes an effort with the first employer, his discounted payoff will be

$$(w-e)\sum_{k=1}^{\infty}\delta^{k-1}=\frac{w-e}{1-\delta}$$

- This payoff is higher than the payoff from deviating, since

$$(w-e)\sum_{k=1}^{\infty}\delta^{k-1}=(w-e)\sum_{k=1}^{m+1}\delta^{k-1}+(w-e)\sum_{k=m+2}^{\infty}\delta^{k-1}$$

$$\geq(1-\delta^{m+1})\frac{w-w\frac{\delta-\delta^{m+1}}{1-\delta^{m+1}}}{1-\delta}+\delta^{m+1}\frac{w_1-e}{1-\delta}=w+\delta^{m+1}\frac{w_1-e}{1-\delta}$$

Efficiency Wage: SPNE

- **Second deviation:** if the employee makes an effort even when the wage w paid to him is smaller than w_1, $w < w_1$, he will not be fired.
- Under the assumption that neither the employer nor the employee will later deviate from their original strategies,
    - The employer will offer the employee a monthly wage of w_1 commencing from the following month, and the employee will regularly exert an effort. His total payoff will be

$$w - e + (w_1 - e) \sum_{k=2}^{\infty} \delta^{k-1} = w - e + \delta \frac{w_1 - e}{1 - \delta}$$

Efficiency Wage: SPNE

- *Not deviating:*
- In contrast, if the employee adheres to his original strategy, he will exert zero effort when $w < w_1$, and will be fired after one month of work, spending the next m months unemployed and looking to find a new job at wage w_1.
  - Therefore, his overall payoff will be:

$$w + (w_1 - e) \sum_{k=m+2}^{\infty} \delta^{k-1} = w + \delta^{m+1} \frac{w_1 - e}{1 - \delta}$$

Efficiency Wage: SPNE

- After some algebra, we can show that his payoff from deviating (which we found two slides ago) is equal to his payoff from adhering to his original strategy (found in the previous slide), that is:

$$w - e + \delta \frac{w_1 - e}{1 - \delta} = w + \delta^{m+1} \frac{w_1 - e}{1 - \delta}$$

- Therefore, the employee will not profit if he deviates from his strategy, according to which he exerts no effort when his wage w is less than w_1.
- Thus, this strategy is a SPNE.