Mixed and behavioral strategies in extensive form games

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EconS 503 - *Microeconomic Theory II*

Pure strategies in extensive form games

- A **pure strategy** in a extensive form game (i.e., a game tree) must be understood as a complete plan of action that specifies what player i should do at every node (or, more generally, for every information set) at which he can be called on to move.
- More formally,
  - A **pure strategy** for player i is a mapping $s_i : H_i \longrightarrow A_i$ that assigns an action $s_i(h_i) \in A_i(h_i)$ for every information set $h_i \in H_i$.

Mixed strategies in extensive form games

- A **mixed strategy** in a extensive form game is a probability distribution over all player i's complete plans of action, i.e., over all his pure strategies $s_i \in S_i$.

Behavioral strategies in extensive form games

- A **behavioral strategy** specifies, for every information set $h_i \in H_i$, an independent probability distribution over actions $A_i(h_i)$, denoted as

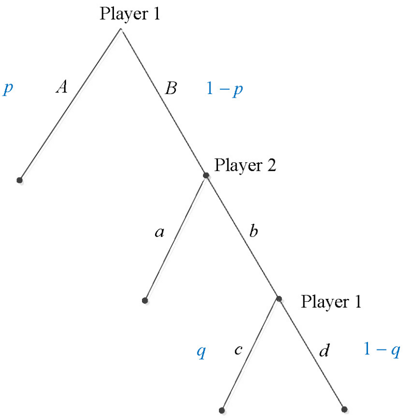
$$\sigma_i : H_i \rightarrow \Delta A_i(h_i)$$

where $\sigma_i\left(a_i\left(h_i\right)\right)$ is the probability that player i selects action $a_i(h_i) \in A_i(h_i)$ when he is at information set h_i.

Behavioral strategies in extensive form games

- *Example:*
  - Consider a game tree with:
    - Player 1 choosing between A and B.
    - If Player 1 chooses B, then Player 2 gets to choose between a and b.
    - If Player 2 chooses b, then Player 1 is called to choose between c and d.
  - **Strategy space for player 1:** $S_1 = \{Ac, Ad, Bc, Bd\}$.

Behavioral strategies in extensive form games



Behavioral strategies in extensive form games

- *Example (cont'd):*
  - A **mixed strategy** for Player 1 is a randomization among the four pure strategies described above,

$$S_1 = \{Ac, Ad, Bc, Bd\}$$

  - A **behavioral strategy** for Player 1:
    - At the first information set at which he is called on to move is a randomization between A and B, e.g., $pA + (1-p)B$
    - At the second information set at which he is called on to move is a randomization between c and d, e.g., $qc + (1-q)d$

Behavioral strategies in extensive form games

- *Example (cont'd):*
  - For instance, a specific **mixed strategy** for Player 1 is

$$\left(0Ac, 0Ad, \frac{1}{2}Bc, \frac{1}{2}Bd\right)$$

  - And a specific **behavioral strategy** for Player 1 is

$$p = \frac{1}{3} \quad \text{and} \quad q = \frac{1}{4}$$

  - Importantly, note that if $p = 0$ and $q = 1/2$, then the mixed and behavioral strategies are equivalent. Otherwise, they don't produce the same outcome.

Behavioral strategies in extensive form games

- *Luce and Raiffa's (1957) analogy:*
  - A **pure strategy** $s_i \in S_i$ can be understood as an instructions manual in which each page tells player i which action to choose when he is called on to move at information set h_i.
    - (Such instruction manual has as many pages as the number of information sets that player i has on the game tree.)
  - A **strategy space** S_i is then a library with all possible instruction manuals.

Behavioral strategies in extensive form games

- *Luce and Raiffa's (1957) analogy (cont'd):*
  - A **mixed strategy** randomly chooses a specific manual from the library S_i.
    - Hence, once the mixed strategy has picked a specific instructions manual, it sticks to it throughout the game.
  - A **behavioral strategy** chooses pages of different manuals with positive probability.
    - It does not necessarily stick to a unique instructions manual for the duration of the game.

Behavioral strategies in extensive form games

- We can easily construct a one-to-one correspondence between mixed and behavioral strategies.
  - That is, equivalently describe a mixed strategy with the use of a behavioral strategy, and vice versa.
  - See Tadelis (section 7.2.2) for examples.