Cheap Talk Games with three types

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So far, in all signaling games we considered...

- There were two types for the privately informed player,
  - e.g., high and low productivity, beneficial or useless test, etc.,
- and the privately informed player only had two possible messages to choose from.
  - e.g., acquire/not acquire college education, recommend/not recommend the MRI test, etc.

What if we extend our analysis to signaling games in which:

- the privately informed player has three possible types, and
- he/she has three available messages to choose from.
Example 2: Stock recommendations

Let us now examine the transmission of information between:

- An analyst who privately observes the future prospects of a stock, and
- An investor who does not observe such information.
Example 2: Stock recommendations

- We will assume that your analyst is none of these two guys, otherwise you wouldn’t be paying attention to his messages (since they are never informative!)
Example 2: Stock recommendations

- First, nature determines whether the stocks of a particular firm will:
  - outperform/underperform/be neutral to average stock market prices.

- This information is only observed by an analyst, after weeks of research, but not by the investor holding the stock.

- The analyst then decides to recommend buy/hold/sell to the investor.

- Finally, the investor observes the recommendation from his analyst, and decides whether to buy/hold/sell.

- Figure.→
Example 2: Stock recommendations

What about the payoffs?
Example 2: Stock recommendations

- Let’s first examine the investor’s payoff (last column)

<table>
<thead>
<tr>
<th>State</th>
<th>Action</th>
<th>Analyst’s Payoff</th>
<th>Investor’s Payoff</th>
</tr>
</thead>
<tbody>
<tr>
<td>Outperform</td>
<td>Buy</td>
<td>$a + 1$</td>
<td>$1$</td>
</tr>
<tr>
<td>Outperform</td>
<td>Hold</td>
<td>$0$</td>
<td>$0$</td>
</tr>
<tr>
<td>Outperform</td>
<td>Sell</td>
<td>$-b - 1$</td>
<td>$-1$</td>
</tr>
<tr>
<td>Neutral</td>
<td>Buy</td>
<td>$a$</td>
<td>$0$</td>
</tr>
<tr>
<td>Neutral</td>
<td>Hold</td>
<td>$1$</td>
<td>$1$</td>
</tr>
<tr>
<td>Neutral</td>
<td>Sell</td>
<td>$-b$</td>
<td>$0$</td>
</tr>
<tr>
<td>Underperform</td>
<td>Buy</td>
<td>$a - 1$</td>
<td>$-1$</td>
</tr>
<tr>
<td>Underperform</td>
<td>Hold</td>
<td>$0$</td>
<td>$0$</td>
</tr>
<tr>
<td>Underperform</td>
<td>Sell</td>
<td>$1 - b$</td>
<td>$1$</td>
</tr>
</tbody>
</table>

- $a$: benefit for your analyst when you buy stock
- $b$: cost for your analyst when you sell stock
Example 2: Stock recommendations

Let us check for the existence of a separating equilibrium where:

- Information is perfectly transmitted from the analyst to the investor.
- That is, the analyst recommends:
  - Buy only when the prospect of the stock are Good.
  - Hold only when the prospect of the stock are Neutral.
  - Sell only when the prospect of the stock are Bad.

This strategy profile is represented in the following figure.
Separating equilibrium (fully informative)
Example 2: Stock recommendations

**Investor:**

After observing a recommendation of "Buy" from his analyst, the investor responds Buying since

\[
1 > 0 \text{ (if Hold), and} \\
1 > -1 \text{ (if Sell)}
\]

We hence shade the branch where the investor responds with "Buy" every time he observes a recommendation of Buy.

- Blue shaded branch in the following tree.
After observing a recommendation of "Buy"
Example 2: Stock recommendations

- **Investor (responder in this game):**

  After observing a recommendation of "Hold" from his analyst, the investor responds Holding since

  \[ 1 > 0 \] (if Buy), and
  \[ 1 > 0 \] (if Sell)

- We hence shade the branch where the investor responds with "Hold" every time he observes a recommendation of Hold.
  - Green shaded branch in the following tree.
After observing a recommendation of "Hold"
Investor (responder in this game):
After observing a recommendation of "Sell" from his analyst, the investor responds Selling since

\[ 1 > 0 \text{ (if Hold), and} \]
\[ 1 > -1 \text{ (if Buy)} \]

We hence shade the branch where the investor responds with "Sell" every time he observes a recommendation of Sell.

Purple shaded branch in the following tree.
After observing a recommendation of "Sell"
Separating equilibrium (Cont.)

Summarizing the investor’s optimal responses found above...
Example 2: Stock recommendations

Analyst:

If the stock will *outperform* the market, then the analyst recommends to Buy if

\[ a + 1 > 0 \] (if Hold), and
\[ a + 1 > -b - 1 \] (if Sell)

which simplify to \( a > -1 \) and \( a + b > -2 \).

Then, both conditions hold since \( a, b > 0 \) by definition.
Example 2: Stock recommendations

- **Analyst:**
- If the stock will be *neutral* relative to the market, then the analyst recommends to Hold if

\[ 1 > a \] (if Buy), and

\[ 1 > -b \] (if Sell)

Always holds since \( b > 0 \)
Example 2: Stock recommendations

- **Analyst:**
  - If the stock will *underperform* the market, then the analyst recommends to Sell if
    
    \[ 1 - b > a - 1 \implies 2 > a + b \] (if Buy), and
    
    \[ 1 - b > 0 \implies 1 > b \] (if Hold)
The conditions that must be satisfied for a fully informative separating equilibrium to exist are hence

\[ a + b < 2 \]
\[ a < 1, \text{ and} \]
\[ b < 1 \]

Let us represent all three conditions in the following figure.
Conditions for a separating equilibrium in the stock recommendations cheap talk game:

- $a + b < 2$, $a < 1$, $b < 1$
- $a$: bonus for the analyst if the investor buys shares
- $b$: penalty for the analyst if the investor sells shares
- $a$ and $b$ being low $\Rightarrow$ preferences of the investor and his analyst (or his investment bank) are very similar.
Example 2: Stock recommendations

- This suggests that the fully informative separating equilibrium can be sustained if:
  - $a$ and $b$ are both low.
  - Intuitively, this implies that the preferences of the analyst and investor are very similar, and thus communication is easy.
Example 2: Stock recommendations

- **What happens if, instead, \( b > 1 \)?**
  - Graphically, this occurs in the upper triangle of the figure.
  - Intuitively, \( b > 1 \) indicates that it is highly detrimental for the analyst to induce clients to sell.
  - We know that the fully informative separating equilibrium cannot be sustained.
Example 2: Stock recommendations

But, can we have at least some information transmission?

Yes, we can have a partially separating equilibrium where:

- The analyst recommends to Buy both when the prospects are Good and Neutral, but
- Recommends to Hold when the prospects are Bad.

(See next figure).
Partially separating equilibrium
Example 2: Stock recommendations

- **Beliefs:**
  - Not so immediate!
  - Beliefs will often be more involved than in signaling games with only two types and messages. (here we have 3 types and messages)
Example 2: Stock recommendations

Beliefs:

After observing a recommendation of "Buy" from his analyst,

\[
\text{prob(outperform} \mid \text{Buy}) = \frac{\frac{1}{3} \cdot 1}{\frac{1}{3} + \frac{1}{3} \cdot 1} = \frac{\frac{1}{3}}{\frac{2}{3}} = \frac{1}{2}
\]

since the recommendation of "Buy" may originate from an analyst informed about Good or Neutral prospects.

\[
\text{prob(neutral} \mid \text{Buy}) = \frac{\frac{1}{3} \cdot 1}{\frac{1}{3} + \frac{1}{3} \cdot 1} = \frac{\frac{1}{3}}{\frac{2}{3}} = \frac{1}{2}
\]

\[
\text{prob(underperform} \mid \text{Buy}) = 0
\]

since the recommendation of "Buy" does not originate from an analyst informed about Bad prospects.
Example 2: Stock recommendations

- **Beliefs:**
  - After observing a recommendation of "Hold" from his analyst,
    
    \[
    \begin{align*}
    \text{prob(outperform|Hold)} &= 0 \\
    \text{prob(neutral|Hold)} &= 0 \\
    \text{prob(underperform|Hold)} &= 1
    \end{align*}
    \]
    
    since the recommendation of "Hold" can only originate from an analyst informed about Bad prospects.
Example 2: Stock recommendations

- **Beliefs:**
  - Finally, after observing a recommendation of "Sell" from his analyst (off-the-equilibrium path),

  \[
  \begin{align*}
  \text{prob}(\text{outperform}|\text{Hold}) &= \gamma_1 \\
  \text{prob}(\text{neutral}|\text{Hold}) &= \gamma_2 \\
  \text{prob}(\text{underperform}|\text{Hold}) &= 1 - \gamma_1 - \gamma_2
  \end{align*}
  \]

  - For simplicity, Harrington assumes that \( \gamma_1 = \gamma_2 = 0 \)
    - (Intuitively, if "Hold" signifies that the stock will underperform, then "Sell" should convey the same information, or worse!).
  
  - Remember that in any case this is an assumption about off-the-equilibrium beliefs.
Example 2: Stock recommendations

- **Investor:** *(Responder in this game)*

- After observing the recommendation of "Buy" from his analyst, the investor obtains

\[
\frac{1}{2} \times 1 + \frac{1}{2} \times 0 = \frac{1}{2} \quad \text{from buying}
\]

\[
\frac{1}{2} \times 0 + \frac{1}{2} \times 1 = \frac{1}{2} \quad \text{from holding}
\]

\[
\frac{1}{2} \times (-1) + \frac{1}{2} \times 0 = -\frac{1}{2} \quad \text{from selling}
\]

and hence the investor Buys.

- *(Shaded in the next figure)*.
Partially separating equilibrium

- After observing the recommendation of "buy"...
Example 2: Stock recommendations

- **Investor (cont.):**

  After observing the recommendation of "Hold" from his analyst, the investor sells since

  \[
  1 > 0 \quad \text{(if the investor holds)}
  \]

  \[
  1 > -1 \quad \text{(if the investor sells)}
  \]

  given that the investor puts full probability on "underperform."

- (Shaded in the next figure).
Partially separating equilibrium

- After observing the recommendation of "hold"...
Example 2: Stock recommendations

- **Investor (cont.):**
  
  After observing the recommendation of "sell" from his analyst, the investor sells since
  
  \[ 1 > 0 \text{ (if the investor holds)} \]
  
  \[ 1 > -1 \text{ (if the investor sells)} \]
  
  given that the investor puts full probability on "underperform."

  (Shaded in the next figure).
Partially separating equilibrium

- After observing the recommendation of "sell"...
Partially separating equilibrium

- Summarizing the optimal responses of the investor we found above...
Example 2: Stock recommendations

- **Analyst:**
  - If the stock will *outperform* the market, then the analyst recommends to Buy if

\[
\begin{align*}
a + 1 &> -b - 1 \text{ (if Hold)}, \\
a + 1 &> -b - 1 \text{ (if Sell)}
\end{align*}
\]

which simplify to \( a + b > -2 \) and \( a + b > -2 \), respectively.

- Hence, these two conditions hold given that \( a > 0, b > 0 \) by definition.
Example 2: Stock recommendations

- **Analyst:**
  - If the stock will be *neutral* relative to the market, then the analyst recommends to Buy as well if
  
  \[
  \begin{align*}
  a &> -b \quad \text{(if Hold), and}
  
  a &> -b \quad \text{(if Sell)}
  \end{align*}
  \]

  \{ \text{satisfied, since} \ a > 0, b > 0 \}
Example 2: Stock recommendations

- **Analyst:**
- If the stock will *underperform* the market, then the analyst recommends to Hold if

\[
1 - b > a - 1 \implies 2 > a + b \quad \text{(if Buy), and}
\]
\[
1 - b = 1 - b \quad \text{(if Hold)}
\]
Example 2: Stock recommendations

- As the interests of investor and analyst diverge more (higher $a$ and $b$), information becomes more difficult to transmit between the parties.

- Only condition now was $a + b < 2$
We can hence conclude that:

- When preferences are very similar \((a, b < 1)\), a fully informative separating equilibrium can be sustained, whereas...
- When preferences are not so similar \((a + b < 2, \text{ but either } a > 1 \text{ or } b > 1)\), only a partially informative equilibrium can be supported.

Note that in the latter equilibrium there is no deception:

- The investor knows that a "Hold" recommendation means that he should sell.
- However, the information content of recommendations deteriorates:
  - The analyst recommended "Buy," but the investor doesn’t know if the stock will be outperforming the market or be neutral.
Example 2: Stock recommendations


- Analysts are uncomfortable making sell recommendations on particular stocks. Often the analysis will cop out with a euphemism: the hold rating. But now hold is getting such a bad name that different terminology is gaining favor on the street. Like strong hold... Just what does strong hold mean? Since most investors assume a hold is really a polite way to say well, does strong hold actually mean strong sell?
Example 2: Stock recommendations

1. No "strong sell", and what does it mean?
2. Few "sell" recommendation
3. This strategy profile resembles the "partially informative separating equilibrium" described above, which emerges when for instance $a > 1$ but $a + b < 2$? Intuition: too big bonuses.

<table>
<thead>
<tr>
<th>Recommendation</th>
<th>Frequency</th>
<th>Cumulative Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strong buy</td>
<td>38</td>
<td>15.2%</td>
</tr>
<tr>
<td>Buy</td>
<td>128</td>
<td>66.4%</td>
</tr>
<tr>
<td>Hold</td>
<td>70</td>
<td>94.4%</td>
</tr>
<tr>
<td>Sell</td>
<td>14</td>
<td>100.0%</td>
</tr>
<tr>
<td>Strong sell</td>
<td></td>
<td>100.0%</td>
</tr>
</tbody>
</table>